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ARITHMETIC

FOR

PUPIL TEACHERS

By GEORGE RICKS, B.Sc., London

INSPECTOR OF SCHOOLS, SCHOOL BOARD FOR LONDON

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PREFACE

THE time which can be devoted to the oral instruction of pupil teachers in any given subject is necessarily very limited.

Without professing to supersede oral instruction, this little work is designed to assist the pupil teacher to master the principles of the science of numbers, and to attain proficiency in the methods and processes of computation with the minimum amount of assistance from the teacher.

With this end in view—

(1) The elucidations and explanations of the *principles* have been made as clear yet concise as possible, so that the student may rely on *reason* rather than *rule*.

(2) Numerous solutions of *typical* examples have been given to serve as models for reference and guidance.

(3) Wherever points occur of more than ordinary importance, or where difficulties are likely to arise, notes have been added to smooth the path of the pupil as much as possible.

(4) Numerous and varied examples are given, so carefully graduated that the student who takes them in order will find little or no difficulty in solving the more complicated problems as they arise.

The first chapter is taken up with simple questions and problems, which involve only a knowledge of the most elementary processes of arithmetic, viz. the notation and the addition, subtraction, multiplication, and division of abstract numbers, money, and weights and measures. We assume that candidates for the office of *pupil teacher* will have, at least, mastered so much.

It is of great importance that the whole of these exercises should be worked before proceeding to the regular course laid down for pupil teachers, because they are designed to form *stepping stones* to the exercises which follow. With the help of the models and explanations, and by the exercise of a little thought and ingenuity, candidates may obtain solutions to all the examples in this chapter without much difficulty.

With one exception the syllabus of the New Code has been followed throughout. In this syllabus* fractions are introduced *after* practice, ratio, and proportion. As it is, however, impossible for the student to acquire an intelligent knowledge of either practice or proportion without understanding some of the fundamental ideas involved in fractions, a short introductory chapter, dealing with this subject in its most elementary form, has been inserted, immediately after the miscellaneous examples on the elementary rules.

Many of the exercises given are original, but many more have been carefully selected for their typical character from every good available source. They number upwards of *two thousand*, and full solutions are given of over *two hundred*.

* Some of the examples given in the papers for pupil teachers of the first year presuppose an elementary knowledge of fractions.

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ARITHMETIC

FOR

PUPIL TEACHERS.

CHAPTER I.

PRELIMINARY EXERCISES ON THE SIMPLE AND COMPOUND RULES.

SECTION I.

Ex. 1. From what number must 263 be taken away to leave 197?

Clearly, if what is taken away be *added to* what is left, the sum must be the original number.

No. taken away = 263
No. left = 197
The original No. = 460

(This is the *proof* to an ordinary subtraction sum.)

Ex. 2. What number exceeds 509 by 727?

Ex. 3. From what number must 638 be subtracted to leave 419?

Ex. 4. Given 3,014 as the remainder, and 6,925 as the minuend; find the subtrahend.

Ex. 5. If from a certain number 168 be taken, 576 are left; find the number.

Ex. 6. When 641, 350, 397, and 58 have been taken from a certain number 1,001 are left; find the number.

SECTION II.

Ex. 1. Divide 4,320 into two parts, such that one is 156 greater than the other.

Set yourself the problem: "Divide 8 into two parts, such that one is

2 greater than the other;" you at once subtract 2 from 8, and divide the 6 by 2, giving 5 and 3 as the numbers. In all exercises, then, of this kind, first subtract the number by which the one is to exceed, or be less than the other, and then divide it into the even parts, and add.

$$\begin{array}{r} 4{,}320 \\ 156 \\ \hline 4{,}164 \end{array} \qquad \begin{array}{r} 2)4{,}164 \\ \hline 2{,}082 \end{array}$$

1st No. $2{,}082 + 156 = 2{,}238$
2nd No. $= 2{,}082$

Ex. 2. Divide 506 into two parts such that one is 60 less than the other.

Ex. 3. Divide 4,607 into three parts such that the first is 37 more than the second, and the second 20 more than the third.

Ex. 4. At an election for a member of Parliament there were two candidates. The successful candidate polled 17,631, viz. a majority of 647 over the defeated candidate. How many persons voted?

Ex. 5. There are two numbers; the less is 735, and their difference is 141; find the greater number.

Ex. 6. What number is that to which 147 must be added to make 10,000?

Section III.

Ex. 1. From what number can 327 be taken exactly 59 times?

Ask yourself: "From what number can you take 6 three times?"

From 18, because $6 \times 3 = 18$

Hence 327×59 or 19,293, is the required number.

Ex. 2. From what number can you take 509 exactly 104 times?

Ex. 3. From what number can 57 be taken 56 times, and leave a remainder of 27?

Ex. 4. What number divided by 620 gives 62 for a quotient?

Ex. 5. What number divided by 392 gives 481 for a quotient, and leaves 63 remainder?

Ex. 6. Out of a basket of nuts I give to each of 96 boys 47. I find I have 293 left; how many nuts were in the basket, and how many more nuts do I want to give each boy 5 nuts more?

Section IV.

Ex. 1. How many times can you subtract 56 from 3,604, and what remains?

We may find the answer to this question by continued subtractions,

but the process would be a long one. How many times can you subtract 2 from 8? *Four* times; because there are exactly four twos in 8, *i.e.* we divide 8 by 2.

And so, in examples like the above, we perform the subtractions by division, for division is a short method of doing subtractions.

```
56)3604(64
   336
   ---
   244
   224
   ---
    20
```

Ans. 64 times, and 20 remains.

Ex. 2. What number multiplied by 67 will give the same product as 201 multiplied by 74?

Ex. 3. How many times must 15 be added to 62 to produce 1,607?

[Subtract 62 from 1,607, and then find how many *fifteens* there are in the remainder.]

Ex. 4. How many times can you take 17 marbles from 695? and how many more marbles do you want to make up another 17?

Ex. 5. A man walking 8 hours per day makes a journey of 264 miles in eleven days; how many miles does he walk per hour?

Ex. 6. A prize of £1,000 has to be divided between 4 officers and 12 sailors. An officer is to receive twice as much as a sailor; how much does each officer and each sailor receive?

Section V.

Ex. 1. A book-keeper, in error, entered £93 15*s.* 3*d.* as loss instead of profit. At the end of the quarter the books showed a profit of £760 15*s.* 4*d.*; what ought they to have shown?

It will not be sufficient in this example simply to add the £93 15*s.* 3*d.* to the quarter's profit, because not only have we *not added* this amount as profit, but we have *subtracted* it as loss; hence we have to add first that which was subtracted in error, and then that which was *not added*, or twice £93 15*s.* 3*d.*

Profit shown	= £760	15	4
Subtracted in error	= 93	15	3
Not before added as profit	= 93	15	3
Correct profit	= £948	5	10

Ex. 2. From what amount must £56 19*s.* 7½*d.* be subtracted to leave £605 17*s.* 11*d.*?

Ex. 3. What amount exceeds £540 16*s.* 9*d.* by £372 15*s.* 10*d.*?

Ex. 4. From a certain field 4 plots were taken, measuring respectively $\frac{3}{4}$ acre, $1\frac{1}{2}$ acres, 3 roods 20 poles, and 2 roods 10 poles; it was then found that $4\frac{3}{4}$ acres were left; find the size of the field.

Ex. 5. From a chest of tea 15 lbs. 12 ozs., 19 lbs. 13 ozs., and 12 lbs. 4 ozs. were taken; 6 ozs. were wasted in weighing, and then it was found that 7 lbs. 13 ozs. were left; how much did the chest contain at the first?

Section VI.

Ex. 1. Divide £172 16*s.* 3*d.* between two persons, giving one £7 13*s.* 1*d.* less than the other.

We must first subtract the £7 13*s.* 1*d.*, and then divide the remainder equally.

£172	16	3
7	13	1
2)165	3	2
£82	11	7

The smaller amount = £82 11 7
The larger amount = £82 11*s.* 7*d.* + £7 13*s.* 1*d.* = £90 4 8
These added should give the original amount = £172 16 3

Ex. 2. What amount is that to which £72 13*s.* $2\frac{1}{2}$*d.* must be added to make £100?

Ex. 3. At a contested election, the successful of two candidates paid £725 6*s.* 9*d.* as expenses; the unsuccessful candidate paid £37 19*s.* 4*d.* less; what were the total expenses of the election?

Ex. 4. Divide £51 19*s.* 3*d.* among 15 men, giving the first 6*s.* 9*d.* more than each of the others.

Ex. 5. I wish to distribute 18*s.* among 2 monitors and 12 scholars, a monitor to receive three times as much as a scholar; find the share of each.

Ex. 6. 8 quarts of oats are to be distributed between 3 horses and 2 ponies. If a horse's share is double that of a pony, what is the share of each?

Section VII.

Ex. 1. From what sum of money can £7 16*s.* 5*d.* be subtracted 11 times, leaving a remainder of 6*s.* 9*d.*?

£7 16*s.* 5*d.* × 11 = £86 0*s.* 7*d.*
And £86 0*s.* 7*d.* + 6*s.* 9*d.* = £86 7*s.* 4*d.*

Ex. 2. From what amount can you take away £141 13*s.* 5*d.* exactly ninety-five times?

Ex. 3. What amount divided by 171 gives £6 3*s.* 5*d.* as a quotient?

Ex. 4. What amount divided by 65 gives £13 0*s.* 10½*d.* as a quotient, and 11½*d.* as a remainder?

Ex. 5. Out of a certain sum of money I pay 56 men £2 16*s.* 5*d.* each, and find I have £6 9*s.* 8*d.* left; find the amount.

Ex. 6. A farmer has 5 fields, each measuring 17½ acres; 6 more measuring 9¼ acres each; and 2 others measuring 2¾ each, and an orchard measuring 1¾ acres; required the size of his farm.

Section VIII.

Ex. 1. What sum of money subtracted five times from £62 17*s.* 5*d.* will leave a remainder of £2 4*s.* 1*d.*?

We must first subtract the remainder, and then divide into five equal parts.

£62 17 5
2 4 1
5)60 13 4
£12 2 8 *Ans.*

Ex. 2. By selling 476 sacks of flour for £1,415 3*s.* a miller made a total profit of £135 18*s.*; find the cost of each sack.

Ex. 3. What sum of money can be subtracted 79 times from £257 9*s.* 9¾*d.*, leaving no remainder?

Ex. 4. Divide 267 lbs. 5 ozs. 17 dwts. by 49.

Ex. 5. What weight of sugar taken 19 times from a hogshead containing 2 cwts. 3 qrs. 24 lbs. will leave no remainder?

Ex. 6. A grocer made up 127 parcels of tea from five chests, each containing 1 cwt. 15 lbs.; what was the weight of each parcel?

Section IX.

Ex. 1. An estate is bought for £68,423. The cost per acre is £32 5*s.* 6*d.* Find the number of acres.

We have here to find how many times £32 5*s.* 6*d.* is contained in £68,423, or, which is the same thing, we must divide the latter by the former. To do this we must first reduce both concrete quantities to the same denomination. In this case reduce to sixpences.

£32	5	6	£68423	
20			20	
645	shillings		1368460	shillings
2			2	
1291	sixpences		2736920	sixpences.

```
1291)2736920(2120
  2582
  ----
   1549
   1291
   ----
    2582
    2582
    ----
       0
```

Ans. 2,120 acres.

This must not be understood to mean that sixpences divided by sixpences produce acres. It means that 1,291 sixpences, the price of one acre, is *contained* in 2,736,920 sixpences, the price of the estate, 2,120 times, each one of those times standing for an acre, and hence together standing for 2,120 acres.

We may note here that though it is possible to *divide* concrete quantities of like kinds, it is not possible to multiply them.

Thus 16 oranges divided by 4 oranges means that 16 oranges are to be divided into a certain number of lots, each containing 4 oranges. We cannot multiply 4 oranges by 4 oranges, because that would mean 4 oranges taken 4 oranges number of times, which would be absurd. In the division the oranges do not divide, but the number 4 does.

Ex. 2. A tax levied at the rate of £3 17*s.* 5*d.* per house produces £917 7*s.* 9*d.*; how many houses are subject to the tax?

Ex. 3. How many sheep at £2 16*s.* 5*d.* each can be purchased for £135 8*s.*?

Ex. 4. Bought a leg of mutton for 12*s.* 5½*d.* at 11½*d.* per lb.; how many pounds did it weigh?

Ex. 5. How many times can you subtract £2 16*s.* 5*d.* from £56 8*s.* 4*d.*?

Ex. 6. How many shirts can be made out of 243 yds. of calico, each shirt requiring 3 yds. 1 qr. 2 nls.?

Section X.

Ex. 1. £44 5*s.* 6*d.* is the exact sum of an equal number of pounds, shillings, and pence; find the number.

The meaning of the problem is that if we take a certain *number* of sovereigns, the same *number* of shillings, and the same *number* of pence, and add the whole, the sum will be £44 5*s.* 6*d.* In other words, we have to find how many times £1 1*s.* 1*d.* are contained in £44 5*s.* 6*d.* The question thus becomes similar to those in the preceding section. We must first reduce to the same denomination, and then divide.

£1 1 1
20
21
12
253 pence.

£44 5 6
20
885
12
10,626 pence.

253)10626(42
1012
506
506

Ans. 42.

Ex. 2. A purse contains £8 7*s.* 6*d.*, consisting of an equal number of sovereigns, half-sovereigns, half-crowns, and shillings ; how many coins are there of each kind ?

Ex. 3. In a certain cotton mill there were an equal number of men, women, boys, and girls ; the men were paid 5*s.* 6*d.*, the women 3*s.* 3*d.*, the boys 2*s.* 3*d.*, and the girls 2*s.* each per day, and the daily wages amounted to £78 ; how many persons were employed in the mill ?

Ex. 4. How many shillings, sixpences, fourpenny-pieces, and threepenny-pieces, and of each an equal number, are there in £26 5*s.* ?

Ex. 5. How many 4½-lb., 3½-lb., 2½-lb., and 1½-lb. parcels, and of each an equal number, can be made up out of a chest of tea weighing 1 cwt. 2 qrs. 12 lbs. ?

Ex. 6. If a coat takes 3¼ yds. of cloth, a pair of trousers 2¼ yds., and a waistcoat 3 qrs. 2 nls., find how many suits of clothes can be made up out of a piece of cloth measuring 46 yds. 1 qr. 2 nls. ?

Section XI.

Ex. 1. Divide £31 10*s.* between A, B, and C, giving to A twice as much as to B, and to B three times as much as to C ; find the amount which each will receive.

The simplest method of solving problems of this kind is to take the smallest division (in this case C's share), and call it *one* share.

Thus if C has 1 share,
B will have 3 shares,
and A will have 6 shares.

We therefore divide the whole sum into 10 shares, and give C one, B three, and A six.

£31 10*s.* ÷ 10 = £3 3 0 = C's share.
£3 3*s.* × 3 = 9 9 0 = B's share.
£9 9*s.* × 2 = 18 18 0 = A's share.
£31 10 0 = Total sum divided.

Ex. 2. Tom has 140 marbles in 3 bags. The first bag contains twice as many as the second, and the third half as many as the second. Find the number of marbles in each bag.

Ex. 3. A father and his three sons together earn 13*s.* 6*d.* per day; the eldest son earns half as much as his father, and the other two sons together earn half as much as their father and eldest brother; find the amount earned by each.

Ex. 4. A father left £10,000 to be divided among 2 sons and 3 daughters; the sons were to have as much as the 3 sisters; find the amount due to each.

Ex. 5. In a game at cricket 4 boys, A, B, C, D, scored 60 runs; A scored half as many as B, B half as many as C, and C half as many as D; find the number of runs scored by each.

Ex. 6. Divide a piece of building land measuring 9 acres into 4 plots, the second plot to be twice as large as the first, and the third and fourth each half as large as the first and second put together.

Section XII.

Ex. 1. 460 persons attended a concert; of these 110 paid 2*s.* 6*d.* for admission, 205 paid 1*s.*, and the rest 6*d.*; find the total receipts.

460	110 at 2*s.* 6*d.* =	£13	15	0
315	205 at 1*s.* 0*d.* =	10	5	0
145	145 at 0*s.* 6*d.* =	3	12	6
	460	£27	12	6

Total amount received = £27 12*s.* 6*d.*

Ex. 2. At a certain lecture 544 people attended; one half paid 1*s.*, one half of the remainder paid 6*d.*, and the remainder paid 3*d.*; what were the total receipts?

Ex. 3. In a school of 980 children 240 pay 3*d.* per week, half the remainder pay 2*d.*, 15 of those left are free, and the remainder pay 1*d.*; what do the fees amount to in one week?

Ex. 4. If I spend half of my money in buying a railway ticket, half of the remainder in purchasing a trunk, and have 16*s.* 2*d.* left, how much money had I at the first?

Ex. 5. After drawing out 18 galls., 27 galls., 15 galls., and 28 galls. from a cistern full of water, it is found to be one-fourth full; how much does the cistern contain?

Ex. 6. I bought a watch and chain for 30 guineas; the price of the watch was five times that of the chain; what was the price of each?

Section XIII.

Ex. 1. Which will cost the more, and by how much, 128 ozs. of coffee at 1*s.* 8*d.* per lb., or 5 lbs. of tea at 2*d.* per oz. ?

128 ozs. = 8 lbs. at 1*s.* 8*d.* per lb. = 13*s.* 4*d.*
5 lbs. at 2*d.* per oz., viz. 2*s.* 8*d.* per lb. = 13*s.* 4*d.*

Hence the one costs the same as the other, viz. 13*s.* 4*d.*

Ex. 2. What is the difference in the cost of 152 lbs. of sugar at 4½*d.* per lb. and 19 lbs. of tea at 2*s.* 8*d.* per lb. ?

Ex. 3. A tea-dealer mixes 20 lbs. of tea at 1*s.* 8*d.*, 16 lbs. at 2*s.* 3*d.*, 14 lbs. at 2*s.* 9*d.*, and 10 lbs. at 3*s.*; at what price per lb. must the mixture be sold, so that there is neither profit nor loss on the transaction ?

Ex. 4. How much money will it take to pay the wages of 75 men for one week at 5*s.* 3*d.* per day ?

Ex. 5. A gold chain weighs 8 ozs. 12 dwts. 5 grs.; what will it cost at 2*d.* per grain ?

Ex. 6. How much will a grocer receive for ¼ ton of currants at 6*d.* per lb. ? If he bought them at 4½*d.* per lb., what are his profits ?

Section XIV.

Ex. 1. If 60 pairs of boots are bought for £38 10*s.* and sold at 16*s.* 6*d.* per pair, what is the gain on each pair ?

First, find the cost price per pair by dividing 60 into £38 10*s.*

£38 10*s.* ÷ 60 = 12*s.* 10*d.*
Hence the gain = 16*s.* 6*d.* − 12*s.* 10*d.* = 3*s.* 8*d.*

Ans. 3*s.* 8*d.*

Ex. 2. A grocer buys sugar at £1 8*s.* per cwt., and sells it at £2 2*s.* per cwt.; what is his profit per lb. ?

Ex. 3. A woman bought ten shillings' worth of eggs at 7 for 6*d.*, and sold them at 5 for 6*d.*; how much did she gain ?

Ex. 4. A grocer bought 365 lbs. of tea at 2*s.* per lb.; 40 lbs. were found to be damaged, and had to be sold at 1*s.* 2*d.* per lb.; the remainder was sold at 3*s.* per lb.; find the gain or loss on the whole.

Ex. 5. If 150 tables are bought for £103 15*s.*, what must be the selling price per table to gain £20 on the whole ?

Ex. 6. A butcher bought meat at £4 8*s.* 8*d.* per cwt., and sold it at 11½*d.* per lb.; what was his gain per cwt. and per lb. ?

Section XV.

Ex. 1. If a servant's wages are £18 5*s.* per year, how much is that per month? If she enters service on the 1st of April and leaves on the 20th of June, what will she have earned as wages?

(1.) £18 5*s.* ÷ 12 = £1 10*s.* 5*d.* per month.

(2.) From 1st April to June 20 there are 81 days :—
£18 5*s.* = 365*s.*, or 1*s.* per day; hence for 81 days the wages = £4 1*s.*

Ex. 2. How much will a man earn in a year of 365 days at 4*s.* 9¾*d.* a day?

Ex. 3. A servant's wages are £16 per year; what is that for 3, 5, and 7 months?

Ex. 4. How much money will it take to pay 57 men for 6 weeks' work at 4*s.* 9*d.* per day?

Ex. 5. A man earns £4 10*s.* 9*d.* per week, and spends £3 11*s.* 11*d.*; how much does he save in a year?

Ex. 6. If a boy earns 10*s.* 8*d.* per week, and saves £3 15*s.* 4*d.* in the year, how much does he spend per week?

Section XVI.

Ex. 1. What will be the annual cost of insuring a house for £1,050 at 1*s.* 6*d.* per cent.?

The meaning of this problem is, that for every £100 we pay 1*s.* 6*d.* 1,050 = 10½ hundred. Hence 1*s.* 6*d.* × 10½ or 15*s.* 9*d.* is the required amount.

Ex. 2. What will be the annual payment to insure a life at £600 at 4*s.* 10*d.* per cent.?

Ex. 3. A tradesman offers 1*s.* 6*d.* in the £ discount* for cash; what will be the amount of discount on a bill amounting to £25 10*s.*?

Ex. 4. What is the nett price of a guinea book, allowing 2*d.* in the shilling discount?

Ex. 5. A coal merchant gains 5*s.* per ton by selling coals at 25*s.*; what is his gain on an outlay of £100?

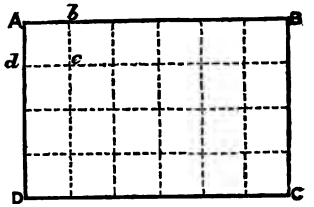
Ex. 6. Goods worth £150 were damaged, and sold at a reduction of 25 per cent.; for how much were they sold?

Section XVII.

Ex. 1. Find the area of a garden, the length of which is 6 poles, and the width 4 poles.

* *Discount is money deducted* from the fixed price in consideration of prompt payment.

We shall best understand the nature of the process in working examples of this kind from a diagram. Mark along A B and A D divisions to represent 6 poles and 4 poles, and draw lines as in the figure. Then the figure A b c d is manifestly a square pole, and there are 24 of such square poles. Hence, to find the area of a rectangle,* *multiply the length by the breadth.* Of course it cannot be true that we (in the given example, for instance) multiply 6 poles by 4 poles, because that would mean 6 poles taken 4 poles times over, which is absurd. What we really do is to multiply the number of *units* of length by the number of *units* of breadth, and this gives the number of *units* of area.



If length × breadth = area, it follows

(1) area ÷ length = breadth.
(2) area ÷ breadth = length.

Thus, in the given example

$$6 \times 4 = 24$$
and (1) $24 \div 6 = 4$
and (2) $24 \div 4 = 6$

NOTE.—Attention to the above facts will help us to solve many problems about carpeting rooms, papering walls, paving streets, &c.

Ex. 2. What length of carpet 3 ft. wide will it take to cover a room 16 ft. by 12 ft.?

Area of room $= 16 \times 12 = 192$ sq. ft.

Hence the area of the carpet wanted must be the same.

But area ÷ width = length
$= 192 \div 3 = 64$

Hence the length required is

64 ft., or 21 yds. 1 ft.

NOTE.—Always reduce length and breadth to the same denomination before multiplying.

Ex. 3. 72 yards of carpet are required to cover a room 15 ft. long and 12 ft. wide; find the width of the carpet.

Ex. 4. How many loads of gravel will it take to cover a garden walk 144 ft. long and 3 ft. wide, when one load covers 3 sq. yards?

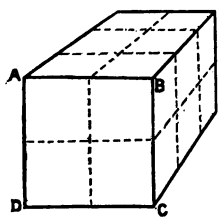
Ex. 5. In making a croquet lawn 60 ft. long and 40 ft. wide, how many *turfs* shall I require, each 1 yd. long and 10 in. wide?

* A rectangle *is a four-sided figure having its angles right angles, as a door, one side of a room, the floor, &c.*

Ex. 6. Which will cost the more, and by how much, painting a wall 17 ft. long and 12 ft. high, at 2*s.* per sq. yard, or papering it with paper 2 ft. 10 in. wide at 5*s.* per piece of 12 yds. ?

Section XVIII.

Ex. 1. The depth of a canal is 7 ft. 3 in., the width 20 ft. 4 in., and the length 10 miles; how many cubic feet of water will it contain ?



We will, first, give a simple illustration of finding the *solid* contents. A block of marble is 3 ft. long, 2 ft. wide, and 2 ft. high; find the number of cubic* feet it contains.

The diagram represents such a block divided into cubic feet. It will be seen that there are 12 such cubic feet; and we get the number 12 according to the following rule :—

The number of cubic units in a solid of uniform length, breadth, and thickness is found by multiplying together the number of units of length, breadth, and thickness. Thus, in the illustration—

length × breadth × height = solid contents ;
that is, $3 \times 2 \times 2 = 12$

From this it is easy to find any one of the above numbers when the other three are given. In the example we have given the length, depth, and width; we must reduce to the same denomination, and multiply together.

7 ft. 3 in. = 87 in.
20 ft. 4 in. = 244 in.
10 miles = 633600 in.
then $87 \times 244 \times 633600$ = number of cubic inches.
Divide by 1728 to reduce to cubic feet.
and $87 \times 244 \times 633600 \div 1728 = 7783600$.

Ans. 7,783,600 cubic feet.

Ex. 2. What must be the length of a trench 5 ft. 6 in. deep and 10 ft. 8 in. wide, that it may contain 7,040 cubic feet ?

Ex. 3. How many cubic feet of earth are taken from a trench 120 ft. long, 5 ft. 6 in. deep, and 10 ft. 8 in. wide ?

Ex. 4. What must be the width of a trench which is 5 ft. 6 in. deep and 120 ft. long to hold 7,040 cubic feet of water ?

Ex. 5. How many cubic inches are there in a cube which measures 13 in. in the side ?

* *A cubic foot, that is,* a block measuring one foot in length, breadth, and height.

Ex. 6. How many cubic feet of air are there in a room which is 17 ft. 6 in. long, 13 ft. 4 in. wide, and 12 ft. 9 in. high?

Section XIX.

Ex. 1. If 12 yds. of velvet cost £14, what is the price of 37 yds.?

We will *first* find the price of 1 yd. If 12 yds. cost £14, 1 yd. will cost one-twelfth.

	£	s.	d.	
12)	14	0	0	
	1	3	4	cost of 1 yd.

Secondly, having the price of 1 yd., we can find the price of 37 yds. by multiplying by 37.

£	s.	d.	
1	3	4	cost of 1 yd.
			$6 \times 6 \times 1 = 37$
7	0	0	cost of 6 yds.
			6
42	0	0	cost of 36 yds.
1	3	4	cost of 1 yd.
43	3	4	cost of 37 yds.

This method of solving little problems with three quantities given—two being similar, and the third like the answer—is called the Unitary, or Unity method, because we first find the value of *one* (unit).

Note.—In working problems of this kind it is important to note that there is always something *given* and something *required*—a *statement* and a *question*. Thus in the given example :—

Statement : 12 yds. cost £14
Question : 37 yds. cost how much?

It is in arranging the statement that pupils sometimes find a difficulty.

We might have made the statement thus :—£14 will buy 12 yds.; but we cannot readily find the value of 1 yd. from this. Practice only will give the required facility in constructing the statement, but the following rule will be of material assistance :—*In the statement place that term last which is of the like kind to the required answer.*

12 yds. cost £14
∴ 1 yd. costs £14 ÷ 12
∴ 37 yds. cost £14 ÷ 12 × 37
or £14 × 37 ÷ 12 = £43 3*s.* 4*d.*

Ans. £43 3*s.* 4*d.*

Ex. 2. If I pay £3 10*s.* for the loan of £100 for one year, how much shall I have to pay for the loan of £325 for one year?

Statement: For the loan of £100 is paid £3 10*s.*

Question: For the loan of £325 how much must be paid?

For £100 is paid £3 10*s.*
∴ for £1 is paid £3 10*s.* ÷ 100
∴ for £325 is paid £3 10*s.* ÷ 100 × 325
or £3 10*s.* × 325 ÷ 100 = £11 7*s.* 6*d.*

Ans. £11 7*s.* 6*d.*

Ex. 3. If 40 bushels of oats keep 65 horses for a week, how long will they keep 13 horses?

65 horses are kept on 40 bushels for 1 week.
∴ 1 horse can be kept on 40 bushels 1 × 65 wks.
∴ 13 horses can be kept 1 × 65 ÷ 13 wks. = 5 wks.

Ans. 5 weeks.

Ex. 4. If the price of 7 lbs. of tea is £1 1*s.* 3½*d.*, what will 63 lbs. cost?

Ex. 5. If 17 lbs. of bacon can be bought for 12*s.* 9*d.*, what will 23 lbs. cost at the same rate?

Ex. 6. If a man earns £36 18*s.* 9*d.* in 6 weeks, how much will he earn in 37 weeks at the same rate?

Ex. 7. Eggs are 16 a shilling; what is that for 123?

Ex. 8. If 18 lbs. of figs cost 13*s.* 6*d.*, what will ½ cwt. cost?

Ex. 9. If I lend £725 for £36 5*s.*, for how much shall I lend £100 for the same time?

Ex. 10. When wheat is 37*s.* 6*d.* per quarter, what shall I pay for 5 bushels?

Ex. 11. If 7½ yds. of cloth cost £8 6*s.* 3*d.*, what is the value of 19½ yds.?

Ex. 12. One cwt. of cheese costs £4 13*s.* 4*d.*; find the cost of 2 qrs. 15 lbs.

MISCELLANEOUS EXERCISES.

1. From what number must three thousand and fourteen be taken away to leave nine hundred and ninety-one?

2. An English sovereign weighs 123 grs.; find the weight of 275 sovereigns.

3. If from a certain number three hundred thousand and ten be taken, three hundred and one thousand and forty are left; find the number.

4. Find the cost of 18 doz. oranges at 2 for 1½*d.*

5. Change 735 sovereigns to guineas.

6. If eggs are sold at 3 for 2*d.*, how many can I buy for £1 8*s.* 8*d.*?

7. Required the cost of travelling 211 miles at 2½*d.* per mile.

8. How many marbles at 1*d.* a dozen can I buy for 5¼*d.*?

9. Divide thirty thousand and forty into two parts, such that one is one thousand and ten less than the other.

10. Find the cost of 6 gross of pens at 2½*d.* per dozen.

11. A man sells 595 pears at 7 for 4*d.*; what are his receipts?

12. Flour is sold at £1 1*s.* per cwt.; what is that per lb.?

13. What money will be required to pay for 17 railway shares at £83 6*s.* 8*d.* each?

14. How many ¾*d.* buns can I buy for £2 1*s.* 3*d.*?

15. Newspapers are bought at 9*d.* per dozen, and retailed at 1*d.* each; find the profit on 141.

16. What is the price of cheese per lb. when a ton costs £37 6*s.* 8*d.*?

17. Given the quotient to be 172, and the dividend 63,984; find the divisor.

18. How many times can 372 be subtracted from 31,992?

19. How many parcels of 7 ozs. can be made from 1 cwt. of tea?

20. In paying a bill of £2 7*s.* 6*d.* a man gave half-a-sovereign in mistake for sixpence; how much did he pay too much?

21. Divide £5 5*s.* 1*d.* equally among 13 persons.

22. How far can I travel for £4 10*s.* at 1¼*d.* per mile?

23. How many yards of calico at 5½*d.* can I buy for £1 9*s.* 4*d.*?

24. Divide 135 marbles among 3 boys, giving the first 18 more than the second, and the second 18 more than the third.

25. After paying away half the money in my purse and 10s. over, I had £2 10*s.* left; what had I at first?

26. How many oranges at 14 for a shilling can I buy for a guinea?

27. The divisor is 207, the quotient is 303, and the remainder 200; find the dividend.

28. The number of letters paying penny postage in 4 weeks was one million nine hundred and seven thousand three hundred and fourteen; what was the sum paid for them?

29. A milkman sells 17 qts. of milk on Monday, 19 on Tuesday, 20 on Wednesday, 16 on Thursday, 24 on Friday, and 18 on Saturday; what is the average sale per day?

30. What change shall I get out of a £5 note after paying for 1 cwt. 3 qrs. 16 lbs. of sugar at £2 2*s.* per cwt.?

31. If 5*s.* 9*d.* be put by weekly, what will it amount to in a year?

32. What is the weekly income of a man who earns £87 8*s.* 6*d.* a year?

33. If 1 yd. of cloth cost 7*s.* 3*d.*, what will 36 yds. cost?

34. Butter is bought at £6 per cwt.; what is that per ton?

35. 75 labourers mend a road at 18*s.* 6*d.* a week each; how much will be required to pay them one week's wages?

36. A man paid 2 half-sovereigns, 2 half-crowns, and 2 sixpences as wages; this was 2*s.* 9*d.* more than the half of the money he had in his purse; what money had he left?

37. How many times is 17 hrs. 3 min. contained in 5 yrs. 186 days?

38. A linendraper bought 300 yds. of calico at $4\frac{1}{2}d.$ a yd., and retailed it at $7\frac{1}{4}d.$ a yd.; what was his profit?

39. Multiply 35 acres 2 roods 19 poles by 68.

40. Reduce £538 2*s.* 6*d.* to half-guineas.

41. A man owes £36 2*s.*; if he earns £3 12*s.* 6*d.* per week, and spends £2 2*s.* 5*d.*, how soon can he pay his debt?

42. A linendraper bought 224 yds. of stuff at 1*s.* 5*d.* per yd.; for what must he sell the whole to gain $3\frac{1}{2}d.$ per yd.?

43. A gross of pens is bought for 1*s.* 8*d.*, and retailed at 3 a penny; find the profit.

44. Find the difference between thirty thousand six hundred and seven inches and 92 miles 867 yds. 5 in.

45. My butcher's bill amounts to £8 3*s.* for 2 months; how many pounds of meat have I bought at 1*s.* per lb.?

46. If I pay 7*d.* income tax out of every £ of my yearly income of £250, how much have I left?

47. Light travels at the rate of 142,000 miles in a second; in what time does a ray of light come from the sun to the earth, a distance of ninety-five millions of miles?

48. A miller bought 345 sacks of flour at £2 3*s.* $2\frac{1}{2}d.$ each, and sold them for £1,000; find his profit.

49. A farmer pays £6 10*s.* for 3 sheep; what will he pay for 25 at the same rate?

50. An acre of wheat produces 7 bush. 2 pks.; what is the produce of 39 acres?

51. A boy earns $9\frac{1}{2}d.$ a day; what does he earn in 313 days?

52. A cart weighs 12 cwt. 2 qrs. 17 lbs., but when loaded with sand it weighs on the machine of the toll-bar 1 ton 7 cwt. 1 qr.; how much does the sand weigh?

53. Multiply 5 cwt. 3 qrs. 21 lbs. 3 ozs. by 17.

54. How much must be added to thirteen times £1 7*s.* $10\frac{1}{2}d.$ to make it equal to seventeen times £1 4*s.* $11\frac{1}{4}d.$?

55. If 3 articles cost 12*s.* 6*d.*, how many can I buy for £4 7*s.* 6*d.*?

56. How many ounces must be added to $\frac{1}{2}$ cwt. to make $\frac{3}{4}$ cwt.?

57. By what number must I multiply 5*s.* $7\frac{1}{2}d.$ to produce £5 1*s.* 3*d.*?

58. An estate is bought for £68,423; the cost per acre is £32 5*s.* 6*d.*; find the number of acres.

59. How many articles can I buy for £17 11*s.* 9*d.*, if 5 cost 50*s.* 3*d.*?

60. A furniture dealer bought 16 doz. chairs at 4*s.* 6*d.* each; for what must he sell the whole to gain 9*d.* per chair?

61. Find the total weight of 435 iron pillars, each weighing 6 cwts. 3 qrs. 22 lbs.

62. The weekly wages of 66 men amount to £137 15*s.* 6*d.*; what is that per man?

63. I spend 15 guineas in 36 days; at the end of the year I find I have saved 100 guineas; find my yearly income.

64. How much is there left out of 150 guineas after paying for 29 sofas at £3 17*s.* 9*d.* each?

65. A carpenter was paid £2 1*s.* 8*d.* as wages; had he worked 3 weeks longer he would have received £9 6*s.* 8*d.*; what were his weekly wages?

66. The cost of 65 doz. pairs of gloves is £53 12*s.* 6*d.*; find the cost of a single pair.

67. A house and garden are rented at 100 guineas; the rent of the house is six times that of the garden; find the rent of the garden.

68. At a halfpenny foot-bridge the toll collector took £439 11*s.* 7*d.* in a year; how many persons passed over the bridge?

69. How many pounds of beef at 10*d.* can be bought for 180 half-crowns?

70. A boy, in walking, covers 1 ft. 10 in. at every step; how many steps does he take in a mile?

71. Subtract 22 poles 16 yds. of land from 5 acres 1 rood.

72. A silk mercer bought 100 pieces of silk for 500 guineas, and sold them at a profit of £5 on every £100; what was the selling price per piece?

73. A servant's wages are £13 19*s.* 6*d.* a year; what will be the amount for 14 weeks (52 weeks = 1 year)?

74. If 19,000 cubic ft. of gas cost £5 1*s.* 4*d.*, what is that per 1,000 ft.?

75. Find the difference between forty-nine tons two quarters seventeen pounds, and fifty thousand nine hundred and seventeen ounces.

76. I want to raise £150 by penny subscriptions; if 17,000 have paid how many more subscribers do I want?

77. If the divisor is twice the quotient, and the quotient twice the remainder, find the dividend when the remainder is 21.

78. A farm of 169½ acres is divided into 12 equal-sized fields; find the size of each.

79. Divide £19 17*s.* 4*d.* among 5 men, giving the last £2 3*s.* 7*d.* more than each of the others.

80. To how many persons can I give £3 5*s.* 7½*d.* out of 100 guineas?

81. Take 2,391 cubic in. from 16 cubic yds. 10 cubic ft. 62 cubic in.

82. How many spoons, each weighing 2 ozs. 8 dwts., can be made out of 5 lbs. 4 ozs. 16 dwts. of silver?

83. A butcher bought 5 cwt. 25 lbs. of beef; 175 lbs. were found to be useless; he sold the rest at 10*d.* per lb., and thereby gained £2 1*s.* 8*d.* on the whole; what did the beef cost him?

84. How many bags of flour can be made up out of 2 cwt. if half the number of bags contain 5 lbs. and the other half 3 lbs.?

85. 112½ lbs. of tea and 3 cwt. 39 lbs. of sugar are to be equally distributed among 150 poor persons; how much tea and sugar will each receive?

86. I paid away £2 5*s.* 6*d.*; had I paid four times that sum I should have paid away one-sixth of my money; how much had I at the first?

87. How many 7½-lb. parcels can be made up out of 1 cwt. 2 qrs. 12 lbs.?

88. If 60 men can build a wall in 8 days, how long will it take 40 men to do it?

89. Find the width of a plank which is 25 ft. long and 75 sq. ft. in area?

90. If 100 sovereigns are coined out of 2 lbs. 1 oz. 12 dwts. 12 grs. of gold, what is the weight of 1 sovereign?

91. If 9 acres can be ploughed in 3 days, how long will it take to plough 39 acres?

92. If the quartern loaf costs 7*d.* when wheat is selling at 56*s.* per qr., what should it cost when wheat is selling at 64*s.* per qr.?

93. If I pay 7½*d.* for 3 lbs. of rice, how many lbs. can I buy for 12*s.* 6*d.*?

94. What is tea worth per lb. when 16 lbs. are exchanged for 24 lbs. of coffee at 1*s.* 8*d.* per lb.?

95. A man has £546; he gives to each of 56 persons as many sovereigns as he can; he then changes the remaining sovereigns to shillings, and gives to each of the 56 as many shillings as he can; what does each person receive, and how many shillings are left?

96. How much will it cost to paint a room 16 ft. long, 12 ft. wide, and 10 ft. high, at 2*d.* per sq. ft.?

97. A cistern which holds 200 galls. of water is supplied by a pipe which lets in 6 galls. a minute; the same cistern supplies another which lets out 4½ galls. per minute; if the first cistern contains 29 galls. of water when both pipes are opened, how long will the cistern be in filling?

98. A ship's cargo worth £25,000 is insured at 5 per cent.; find the amount of the insurance.

99. A man pays 5*d.* in the £ income tax on £250; what is the amount?

100. Will a grocer gain or lose, and how much, by purchasing 17 lbs. of tea for £1 6*s.* and selling it at 2*s.* 6½*d.* per lb.?

101. A person bought 60 yds. of cloth at the rate of 5 yds. for a guinea, and 70 more at 7 yds. for a guinea; he sold the whole at 6 yds. for a guinea; how much did he gain or lose?

102. How many subscriptions of 1*s.* 4*d.* each do I want to make £1,050 ?

103. Mix 1 cwt. of sugar at 5*d.* per lb. with 96 lbs. at 4½*d.*, and find at what price per lb. the mixture must be sold to gain £1 1*s.* 4*d.* on the whole.

104. The iron rails round St. Paul's weighed 200 tons 81 lbs. ; what did they cost if iron is charged 7*d.* per lb. ?

105. If I lend a man £100 for 12 months for £5, what should he charge me for the loan of £550 for 2½ years at the same rate ?

106. A train runs at the rate of 60 ft. in a second ; what is that per hour ?

107. What is the annual premium on a life insurance of £635 at £3 5*s.* per cent. ?

108. The circumference of the large wheel of a bicycle is 12 ft. 6 in., that of the small wheel is 4 ft. 2 in. ; how many times will each turn round in going a distance of 2½ miles ?

109. What is the gain on £50 at 2½*d.* in the shilling ?

110. How many pieces of cloth 2½ yds. long can be cut from 19 pieces, each containing 27 yds. 2 qrs. ?

111. How many guineas, sovereigns, half-crowns, and shillings, and of each an equal number, are there in £623 ?

112. Divide £42 14*s.* between A, B, and C, giving B twice as much as A, and C twice as much as B.

113. A room is 20 ft. broad and 60 ft. long ; how many scholars will it accommodate, allowing 10 sq. ft. to each ?

114. How many yards of cloth at 7*s.* 6*d.* a yd. must be given in exchange for 27 yds. at 11*s.* 3*d.* a yd. ?

115. A room is 20 ft. square and 12 ft. high ; how many persons will it accommodate, allowing 100 cubic ft. of air to each ?

116. How many square yards are there in a square whose side measures 19 yds. ?

117. How many cubic feet are there in a block of marble 3 ft. long, 3 ft. high, and 3 ft. broad ?

118. Two plots of land measure together 1 rood 12 per. 9½ sq. yds. ; one of the plots is four times as large as the other ; find the size of each.

119. How many allotments of 36 poles each can be made from a field containing 38 acres 1 rood ?

120. A yard measures 20 feet each side ; how many square tiles 15 in. in the side will it take to cover it ?

121. A street 500 yds. long and 20 ft. wide is to be paved with wood ; if the blocks are 9 in. long and 6 in. wide, how many will it take ?

122. If a stream runs at the rate of 2 miles an hour, and a man

can row 5 miles an hour "up" the stream, how fast will he row "down" the stream?

123. A corn chandler bought 8 bush. of oats at 4*s.* 6*d.* a bush., 16 bush. at 4*s.* 3*d.*, and 24 at 4*s.*; at what price per bush. must he sell the mixture to gain £2 on the whole?

124. A bankrupt owes £5,460, and can pay 15*s.* in the £; find the amount of his assets.

125. A miller buys 5 cwt. 3 qrs. of flour at 18*s.* per cwt., and 4 cwts. 2 qrs. at 19*s.* per cwt.; at what price must he sell the mixture per lb. to gain £1 6*s.* 3*d.* on the whole?

126. Sound travels at the rate of 1,140 ft. per second; I see the flash from the firing of a cannon 15 secs. before I hear the report; how far distant is the cannon?

127. A train which travels at the rate of 20 miles an hour starts from a certain station at 6 A.M.; a second train starts at 9 o'clock, travelling 50 miles an hour; what distance will the trains have gone over when the second overtakes the first?

CHAPTER II.

FRACTIONS—INTRODUCTORY.

SECTION I.

Take an apple, and cut it into *two equal* parts. Each part is called *one-half*. Cut the apple into *four equal* parts. Each part is called *one-quarter*.

The half is written thus, $\frac{1}{2}$
one-quarter „ „ $\frac{1}{4}$
two-quarters „ „ $\frac{2}{4}$
three-quarters „ „ $\frac{3}{4}$

Each of these parts is said to be a FRACTION, or *even part* of the apple.

We will take one of these fractions, say $\frac{3}{4}$, and look at it a little more closely.

(1) We first *divided* the apple into four equal parts. Now the sign ÷ indicates *division*, and we use this sign in the fraction to indicate division, but we omit the dots. The line, then, between the 3 and the 4 in the fraction $\frac{3}{4}$ indicates division.

(2) We divided the apple into *four* equal parts, and we place the 4 *below* the line to show this particular division.

(3) We took *three* of those parts to make the three-fourths, and to show this number we place the 3 *above* the line.

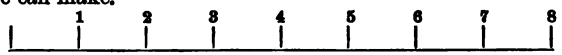
A FRACTION then consists of three parts.

(*a*) A line to indicate division.

(*b*) A figure placed *below* the line to show the number of equal parts into which the whole or unit has been divided. This figure is called the DENOMINATOR.

(*c*) A figure placed above the line to show how many of the equal parts have been taken to form the particular fraction. This figure is called the NUMERATOR.

We will now divide a *line* into 8 equal parts, and see how many fractions we can make.



The denominator in each case is 8, and we may take as numerators 1, 2, 3, 4, 5, 6, 7, and 8.

Hence, the fractions are

$$\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}.$$

In the last of these fractions, $\frac{8}{8}$, we have taken the whole of the parts of the line, and we see that when the numerator and denominator are equal, the fraction is a unit or a whole number.

Now take a second line of equal length to the first, and divide similarly into 8 equal parts: take 3 of these parts together, with the 8 from our first line, viz. in all 11 parts. How shall we write this fraction?

Our unit or whole is still divided into 8 equal parts, hence 8 will be the denominator; and since we have taken 11 parts, 11 will be the numerator, and the fraction is written $\frac{11}{8}$.

This is plainly a different sort of fraction from $\frac{5}{8}$ or $\frac{7}{8}$. It is not, properly speaking, a part, but a *whole* and an *even part*. It is called an IMPROPER FRACTION. $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$, being all *parts* of one whole, are called PROPER FRACTIONS. We may write $\frac{11}{8}$ as a whole, and $\frac{3}{8}$ of another, thus, $1 + \frac{3}{8}$, or $1\frac{3}{8}$ (the sign of addition being understood). When the improper fraction is written in the form of a whole number and a fraction, we call it a MIXED FRACTION.

Definitions—

(1) A FRACTION is an even part of a whole.

(2) A PROPER FRACTION is one which has the numerator *less than* the denominator, as $\frac{5}{8}$.

(3) An IMPROPER FRACTION is one which has the numerator *not less than* the denominator, as $\frac{12}{8}$.

(4) A MIXED FRACTION is one which is made up of a whole number and a proper fraction, as $5\frac{3}{7}$.

Exercises.

1. Find the value of $\frac{5}{8}$ of £53 5*s.* 6*d.*

What is the meaning of the fraction $\frac{5}{8}$? Just this, that the whole is to be divided into 8 equal parts, and that 5 of them are to be taken. (In this case £53 5*s.* 6*d.* is our whole, or unit.) We must divide by 8 to find $\frac{1}{8}$, and then multiply by 5 to find $\frac{5}{8}$.

£	*s.*	*d.*	
8)53	5	6	= whole.
6	13	$2\frac{1}{4}$	$=\frac{1}{8}$ of the whole.
		5	
£33	5	$11\frac{1}{4}$	$=\frac{5}{8}$ of the whole.

Note.—The result will be the same if we first multiply by 5, and then divide by 8.

2. Two-thirds of the money in my purse is £10 2*s.* $2\frac{1}{2}$*d.*; find the amount in the purse.

£	*s.*	*d.*	
2)10	2	$2\frac{1}{2}$	$=\frac{2}{3}$ of money.
5	1	$1\frac{1}{4}$	$=\frac{1}{3}$ of money.
		3	
£15	3	$3\frac{3}{4}$	$=\frac{3}{3}$ or the whole.

3. Find the value of 3,054 articles at $\frac{5}{6}$*d.* each.

6)3054 = value at 1*d.*
509 = value at $\frac{1}{6}$*d.*
5
12)2545 = value at $\frac{5}{6}$*d.*
20)212 1*d.*
£10 12 1

Exercises.

1. Find the value of $\frac{1}{8}$ of four guineas.
2. Find the amount of $\frac{3}{7}$ of £7 12*s.* 3*d.*
3. By how much is $\frac{5}{8}$ of £10 12*s.* 3*d.* greater than $\frac{7}{11}$ of £8 16*s.*?
4. Find the value of $27\frac{3}{4}$ yds. of cloth at 5*s.* per yd.
5. My father's age is 65; I am $\frac{2}{5}$ of my father's age; how old am I?
6. If $\frac{1}{8}$ of an estate is worth 1,000 guineas, what is the value of $\frac{3}{8}$, and of the whole?
7. What is the difference between $\frac{5}{12}$ of a guinea and $\frac{5}{12}$ of a £?
8. Four-fifths of my house is worth £567; what is the value of the house?

9. How many lbs. are there in $\frac{5}{7}$ of a ton. and $\frac{3}{5}$ of a cwt.?

10. Multiply £47 13*s.* 2*d.* by $27\frac{3}{7}$.

11. My income is £250; if I spend $\frac{4}{5}$ of it, how much do I save?

12. Add together $\frac{3}{7}$ of a guinea, $\frac{3}{5}$ of a £, $\frac{3}{5}$ of a shilling, and $3\frac{1}{3}$ of sixpence.

13. Find the value of $29\frac{1}{4}$ lbs. of tea at 3*s.* 2*d.* a lb.

14. What is the cost of $\frac{3}{5}$ of a cwt. of sugar at $3\frac{1}{2}d.$ per lb.?

15. Find the sum in lbs. of $\frac{1}{4}$ of a ton, $\frac{4}{15}$ of a cwt., and $17\frac{1}{2}$ lbs.

16. Find the value of $312\frac{1}{2}$ yds. of silk at 4*s.* 6*d.* per yd.

17. In copying an example I wrote $\frac{3}{5}$ of £10, instead of $\frac{3}{8}$; what was the error in the amount?

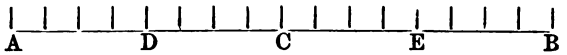
18. Find the cost of 321 articles at $\frac{3}{4}d.$ each.

19. Multiply £3 2*s.* $5\frac{2}{5}d.$ by 35.

20. Multiply £64 13*s.* $7\frac{3}{4}d.$ by $329\frac{3}{4}$.

Section II.

The numerator *and* denominator of a fraction may be multiplied or divided by any number without altering the value of the fraction.



Let the line A B be divided into 4 equal parts; then—

$$A\ D = \frac{1}{4}$$

Now divide into 8 equal parts; then—

$$A\ D = \frac{2}{8}$$

Next divide into 16 equal parts; then—

$$A\ D = \frac{4}{16}$$

And so if we divide into 32 or 64 equal parts; then—

$$A\ D = \frac{8}{32} \text{ or } \frac{16}{64}$$

That is—

$$\frac{1}{4} = \frac{2}{8} = \frac{4}{16} = \frac{8}{32} = \frac{16}{64} \text{ and so on;}$$

and these fractions are all convertible, that is, by multiplying or dividing both numerator and denominator by the same number, we can get any one fraction from any one of the others.

For example—

$$(1)\ \frac{2}{8} = \frac{2 \times 4}{8 \times 4} = \frac{8}{32} \text{ and } \frac{8 \times 4}{32 \times 4} = \frac{32}{128}$$

$$\text{or } (2)\ \frac{16}{64} = \frac{16 \div 8}{64 \div 8} = \frac{2}{8} \text{ and}$$

$$\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$$

The second example, viz. reducing $\frac{16}{64}$ to $\frac{1}{4}$ by division, is called reducing to its lowest terms, or *cancelling*, and this process we shall find to be of great value as we proceed. The process shown in the first example, viz. by multiplication, has its chief use in comparing the value of fractions, and in addition and subtraction. If I am asked to add $\frac{1}{4}$ of an apple to $\frac{1}{8}$, I cannot give any satisfactory reply; but if I change $\frac{1}{4}$ to $\frac{2}{8}$, I can add the pieces of equal size, viz. $\frac{1}{8}$ to $\frac{2}{8}=\frac{3}{8}$. If I am asked to compare $\frac{5}{6}$ and $\frac{6}{7}$, I am puzzled to give a correct answer, but I make the first $\frac{35}{42}$ by multiplying by 7, and the second $\frac{36}{42}$ by multiplying by 6; and I can at once say the one is $\frac{1}{42}$ larger than the other.

We can readily convert improper fractions to mixed fractions, and mixed fractions to improper.

Thus $\frac{16}{9}=\frac{9}{9}+\frac{7}{9}$ but $\frac{9}{9}=1$

Therefore $\frac{16}{9}=1+\frac{7}{9}$ or $1\frac{7}{9}$

Again $\frac{9}{2}=\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{1}{2}$

or $=4+\frac{1}{2}=4\frac{1}{2}$

We may get the same result by dividing the numerator by the denominator; the number of times the denominator is contained in the numerator gives the whole number, and what is over gives the numerator of the proper fraction.

The converse process changes the mixed fraction to an improper fraction.

Thus $4\frac{1}{2}=\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{2}{2}+\frac{1}{2}$

or $\frac{8}{2}+\frac{1}{2}=\frac{9}{2}$

Hence the rules:—

1. To reduce an improper fraction to its equivalent whole or mixed number, *Divide the numerator by the denominator, and place the divisor under the remainder (if any) of the fraction.*

2. To reduce a mixed number to an improper fraction, *Multiply the whole number by the denominator of the fraction, add the numerator, and place the denominator under the sum.*

Exercises.

1. Draw a square, and divide it to show that $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$.
2. Show by the division of a line that $\frac{1}{6}$ subtracted from $\frac{1}{3}$ leaves $\frac{1}{6}$.
3. Show by the division of a line that $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$.
4. Change to whole numbers and fractions

   (1) $\frac{9}{2}$, $\frac{15}{7}$, $\frac{8}{7}$, $\frac{3}{2}$, $\frac{16}{9}$, $\frac{315}{17}$

   (2) $\frac{32}{5}$, $\frac{2012}{76}$, $\frac{3764}{376}$, $\frac{80000}{200}$
5. Reduce the following mixed numbers to improper fractions:—

   (1) $4\frac{1}{2}$, $2\frac{1}{7}$, $1\frac{1}{7}$, $1\frac{1}{2}$, $1\frac{7}{9}$, $18\frac{9}{17}$

   (2) $12\frac{11}{13}$, $19\frac{7}{16}$, $231\frac{3}{4}$, $89\frac{1}{7}$, $100\frac{5}{8}$

6. Change $\frac{1}{5}$ and $\frac{3}{4}$ to fractions having 20 for their denominator.

7. Change $3\frac{15}{35}$ to a fraction having 7 for its denominator.

8. Which is the greater, and by how much, $\frac{5}{8}$ or $\frac{6}{7}$?

9. Compare $2\frac{7}{9}$ and $2\frac{23}{27}$, and say how much the second is greater than the first.

10. Reduce the following fractions to their lowest terms :—

(1) $\frac{18}{20}$, $\frac{27}{30}$, $\frac{30}{3}$, $1\frac{5}{15}$, $\frac{370}{74}$

(2) $\frac{32}{54}$, $\frac{544}{676}$, $\frac{275}{350}$, $\frac{81}{108}$

11. What is the difference between $\frac{3}{5}$ and $\frac{7}{10}$?

12. If $\frac{1}{3}$ of an estate is worth £5,000, what is the value of $\frac{1}{5}$?

13. Take $\frac{1}{4}$ from $\frac{3}{8}$. How much is the remainder less than $\frac{1}{4}$?

14. Change $\frac{2}{3}$ and $\frac{5}{9}$ to fractions with the same denominator.

15. Find the sum and difference of $\frac{1}{6}$ and $\frac{2}{3}$.

16. Reduce to whole, or mixed numbers, $\frac{27}{3}$, $\frac{59}{7}$, and change $6\frac{2}{5}$, and $16\frac{1}{3}$, and $7\frac{5}{11}$ to improper fractions.

17. Find the difference between $6\frac{1}{3}$ shillings and $6\frac{1}{4}$ shillings.

18. One cistern $\frac{2}{5}$ full contains 36 gallons, another when $\frac{3}{5}$ full contains 66 gallons ; how many gallons does the second cistern hold more than the first?

19. A cistern $\frac{3}{5}$ full contains 72 gallons ; when 8 gallons are added it is found to be $\frac{2}{3}$ full ; how much does the cistern hold?

CHAPTER III.

PRACTICE AND BILLS OF PARCELS.

Section I.—Simple Practice.

Practice is another and a shorter mode of performing compound multiplication. It is the method most frequently used in *practice* to find the value of a *number* of articles when the price of one is given. Suppose a grocer is selling 10 lbs. of tea at 1*s.* 6*d.* a lb., he can find the value of the tea by multiplying 1*s.* 6*d.* by 10, but he is much more likely to say—

10 lbs. at 1*s.* = 10*s.*
and 10 lbs. at 6*d.* will be half as much as at 1*s.* = 5*s.*
and adding the two = 15*s.*

This is a method of *partial payments*; we find the total value by part payments. In the example the first payment is at 1*s.* and the second at 6*d.*

We will take another example, and compare with the longer method of multiplication.

Ex. 1. Find the value of 376 tons of coals at £1 15*s.* per ton.

(1) By multiplication :—

£1	15	0 × 6
		10
17	10	0 × 7
		10
175	0	0
		3
525	0	0
122	10	0
10	10	0
£658	0	0

(2) By partial payments :—

376 tons at £1 per ton = £376
376 tons at 10*s.* per ton = half the price at £1 = 188
376 tons at 5*s.* per ton = half the price at 10*s.* = 94
Total = £658

The second method is not only shorter, but much easier in working than the first method.

The following plan is the one recommended for indicating all the particulars in the example :—

Of £1	10*s.*	= ½	£376 = value at	£1	0	0	per ton.
Of 10*s.*	5*s.*	= ½	188 = „	0	10	0	„
			94 = „	0	5	0	„
			£658 = value at	£1	15	0	per ton.

When a concrete number is contained an exact number of times in another concrete number, the first is said to be an *even* or *aliquot* part of the second. Thus 5*s.* is contained exactly four times in 20*s.*; hence 5*s.* is an aliquot part of 20*s.* For a like reason 3*s.* 4*d.* is an aliquot part of 10*s.*, 2*s.* 6*d.* of 12*s.* 6*d.*, and so on; but 5*s.* is not an aliquot part of 12*s.*, nor is 5 ozs. an aliquot part of a lb.

Practice alone will give facility in selecting the best *partial payments*, viz. the most convenient *aliquot parts*.

Ex. 2. Find the value of 6 score sheep at £2 19*s.* 11½*d.* each.

First Method.

Of £1	10*s.*	$=\frac{1}{2}$	£120	0	value at £1 each.				
				2					
			240	0	value at £2	0	0	each.	
Of 10*s.*	5*s.*	$=\frac{1}{4}$	60	0	„	0	10	0	„
Of 5*s.*	2*s.* 6*d.*	$=\frac{1}{8}$	30	0	„	0	5	0	„
Of 2*s.* 6*d.*	1*s.* 3*d.*	$=\frac{1}{16}$	15	0	„	0	2	6	„
Of 1*s.* 3*d.*	7½*d.*	$=\frac{1}{32}$	7	10	„	0	1	3	„
Of 1*s.* 3*d.*	5*d.*	$=\frac{1}{48}$*	3	15	„	0	0	7½	„
Of 5*d.*	1*d.*	$=\frac{1}{5}$	2	10	„	0	0	5	„
Of 5*d.*	1*d.*	$=\frac{1}{5}$	0	10	„	0	0	1	„
			0	10	„	0	0	1	„
			£359	15	value at £2	19	11½	each.	

Second Method.

Of £1	10*s.*	$=\frac{1}{2}$	£120	value at £1 each.					
			2						
			£240	0	value at £2	0	0	each.	
Of 10*s.*	5*s.*	$=\frac{1}{4}$	60	0	„	0	10	0	„
Of 10*s.*	3*s.* 4*d.*	$=\frac{1}{6}$	30	0	„	0	5	0	„
Of 5*s.*	1*s.* 3*d.*	$=\frac{1}{16}$	20	0	„	0	3	4	„
Of 1*s.* 3*d.*	3*d.*	$=\frac{1}{5}$	7	10	„	0	1	3	„
Of 3*d.*	1½*d.*	$=\frac{1}{2}$	1	10	„	0	0	3	„
			0	15	„	0	0	1½	„
			£359	15	value at £2	19	11½	each.	

Third Method.

	£120	= value at £1 each.					
	3						
120 half-pence	£360	0 = value at £3	0	0	each.		
=5*s.*	0	5 = „	0	0	0½	„	
	£359	15 = value at £2	19	11½	each.		

Of these three methods the second is preferable to the first, because it necessitates less divisions, and the third (viz. by subtraction) is preferable to either for shortness and simplicity.

* Note.—One third of the value at 1*s.* 3*d.* is taken, viz. of £7 10*s.*

Ex. 3. How much money will it take to pay 144 boys one week's wages at 11*s.* 10½*d.* each?

First Method.

			s.			
Of 1*s.*	6*d.*	=½	144 = one week's wages at 1*s.*			
			11			
			1584 = one week's wages at 11		0	
Of 6*d.*	3*d.*	=½	72 = „ „		0	6
Of 3*d.*	1½*d.*	=½	36 = „ „		0	3
			18 = „ „		0	1½
			2,0)171,0 = one week's wages at		11	10½
			£85 10*s.* „ „		11	10½

Second Method.

Of £1	10*s.*	=½	£144 = one week's wages at £1.			
Of 10*s.*	1*s.* 3*d.*	=⅛	72 0 = one week's wages at 10		0	
Of 1*s.* 3*d.*	7½*d.*	=½	9 0 = „ „		1	3
			4 10 = „ „		0	7½
			£85 10 = one week's wages at		11	10½

Ex. 4. How much money will a boy earn in 313 days at 7½*d.* per day?

First Method.

			d.			
Of 1*d.*	½*d.*	=½	313 amount at 1*d.* per day.			
			7			
			2191 amount at 7*d.* per day.			
			156½ „ ½*d.* „			
			12)2347½ amount at 7½*d.* per day.			
			2,0)19,5 7½ „ „			
			£9 15 7½ „ „			

Second Method.

			s.			
Of 1*s.*	6*d.*	=½	313 amount at 1*s.* per day.			
Of 6*d.*	1½*d.*	=¼	156 6 amount at 6*d.* per day.			
			39 1½ „ 1½*d.* „			
			2,0)19,5 7½ amount at 7½*d.* per day.			
			£9 15 7½ „ „			

Ex. 5. Find the value of 1,000 oranges at $\frac{3}{4}d.$ each.

			d.	
Of 1*d.*	$\frac{1}{2}d.$	$=\frac{1}{2}$	1000	value at 1*d.* each.
Of $\frac{1}{2}d.$	$\frac{1}{4}d.$	$=\frac{1}{2}$	500	value at $\frac{1}{2}d.$ each.
			250	„ $\frac{1}{4}d.$ „
			12)750	value at $\frac{3}{4}d.$ each.
			2,0)6,2 6	„ „
			£3 2 6	„ „

Ex. 6. Find the cost of 8,976$\frac{5}{7}$ articles at £12 9*s.* 4$\frac{1}{2}$*d.* each.

In examples of this kind first find the value of the articles without the fraction in the ordinary way; then find the value of the fraction and add.

			£					
			8976	= value at £1 each.				
			12					
			107712	0 = value at £12	0	0	each.	
Of £1	3*s.* 4*d.*	$=\frac{1}{3}$	2244	0 = „	0	5	0	„
Of 3*s.* 4*d.*	10*d.*	$=\frac{1}{4}$	1496	0 = „	0	3	4	„
Of 10*d.*	2$\frac{1}{2}d.$	$=\frac{1}{4}$	374	0 = „	0	0	10	„
			93	10 = „	0	0	$2\frac{1}{2}$	„
			£111919	10 = value at £12	9		$4\frac{1}{2}$	each.

£	s.	d.	
7)12	9	$4\frac{1}{2}$	= value of 1 article.
1	15	$7\frac{1}{2}$	= „ $\frac{1}{7}$ „
		5	
£8	18	$1\frac{1}{2}$	= „ $\frac{5}{7}$ „
£111919	10	0	= value of 8,976 articles.
	8	18 $1\frac{1}{2}$	= „ $\frac{5}{7}$ „
£111928	8	$1\frac{1}{2}$	= value of 8,976$\frac{5}{7}$ articles.

Ex. 7. Find the value of 6,413 articles at 4*s.* 10$\frac{7}{16}d.$ each.

In this example we must find the value at 4*s.* 10*d.*, and then deal with the $\frac{7}{16}d.$ separately.

			£					
			6413	= value at £1 each.				
Of £1	4*s.*	$=\frac{1}{5}$						
Of 4*s.*	8*d.*	$=\frac{1}{6}$	£1282	12 0 = value at	4	0	each.	
Of 8*d.*	2*d.*	$=\frac{1}{4}$	213	15 4 = „	0	8	„	
			53	8 10 = „	0	2	„	
			*11	13 $9\frac{11}{16}$ = „	0	$0\frac{7}{16}$	„	
			£1561	9 $11\frac{11}{16}$ = value at	4	$10\frac{7}{16}$	each.	

* See next page.

To find the value of 6,413 articles at $\frac{7}{16}d.$ each :—

£	s.	d.	
2)53	8	10	= value at 2*d.* each.
16)26	14	5	= „ 1*d.* „
1	13	$4\frac{13}{16}$	= „ $\frac{1}{16}d.$ „
		7	
*£11	13	$9\frac{11}{16}$	= „ $\frac{7}{16}d.$ „

EXERCISES IN PRACTICE.

1. What parts of a sovereign are 6*s.* 8*d.*, 3*s.* 4*d.*, 2*s.* 6*d.*, 1*s.* 8*d.*, a 1*s.* 3*d.* ?

2. What parts of half-a-sovereign are 3*s.* 4*d.*, 2*s.* 6*d.*, 1*s.* 8*d.*, and 1*s.* 3

3. Write out the aliquot parts of a shilling and of half-a-crown.

4. What part of 7*s.* is 1*s.* 9*d.* ; of 6*s.* 9*d.* is 2*s.* 3*d.* ; of 7*s.* 8*d.* is 1*s.* 11 of 12*s.* 8*d.* is 1*s.* 7*d.* ; of 8*s.* 3*d.* is 9*d.* ; of 9*d.* is $\frac{3}{4}d.$?

5. Select the most convenient aliquot parts of 17*s.* $11\frac{1}{2}d.$, 18*s.* 11 16*s.* $9\frac{1}{4}d.$, 13*s.* $6\frac{1}{2}d.$, 19*s.* 10*d.*, and $9\frac{1}{2}d.$

6. 360 articles at 5*s.* 9*d.* and 7*s.* 11*d.*
7. 144 „ 9*s.* 6*d.* and 11*s.* 5*d.*
8. 1,060 „ 17*s.* 6*d.* and 16*s.* 9*d.*
9. 426 „ £1 17*s.* 6*d.* and £1 16*s.* 9*d.*
10. 3,960 „ $7\frac{1}{2}d.$ and £1 3*s.* 5*d.*
11. 2,604 „ $1\frac{3}{4}d.$ and £2 5*s.* 6*d.*
12. 444 „ $6\frac{1}{2}d.$ and £3 15*s.* 4*d.*
13. 3,072 „ $9\frac{1}{4}d.$ and 1*s.* $11\frac{1}{2}d.$
14. 6,254 „ $\frac{1}{2}d.$ and £4 6*s.* 9*d.*
15. 6,072 „ 8*s.* 6*d.* and £5 7*s.* 11*d.*
16. 309 „ 9*s.* $11\frac{1}{2}d.$ and £6 3*s.* $7\frac{1}{4}d.$
17. 60 „ £5 7*s.* 6*d.* and 3*s.* $9\frac{1}{4}d.$
18. 4,269 „ 9*d.* and £6 3*s.* 4*d.*
19. 555 „ 8*s.* $4\frac{1}{2}d.$ and £17 17*s.*
20. 10,060 „ 4*s.* $4\frac{1}{2}d.$ and 2*s.* $2\frac{1}{2}d.$
21. 7,954 „ $3\frac{3}{4}d.$ and 6*s.* $11\frac{1}{2}d.$
22. 860 „ $4\frac{1}{4}d.$ and £7 7*s.* 9*d.*
23. 130 „ 7*s.* $11\frac{3}{4}d.$ and 7*s.* $9\frac{1}{2}d.$
24. 1,000 „ £5 5*s.* 11*d.* and £27 19*s.* 11*d.*
25. 3,000 „ £1 6*s.* $9\frac{1}{2}d.$ and £1 17*s.* $5\frac{1}{4}d.$
26. 250 „ £17 16*s.* 11*d.* and £19 13*s.* 11*d.*
27. 61,113 „ 2*s.* $1\frac{1}{2}d.$ and 3*s.* $7\frac{1}{4}d.$
28. 8,250 „ 16*s.* 10*d.* and 17*s.* $5\frac{1}{4}d.$

29. 437 articles at 13*s.* 7½*d.* and 14*s.* 6¼*d.*

30. 999 „ £9 9*s.* 9*d.* and £11 11*s.* 11*d.*

31. Find the cost of 1½ cwt. of beef at 11½*d.* per lb.

32. 746 articles at 5¾*d.* each.

33. 235 articles at £8 4*s.* 10*d.* each.

34. A man earns 5*s.* 6½*d.* per day; what does he earn in 313 days?

35. What must be paid for 79 horses at £39 17*s.* 8¼*d.* each?

36. A bankrupt pays 6*s.* 9*d.* in the £; what will he pay on a debt of £750 10*s.* 6*d.*?

37. Find the cost of 15,134 articles when one costs £59 14*s.* 11¼*d.*

38. What is the amount of 526 subscriptions of 2½ guineas each?

39. Find the value of 7,263 articles at £2 12*s.* 4½*d.* each.

40. A merchant invests 1,000 guineas; if he makes a profit of 3*s.* 9*d.* on each £, what are his total profits?

41. Find the cost of 609 articles at £7 18*s.* 9¼*d.* each.

42. What amount of tax will be paid on £550 at 5½*d.* in the £?

43. Find the value of 8,097 articles at £7 17*s.* 10¼*d.* each.

44. Find the value of a chest of tea weighing 1 cwt. 10 lbs. at 2*s.* 6½*d.* per lb.

45. Find the cost of 847 tons at £1 10*s.* 8¾*d.* per cwt.

46. What should a company charge to carry goods 240 miles at £3 17*s.* 9½*d.* a mile?

47. Find the dividend on £4,060 at 15*s.* 9*d.*

48. Find the cost of 1,111 things at £11 11*s.* 11¾*d.* each.

49. How much money will be required to pay the wages of 46 men for a week at 5*s.* 6½*d.* per day?

50. Find the value of 7,819 articles at 17*s.* 9¾*d.* each.

51. What is the rent on 5,270 acres at an average of £2 19*s.* 7½*d.* per acre?

52. What is the value of 15,638 things at 17*s.* 9¾*d.* each?

53. A bankrupt pays 12*s.* 9*d.* in the £; what will be received on a debt of £3,754 10*s.*?

54. 3,158 articles at £15 3*s.* 10½*d.* each.

55. Find the value of 7 cwt. of sugar at 3¾*d.* per lb.

56. Find the value of 7,487 articles at 13*s.* 11¼*d.* each.

57. 273⅓ articles at £3⅜.

58. Required the value of 2,937½ articles at 10¾*d.* each.

59. Find the value of 3,276⅐ at £1 13*s.* 2¼*d.* each.

60. Find the value of 16 doz. pairs of boots at 19*s.* 7½*d.* a pair.

61. Find the value of 7,698⅝ articles at £2 11*s.* 7½*d.* each.

62. Find the cost of 5 cwt. of tea at 2*s.* 6½*d.* a lb.

63. What will be the cost of building a wall 65¾ yds. long at 13*s.* 4*d.* a yard?

64. Find the value of 305⅔ cwt. of sugar at 3½ guineas per cwt.

65. Find the cost of 70 doz. yds. of braid at 2 yds. for 1½*d.*

66. Find the cost of 8,976⅘ articles at eight shillings and eleven pence halfpenny each.

67. Find the value of 217¼ articles at £2 17*s.* 7½*d.*

Section II.—Compound Practice.

In Simple Practice we have to find the value of a quantity expressed in *one* denomination, as, 250 lbs. of sugar at 3½*d.* per lb. In Compound Practice we have to find the value of a quantity expressed in *several* denominations, as, 2 cwt. 2 qrs. 15 lbs. of sugar at £1 8*s.* per cwt.

Ex. 1. Find the cost of 1 lb. 8 ozs. of tea at 2*s.* 6*d.* per lb.

1 lb. costs . . . . . 2*s.* 6*d.*
8 ozs. or ½ lb. costs half of 2*s.* 6*d.* = 1*s.* 3*d.*
1 lb. 8 ozs. costs . . . . . 3*s.* 9*d.*

The example is best written thus :—

Of 1 lb.	8 ozs.	= ½	2*s.* 6*d.* = value of 1 lb. 0 oz.	
			1 3 = „ 0 lb. 8 ozs.	
			3*s.* 9*d.* = value of 1 lb. 8 ozs.	

Ex. 2. Find the cost of 4 tons 6 cwt. 2 qrs. 14 lbs. at £10 10*s.* per ton.

Of 1 ton.	5 cwt.	= ¼	£10	10	= value of 1 ton.					
				4			tons.	cwt.	qrs.	lbs.
			£42	0	0	= value of 4	0	0	0	
Of 5 cwts.	1 cwt.	= ⅕	2	12	6	= „	0	5	0	0
Of 1 cwt.	2 qrs.	= ⅛	0	10	6	= „	0	1	0	0
Of 2 qrs.	14 lbs.	= ¼	0	5	3	= „	0	0	2	0
			0	1	3¾	= „	0	0	0	14
			£45	9	6¾	= value of 4 tons 6 cwt. 2 qrs. 14 lbs.				

Ex. 3. Find the rent of 12 acres 3 roods 25 poles at £2 12*s.* 8*d.* per acre.

Of 1 acre	2 roods	= ½	£2	12	8	= rent of 1 acre.				
				12			ac.	rd.	po.	
			£31	12	0	= „	12	0	0	
Of 2 roods	1 rood	= ¼	1	6	4	= „	0	2	0	
Of 1 rood	20 poles	= ⅛	0	13	2	= „	0	1	0	
Of 20 poles	5 poles	= ¼	0	6	7	= „	0	0	20	
			0	1	7¾	= „	0	0	5	
			£33	19	8¾	= rent of 12 acres 3 roods 25 poles.				

Ex. 4. What is the cost of excavating 200 cub. yds. 9 cub. ft. 1,296 cub. in. of rock at £1 10*s.* per cub. yd. ?

Of 1 cub. yd.	9 cub. ft.	$=\frac{1}{3}$	£1	10	0	=cost of 1 cub. yd.				
					200		c. yd.	c. ft.	c. in.	
			£300	0	0	=„	200	0	0	
Of 9 cub. ft.	1296 cub. in.	$=\frac{1}{13}$	0	10	0	=„	0	9	0	
			0	0	10	=„	0	0	1,296	
			£300	10	10	=cost of 200 cub. yds. 9 cub. ft. 1,296 cub. in.				

EXERCISES IN COMPOUND PRACTICE.

1. 5 cwts. 1 qr. 14 lbs. at £1 10*s.* 4*d.* per cwt.
2. 2 cwts. 3 qrs. 7 lbs. at £2 15*s.* 6*d.* per cwt.
3. 3 tons 15 cwt. 3 qrs. 7 lbs. at £1 12*s.* per cwt.
4. 17 tons 3 cwt. 3 qrs. at 20 guineas per ton.
5. 13 lbs. 10 ozs. 10 dwts. at £5 per lb.
6. 3 lbs. 2 ozs. 15 dwts. at 6*d.* per oz.
7. 9,072 lbs. 2 ozs. 15 dwts. at 39*s.* 7*d.* per lb.
8. 31 galls. 2 qts. at 17*s.* 9*d.* per gall.
9. 207 galls. 3 qts. 1 pt. at 4*s.* 9½*d.* per qt.
10. 87 yds. 1 qr. 3 nls. at 13*s.* 10*d.* a yd.
11. 3 qrs. 3 nls. at 4 guineas per yd.
12. 2 furlongs 25 poles at £4 10*s.* per mile.
13. What should be paid for 5 cwt. 3 qrs. 14 lbs. at 9*s.* 6½*d.* per qr. ?
14. Find the cost of 9 reams 15 quires of paper at 7*s.* 6*d.* per ream.
15. What is the cost of 5 bush. 2 pks. 1 gall. of oats at 5*s.* 6*d.* per bush. ?
16. Find the cost of 7 tons 15 cwt. 1 qr. 17 lbs. at 4*s.* 8*d.* per cwt.
17. What will a man earn in 17 wks. 4 days 7 hrs. at £2 10*s.* a week (6 days a week and 10 hrs. a day) ?
18. Find the price of 39 qrs. 3 bush. 2 pks. at £1 4*s.* 2*d.* per bush.
19. Find the value of 6 tons 12 cwt. 3 qrs. 10 lbs. 8 ozs. at £3 14*s.* 8¼*d.* per cwt.
20. Find the cost of 7 galls. 3 qts. 1 pt. of milk at 2½*d.* per qt.
21. Find the cost of 11 tons 17 cwt. 1 qr. 19 lbs. at £7 per ton.
22. What is the value of 27 yds. 1 ft. 10 in. of silk at 5*s.* 6*d.* a yd. ?
23. Find the cost of 11 ozs. 17 dwts. at 10*s.* per lb.
24. What is the value of 3 cwt. 2 qrs. 16 lbs. at £3 7*s.* 8*d.* per cwt. ?
25. Find the cost of mowing 59 acres 3 roods 16 poles at 5*s.* per acre.

26. If a builder charges £310 3*s.* 4*d.* for erecting a wall a mile long, what should be his account for enclosing an estate 25 miles 3 fur. 110 yds. round, at the same rate?

27. Find the value of 319 cwt. 3 qrs. 16 lbs. at £2 12*s.* 6*d.* per cwt.

28. Find the cost of 12 dwts. 20 grs. at £3 per oz.

29. Find the cost of making a road 19 miles 2 fur. 88 yds. long at £25 8*s.* 4*d.* a mile.

30. What is the rent of a farm of 317¾ acres at £2 5*s.* per acre?

31. Find the cost of reaping 37 acres 1 rood 15 per. of wheat at half-a-guinea an acre.

32. Find the cost of 39 yds. 3 qrs. 3 nls. of cloth at 5*s.* a yd.

33. Find the cost of 16 cwt. 3 qrs. 17½ lbs. at £42 per ton.

34. What would be the expense of constructing a railway 115 miles 3 fur. long at £5,000 per mile?

35. What is the cost of 22 qrs. 4 bush. 3 pks. at 5*s.* 8¼*d.* per bush.?

36. What is the value of 14 lbs. 8 ozs. at £1 8*s.* per cwt.?

37. Find the cost of 11 ozs. 16 dwts. 16 grs. of gold at £3 17*s.* 10½*d.* per oz.

38. What must a grocer pay for 12 cwt. 2 qrs. 17 lbs. of sugar at £2 2*s.* per cwt.?

39. Find the drainage tax on 780 acres 3 roods 19 poles at 10*s.* 6*d.* per acre.

40. The planting of a rood of ground cost £2 10*s.* 6*d.*; how much must be paid for planting 3 acres 25 poles at the same rate?

41. Find the price of a silver snuff-box weighing 2 ozs. 12 dwts. 12 grs. at 15*s.* 6*d.* per oz.

42. Find the value of 27 yds. 3 qrs. 1 nl. of cloth at £2 5*s.* 4*d.* per yd.

43. Which will cost the more, and how much, 5 cwt. 2 qrs. 17 lbs. at £5 10*s.* per cwt., or 7 cwt. 3 qrs. 20 lbs. at £4 10*s.* per cwt.?

44. Find the cost of constructing a wire fence 2 fur. 165 yds. long at 1*s.* 6½*d.* per yd.

45. Find the expense of mending a road 97 miles 1 fur. 6 chains 18 yds. at £1 16*s.* 8*d.* per mile.

46. Find the value of 2 qrs. 8 lbs. of sugar at £2 4*s.* 4*d.* per cwt.

47. Find the price of 3 lbs. 5 ozs. 14 dwts. 12 grs. at 17*s.* 6*d.* per oz.

48. If gold is valued at £3 15*s.* 6*d.* per oz., find the value of 8 lbs. 7 ozs. 10 dwts.

49. What is the freight of 7 trucks, each 6 tons 3 cwts. 2 qrs., at £1 6*s.* 7½*d.* per ton?

50. What is the value of 1 qr. 23 lbs. 2 ozs. 4 drs. of borax at 1*s.* 4*d.* a lb.?

51. Find the rent of 140 acres 3 roods 25 poles of land at £1 10*s.* per acre.

52. What is the value of 7 sq. yds. 8 sq. ft. and 54 sq. in. at 13*s.* 6*d.* per sq. yd. ?

53. Find the rent of a farm of 279 acres 3 roods 36 poles at £2 11*s.* 8*d.* per acre.

54. What is the value of 10 lbs. 11 ozs. 16 dwts. 16 grs. of gold at £3 17*s.* 10½*d.* per oz. ?

55. What will be the rent of 1 acre 2 roods 8 poles of land if 3 acres are let for £9 5*s.* ?

56. Find the cost of 7 poles 27 sq. yds. at the rate of £2 7*s.* per sq. yd.

57. Find the value of 301 cwt. 2 qrs. 16 lbs. at £6 6*s.* 7*d.* per cwt.

58. Find the value of 44 acres 2 roods 25 poles at £55 17*s.* 6½*d.* an acre.

59. Find the value of 2 roods 19 poles 12 yds. of land at £80 13*s.* 4*d.* per acre.

60. What will a man earn in 5 yrs. 30 wks. 5 days at £117 a year ?

Section III.—Bills of Parcels.

Bills of Parcels consist of two or more of such little problems as are given in the preceding sections, arranged in a particular form—called a *bill*—so that the amounts may be shown in a column for convenience of addition, &c.

Ex. 1. Mrs. Richards made the following purchases at Peter Robinson & Co's.—25 yds. of silk at 5*s.* 6*d.*, 19 yds. French merino at 3*s.* 9*d.*, 30 yds. of velveteen at 3*s.* 7d., and 72 buttons at 2 for 1½*d.*; make a bill of the transaction.

London, *Jan.* 3*rd*, 1878.

Mrs. Richards,

Bought of Peter Robinson & Co.,
Silk Mercers, &c.

	£	s.	d.
25 yds. of silk at 5*s.* 6*d.* a yd. . .	6	17	6
19 „ merino at 3*s.* 9*d.* a yd. .	3	11	3
30 „ velveteen at 3*s.* 7*d.* a yd.	5	7	6
72 buttons at 2 for 1½*d.* . . .		4	6
	£16	0	9

Received, *Jan.* 3*rd*, 1878,
Peter Robinson.

Ex. 2. I bought from the butcher on Jan. 1st—6 lbs. 10 ozs. of mutton at 1*s.* 2*d.* a lb.; Jan. 2nd, 12 lbs. 9 ozs. of beef at 1*s.* per lb.; Jan. 3rd,

11 ozs. steak at 1*s.* 4*d.* a lb. ; and Jan. 4th, loin of mutton 5 lbs. 10 ozs. at 10*d.* a lb. ; make out the bill.

LONDON, *Jan. 8th*, 1878.

G. F. TOMS, Esq.,

Bought of E. JONES,
Butcher.

1878		£	s.	d.
Jan 1.	6 lbs. 10 ozs. mutton at 1*s.* 2*d.* .		7	8½
„ 2.	12 lbs. 9 ozs. beef at 1*s.* . . .		12	6¾
„ 3.	11 ozs. steak at 1*s.* 4*d.* . . .			11
„ 4.	5 lbs. 10 ozs. mutton at 10*d.* . .		4	8¼
		£1	5	10¾

Paid, *Jan. 8th*, 1878,
E. JONES.

NOTE.—Most of the food we purchase is sold at so much per lb. weight. It is easy enough for the grocer to weigh exactly a lb. of tea or coffee ; but it is not so easy for the butcher to cut off a lb. of steak, or the cheeseman a lb. of cheese. It is pretty certain that in the latter cases there will be some ozs. too much or too little. Bacon, cheese, butter, and all kinds of meat vary in price from 4*d.* to 2*s.* per lb., and we should be able readily to calculate the price of the odd ozs. at these rates per lb.

16 ozs. = 1 lb.

Therefore—

16*d.* per lb. = 1*d.* per oz.
16 ÷ 2 = 8*d.* „ = ½*d.* „
16 ÷ 4 = 4*d.* „ = ¼*d.* „
8 + 4 = 12*d.* „ = ¾*d.* „
24*d.* „ = 1½*d.* „
and 6*d.* „ = ¾*d.* for 2 ozs.
„ 18*d.* „ = 2¼*d.* „
6*d.* + 4*d.* = 10*d.* „ = 1¼*d.* „
8*d.* + 6*d.* = 14*d.* „ = 1¾*d.* „
12*d.* + 8*d.* = 20*d.* „ = 2½*d.* „
12*d.* + 10*d.* = 22*d.* „ = 2¾*d.* „

Intermediate prices per lb. do not give exact prices per oz., and we must calculate to the nearest price we can.

EXERCISES IN BILLS OF PARCELS.

1. Bought 2 doz. pairs of stockings at 2*s.* 8*d.* per pair, 8 pairs of gloves at 2*s.* 9½*d.* per pair, and 11 yds. of silk at 8*s.* 3½*d.* per yd. ; make out the bill.

2. Make a bill of the following :—18 plates at 9½*d.* each, 3 doz. tumblers

at 5¾*d.* each, 4¼ doz. small plates at 2¾*d.* each, and 1½ doz. wine-glasses at 9*s.* per doz.

3. Make a bill of the following :—13 lbs. of flour at 2¾*d.*, 18 loaves at 5¼*d.*, 12 small loaves at 2¾*d.*, and 81 buns at 9*d.* per doz.

4. Make out an invoice for 12½ doz. brushes at 3*s.* 7½*d.* each, 162 plates at 4*s.* 11½*d.* per doz., and 3 gross of spoons at 4*s.* 4*d.* per doz.

5. Mr. John Wilkins buys of John James, butcher—7½ lbs. of beef at 11½*d.* per lb., 9 lbs. 6 ozs. of mutton at 10*d.* per lb., 8 lbs. 14 ozs. of veal at 1*s.* 2*d.* per lb., and 7 lbs. 5 ozs. of lamb at 1*s.* per lb. ; make out the bill.

6. 15 loaves at 7*d.* each, 6 doz. biscuits at 6*d.* a doz., 14 rolls at 6*d.* a doz., 12 cheesecakes at 1½*d.* each, 9 doz. rusks at 10*d.* a doz., and 13 lbs. of ginger nuts at 9*d.* per lb. ; make out the bill.

7. I owe the greengrocer for ½ cwt. and 21 lbs. of potatoes at 7 lbs. for 6*d.*, and for 3 qrs. 17 lbs. at ¾*d.* per lb. I tender a £5 note to pay the bill ; what change ought I to receive ?

8. Make out the following bill :—8 doz. and 8 eggs at 1*s.* 1½*d.* per doz., 17½ lbs. of butter of 1*s.* 3*d.* per lb., 6¼ lbs. of cheese at 9*d.* per lb., 13 lbs. of soap at 4¼*d.* per lb., 14 lbs. of bacon at 9½*d.* per lb., and 15 lbs. of lard at 7¾*d.* per lb.

9. Find the total amount of 137 lbs. of tea at 3*s.* 10*d.* per lb., 237 lbs. at 3*s.* 5*d.* per lb., 420 lbs. at 2*s.* 11*d.* per lb., and 315 lbs. at 2*s.* 6*d.* per lb.

10. Make out the following bill :—4 lbs. of tea at 3*s.* 6*d.* per lb., 12 lbs. of sugar at 5*d.*, 7 lbs. of butter at 1*s.* 1½*d.*, 10½ lbs. of rice at 3½*d.*, 2 lbs. of currants at 8*d.*, and find what change I ought to receive out of £2.

11. Correct, if necessary, the following invoice :—

	s.	*d.*
40 lbs. of soda at 11*s.* 8*d.* per cwt.	=4	2
14 lbs. of soap at £3 2*s.* 8*d.* „	=4	1
8 doz. eggs at ¾*d.* each	=1	2
	9	5

12. Make out the following bill :—4 lbs. of tea at 4*s.* 3*d.*, 6 lbs. of sugar at 4¾*d.*, 7 lbs. of sugar at 6½*d.*, 11 lbs. of candles at 10*d.*, and 6 lbs. at 8½*d.*, 2 galls. of oil at 4*s.* 9*d.*, and 7 lbs. of soap at 7½*d.*

13. Make out an invoice for 7 quarters 3 pecks of wheat at 7*s.* a bushel, 5 quarters 7 bushels 2 pecks of oats at £1 12*s.* per quarter, 3 bushels 3 pecks 1 gallon of barley at 5*s.* per bushel, and 15 pecks of peas at 3*s.* 6*d.* per bushel.

14. Make out a bill for 37,000 ft. of gas at 4*s.* 6*d.* per 1,000 ft., one year's rent of meter at 1*s.* 6*d.* per quarter, and man's time for 13 hours at 7½*d.* per hour.

15. A lady bought at a draper's shop 23 yds. of silk at 5*s.* 6*d.* a yd., 33 yds. of calico at 11½*d.* a yd., 2 doz. pairs of gloves at 2*s.* 7*d.* a pair, and 10 yds. of ribbon at 1*s.* 10*d.* a yd. The draper made the bill amount to

£11 17*s.* 8½*d.* Did he charge too much or too little, and what was the amount of the error?

16. The following is a leaf from my butcher's book; examine and correct if necessary :—

1878		£	*s.*	*d.*
	Brought forward	1	17	5
Jan. 3	Mutton chops, 3 lbs. 6 ozs. @ 1*s.* 2*d.*		3	11¼
„ 5	Beef, 9 lbs. 11 ozs. @ 1*s.*		9	9¼
„ 7	Loin mutton, 10 lbs. 12 ozs. @ 1*s.* 2*d.*		11	11½
„ 9	Steak, 3 lbs. 15 ozs. @ 1*s.* 4*d.*		5	3
„ 10	3 kidneys @ 3½*d.* each			10½
„ 12	Ribs of beef, 15 lbs. 8 ozs. @ 11½*d.*		14	10¼
„ 14	Lamb cutlets, 2 lbs. 2 ozs. @ 1*s.* 6*d.*		3	2¼
„ 17	Leg of pork, 8¼ lbs. @ 10*d.*		6	8
„ 18	Calf's heart, 1*s.* 5½*d.*		1	5½
„ 19	Shoulder of mutton, 10½ lbs. at 11½*d.*		10	0¼
„ 22	Suet, 10 ozs. @ 1*s.*			7
		£5	3	11¾

17. Make out a bill for 5 chests of tea, each 2 qrs. 11 lbs., at 3*s.* 8*d.* per lb., 3 hhds. of sugar, each 13 cwt. 2 qrs., at 39*s.* 4*d.* per cwt., 3 cwt. 1 qr. 14 lbs. of coffee at 49*s.* 6*d.* per cwt., and 14 cwts. 2 qrs. 3 lbs. of cheese at 5½*d.* per lb.

18. Mrs. Jones buys of Mr. J. Hinds beef to the value of 11*s.* 5½*d.* at 10*d.* per lb., mutton to the value of 10*s.* 0¾*d.* at 10½*d.* per lb., veal to the value of 11*s.* 9¾*d.* at 1*s.* per lb., and lamb to the value of 10*s.* 2½*d.* at 1*s.* 2*d.* per lb.; make out the bill, putting in the different weights.

19. A grocer bought tea to the value of £95 16*s.* 8*d.* at £10 14*s.* 8*d.* per cwt., sugar to the value of £18 15*s.* at £1 17*s.* 4*d.* per cwt., and rice to the value of 19*s.* 4¾*d.* at 3½*d.* per lb.; make out the bill, putting in the proper weights.

20. Find the amount of the following bill:—12½ tons of hay at £3 15*s.* per ton, 8¼ qrs. of beans at 2 gs. a qr., 11¾ qrs. of oats at £1 6*s.* a qr., and 5 sacks of flour at 27*s.* 6*d.* a sack.

Miscellaneous Exercises.

1. Find the value of 5,198 articles at 14*s.* 7*d.* each.
2. What will be the cost of 1,275⅔ yds. of rep at 2*s.* 3¾*d.* a yd.?
3. How many persons may receive 4*s.* 4*d.* out of £537 11*s.*?
4. If a man buys 100 tons of coals at 14*s.* 8*d.* per ton, and sells them at 11*d.* per cwt., how much will he gain?
5. Find the value of 5,721 ft. at 16*s.* 9*d.* per ft.

6. What will be the cost of 2,395$\frac{7}{8}$ articles at £2 17*s.* 10*d.* each?

7. Find the cost of a telegram consisting of 425 words at £1 12*s.* 6*d.* for 20 words.

8. Make out the following bill:—5 lbs. of tea at 4*s.* 4*d.*, 12 lbs. of loaf sugar at 5$\frac{1}{4}$*d.*, 8 lbs. of rice at 3$\frac{1}{2}$*d.*, 3 lbs. of currants at 5$\frac{1}{4}$*d.*, 4 lbs. of coffee at 1*s.* 8*d.*, and 3 lbs. of raisins at 1*s.* 2*d.*

9. Find the cost of 15 qrs. 7 bush. 3 pks. 3 qts. at £1 17*s.* 4*d.* a qr.

10. How many miles can I travel in a train for £2 2*s.* at the rate of 1$\frac{1}{8}$*d.* per mile?

11. What amount must be added to £490 16*s.* 9*d.* to make it exactly divisible by 2,070?

12. Find the value of 319 cwt. 3 qrs. 16 lbs. at £2 12*s.* 6*d.* per cwt.

13. Find (by practice) the dividend on £6,050 at 17*s.* 9$\frac{1}{2}$*d.* in the £.

14. 25,041 railway tickets were sold at a station, $\frac{1}{3}$ were sold at 8*d.* each, and the rest at 4*d.*; what was the amount received for the tickets?

15. A dealer bought 481 chairs at £1 9*s.* 5$\frac{1}{2}$*d.* each, and sold them at 30*s.* each; what was his total gain?

16. What will be the purchase money of an estate containing 191 acres, 3 roods, 37 poles at £42 3*s.* 4*d.* an acre?

17. Find the cost of carpeting a room 20 ft. sq. with carpet 2 ft. wide at 5*s.* 3*d.* per yd.

18. Find the dividend on £5,398 at 14*s.* 3*d.* in the pound.

19. Find the value of 102 cwt. 3 qrs. 8 lbs. at £1 19*s.* 8*d.* per cwt.

20. How many subscriptions of £5 17*s.* 4$\frac{1}{2}$*d.* each will it take to make up £223 0*s.* 3*d.*?

21. Find the cost of 16 tons 6 cwt. 12 lbs. at £8 9*s.* 9*d.* per cwt.

22. Write out an invoice for 17 yds. of calico at 7$\frac{3}{4}$*d.* per yd., 19$\frac{1}{2}$ yds. of trimming at 2*s.* 7$\frac{1}{4}$*d.* per yd., and 19$\frac{1}{2}$ yds. of flannel at 1*s.* 10$\frac{1}{2}$*d.* per yd.

23. Find the value of 23 cwt. 3 qrs. 8 lbs. of cheese at £3 19*s.* 11*d.* per cwt.

24. Make a bill of the following:—15$\frac{1}{4}$ lbs. of beef at 1*s.*, 16$\frac{1}{2}$ lbs. of mutton at 10*d.*, 11 ozs. of steak at 1*s.* 4*d.*, and 10 ozs. of suet at 1*s.*

25. A boy earns 18*s.* 6*d.* per week, and spends £20 per year; what does he save per week?

26. Find the rent for 6 mo. 3 wks. 2 days at £3 0*s.* 6*d.* a month.

27. Find the amount of the following:—12 ozs. of tea at 3*s.* 6*d.* per lb., 14 lbs. of sugar at 30*s.* per cwt., 8 doz. eggs at 1$\frac{3}{4}$*d.* each, 27$\frac{1}{2}$ lbs. of bacon at £5 1*s.* 6*d.* per cwt., and 17 ozs. of butter at 1*s.* 8*d.* per lb.

28. How many yds. of cloth at 2*s.* 2$\frac{1}{2}$*d.* per yd. should be given for 26 yds. 2 qrs. at 2*s.* 9$\frac{1}{4}$*d.* a yd.?

29. What is the area of a lake which measures, on the average, 3 miles long and 7 furlongs broad?

30. A bankrupt, having £645 5*s.* 3$\frac{1}{2}$*d.*, can pay 7*s.* 8$\frac{1}{2}$*d.* in the £; what is the amount of his debts?

31. There are 3 partners in a business; the senior partner has 5 shares, the second 2, and the junior partner 1 share; if the profits are 800 gs., how much will each partner receive?

32. If half-a-dozen eggs are sold for $3\frac{1}{2}d.$, what should be paid for 27, and also for 99 eggs?

33. A grocer bought $3\frac{1}{2}$ cwt. of sugar at £1 15*s.* per cwt.; he sold 1 cwt. at 4*d.* a lb., and the remainder at $4\frac{1}{2}d.$ a lb.; what was his profit?

34. A bankrupt's liabilities amount to £15,007; he pays his creditors 7*s.* $11\frac{1}{2}d.$ in the £; find the value of his assets.

35. Divide £340 among 4 persons, A, B, C, and D, giving A twice as much as B, B twice as much as C, and C twice as much as D.

36. How many sq. ft. are there in a grass plot which measures 27 yds. long and 21 yds. broad?

37. If 1,000 pilchards cost 10*s.* 5*d.*, what quantity can be bought for $3\frac{1}{4}d.$, and what quantity for 2*s.* $5\frac{1}{4}d.$?

38. Make out a bill for $\frac{3}{4}$ lb. of butter at 1*s.* 3*d.* per lb., $3\frac{1}{4}$ lbs. of candles at 9*d.* per lb., $1\frac{1}{4}$ lb. of coffee at 1*s.* 6*d.* per lb., and $\frac{1}{2}$ lb. tea at 3*s.* 10*d.* per lb.

39. How much tea at 3*s.* 6*d.* per lb. should be exchanged for half a cwt. of sugar at $4\frac{1}{2}d.$ per lb.?

40. The rates of a parish amount to £2,250, and the ratable property is valued at £20,000; how much is the rate in the £?

41. A man buys coals at 1*s.* $1\frac{1}{2}d.$ per cwt., and sells them at 25*s.* per ton; how many tons must he sell to gain £1 12*s.* 6*d.*?

42. If 178 lbs. of coffee are given in exchange for 1 cwt. 4 lbs. 8 ozs. of tea worth 2*s.* 2*d.* per lb., find the price of the coffee per lb.

43. The annual rent of a parish is £13,793, and the poor rate is 2*s.* $7\frac{1}{2}d.$ in the £; to how much does the rate amount?

44. Find (by practice) the gross rental of the following 2 farms:—263 acres 38 poles at £1 11*s.* 6*d.* per acre, and 457 acres and 39 poles at £1 5*s.* per acre.

45. What is the expense of digging a ditch of which the solid contents are 5,755 cub. yds. at the rate of 1*s.* $7\frac{3}{4}d.$ per cub. yd.?

46. A bankrupt owes £3,600; what must his assets be worth to produce 17*s.* 3*d.* in the £?

47. It takes 36 yds. of carpet to cover a room 18 ft. long and 12 ft. wide; required the width of the carpet.

48. A rabbit is worth $8\frac{1}{2}d.$, a hare 3*s.* $6\frac{1}{2}d.$; how many scores of rabbits will be equal in value to $13\frac{1}{2}$ doz. hares?

49. A undertakes to give to a charity one guinea for every half-crown B gives; B gives £1 0*s.* 8*d.*; what must A give?

50. If a man spends £1 17*s.* $10\frac{1}{2}d.$ a day out of an annual income of £700, how much will he have saved in 9 years of 365 days in length?

51. What will be the cost of papering a room 15 ft. long, 13 ft. wide,

and 11 ft. high with paper 2 ft. wide at 3*d.* per yd. (omit doors and windows)?

52. If the ratable value of a parish is £5,795, what rate is required to produce £537 4*s.* 10¾*d.*?

53. After paying an income tax of 7*d.* in the £, I have £248 10*s.* 8*d.* left.; what is my gross income?

54. How much money shall I have left if, having been paid the average price of 47½ gs. each for 17 horses, I buy 374 qrs. 7 bush. 2 pks. of wheat at £2 3*s.* 8*d.* a qr.?

CHAPTER IV.

RATIO AND PROPORTION.

One desk is 12 ft. long, another is 3 ft. long; compare their lengths. We may do this in two ways; we may say that one is 9 ft. longer than the other; or that one is *four times* as long as the other. The former is a comparison by *subtraction*, the latter by *division*. Ratio deals only with comparison by division. We may ask the question, "What *relation* does the length of one desk bear to the length of the other?" and we answer, "The one is four times longer than the other." That is, the *relation*, or RATIO, of 12 ft. to 3 ft. is the abstract number 4; and we get the number 4 by dividing 12 ft. by 3 ft.

In the same way the ratio—

Of 12*d.* to 4*d.* = 3.
Of 20 cwts. to 5 cwts. = 4.
Of 27 lbs. to 3 lbs. = 9.
Of 4*d.* to 12*d.* = ⅓.
Of 5 cwts. to 20 cwts. = ¼.
Of 3 lbs. to 27 lbs. = ⅑.

Ratio can only subsist between quantities of like kinds; we cannot compare 5 stones with 10 cows, or 20 pence with 15 yds.; but we can compare 5*s.* with £1 or 5*d.*, because we can reduce to the same denomination.

Definition.—"*Ratio is the relationship which subsists between two quantities of like kinds, with respect to the number of times one contains the other.*"

From this we see that a ratio cannot be a *concrete* number; it is an *abstract* number derived from a comparison of two magnitudes.

It is not usual in practice to perform the actual division, and so find the ratio; we simply indicate the ratio by a symbol, just as we express a fraction by a symbol.

To find the ratio of 12 to 4, we divide the *first* number by the *second*, and we may indicate this division as $12 \div 4$, or $\frac{12}{4}$. In practice we use the first sign $\div$, but omit the line, so that the ratio of 12 to 4 is shown as 12 : 4. It is of course equally correct to write it $12 \div 4$, or $\frac{12}{4}$.

The first *term* of the ratio is called the *antecedent*, and the second the *consequent*, so that—

$$\frac{\text{antecedent}}{\text{consequent}} = \frac{\text{numerator.}}{\text{denominator.}}$$

We have seen that we may multiply the numerator and denominator of a fraction by the same number without altering its value; hence in the same way we may multiply or divide the antecedent and consequent of a ratio by the same number without altering the value of the ratio.

Thus $\frac{2}{4} = \frac{6}{12}$

Hence 2 : 4 = 6 : 12

We see that this is true in another way, viz., by finding the actual ratio in each case to be the same, that is $\frac{1}{2}$.

This *equality of ratios* is indicated thus:—

2 : 4 :: 6 : 12

And is read—

2 bears the same relation to 4 as 6 does to 12.

Or, shortly—

2 is to 4 what 6 is to 12 (*a*)

Again $\frac{4}{2} = \frac{12}{6}$

or 4 : 2 :: 12 : 6 (*b*)

It will be seen from these expressions of equal ratios, that the *first* term is as many times greater or less than the *second*, as the *third* term is greater or less than the *fourth*. In (*a*) the *first* term is half the *second*, and so is the *third* half the *fourth*. In (*b*) the *first* term is twice the *second*, and so is the *third* term twice the *fourth*.

Definition.—*Four terms are said to be in* PROPORTION *when the first term is as many times greater or less than the second as the third is greater or less than the fourth.*

The end terms are called *extremes*, and the middle terms are called *means*.

RULE.—*When four numbers are in proportion, the product of the extremes is equal to the product of the means.*

Thus—

(1) 3 : 1 :: 9 : 3

and $3 \times 3 = 9 \times 1 = 9$ (*a*)

(2) 36 : 12 :: 6 : 2

and $36 \times 2 = 12 \times 6 = 72$ (*b*)

(3) 4 : 20 :: 2 : 10

and $4 \times 10 = 20 \times 2 = 40$ (*c*)

And so of all proportions.

This rule enables us to find any one of the terms when the other three are given.

Let the proportion be

$$2 : 10 :: 7 : 35$$

(1) Let it be required to find the first term, given the others.

Put x to stand for the unknown term

$$x : 10 :: 7 : 35$$

The product of the means = 70.

Hence the product of the extremes = 70.

But one factor of the extremes is 35, therefore the other must be $\frac{70}{35} = 2$.

(2) Let it be required to find the second term, given the other terms thus—

$$2 : x :: 7 : 35$$

Product of the extremes = 70.

Hence the product of the means = 70.

But one factor of the means is 7, hence the other is $\frac{70}{7} = 10$.

And exactly in the same way we may find the third or fourth terms, given the other three terms.

NOTE.—The doctrine of proportion affects more or less every part of Arithmetic, and it is of very great importance that the student should not pass beyond this point until he thoroughly understands the idea of a *ratio* and of a *proportion*, or equality of ratios. The facts to be borne in mind are these :—

(1) The ratio which one number bears to another is expressed by dividing the first by the second.

(2) Ratio can only subsist between quantities of like kind.

(3) When two ratios are equal, if we equate them we form a proportion, or equality of ratios.

(4) Although the terms of a ratio when concrete must be of like kind, the two ratios in a proportion may be of different kinds; thus, 6 yds. bears the same ratio to 2 yds. as 6*d.* does to 2*d.*, viz. in each case 3.

(5) The product of the extremes is equal to the product of the means; hence, if the product of any two numbers equals the product of any other two numbers, one pair may be used as the extreme terms, and the other as the mean terms of a proposition.

Thus, from $9 \times 6 = 27 \times 2$ we may arrange 4 proportions.

$$9 : 27 :: 2 : 6$$
$$27 : 9 :: 6 : 2$$
$$9 : 2 :: 27 : 6$$
$$2 : 9 :: 6 : 27$$

Ex. 1. Find a fourth proportional to 6, 9, 15.

$$6 : 9 :: 15 : x$$
$$x = 9 \times 15 \div 6 = 22\tfrac{1}{2}. \quad \textit{Ans.}$$

Ex. 2. To what number has 19 the same ratio as 3 has to 9?

$$3 : 9 :: 19 : x$$
$$x = 9 \times 19 \div 3 = 57. \quad \textit{Ans.}$$

Ex. 3. One tradesmen allows 5 per cent. discount for cash, and another 1*d.* in the shilling; compare their respective discounts.

$$\begin{array}{rcl} & \pounds & \pounds \\ \text{1st} = & 5 & : 100 = \\ & \text{pence} & \text{pence} \\ & 1{,}200 & : 24{,}000 = \\ & 12 & : 240 = \\ & 1 & : 20 \\ \text{2nd} = & 1 & : 12 \end{array}$$

The first allows 1*d.* in 20*d.*
The second allows 1*d.* in 12*d.*

EXERCISES IN RATIO AND PROPORTION.

1. What is the ratio of 3*d.* to 1*s.*? Of 15*s.* to 3*s.* 6*d.*? Of 14 lbs. to 6 lbs.? Of 3 ozs. to 7 ozs.?

2. To what number has 15 the same ratio as 10 has to 50? To what number has 3 the same ratio as 15 has to 5?

3. Find the missing term—marked x—in each of the following proportions:—

$$6 : 4 :: x : 12$$
$$9 : x :: 27 : 81$$
$$x : 7 :: 3 : 2$$

4. Complete the following proportions:—

$$5 : 3 :: 120 : x$$
$$160 : 14 :: 80 : x$$
$$1020 : 204 :: 85 : x$$

5. Find a fourth proportional to 15, 27, and 600.

6. Find a fourth proportional to 64, 25, and 576.

7. Find the number which bears the same ratio to 56 as 7 does to 4.

8. A father's age bears the same proportion to his son's as 9 bears to 3; if the father's age is 54, how old is the son?

9. One girl earns 6*d.*, whilst another earns 5*d.*; when the second has earned 190*d.* what will the first have earned?

10. Two boys were picking up potatoes; while one picked up 10 the other picked up 11; when the first boy had eleven score how many had the other?

11. One man travels at the rate of 4 miles an hour, another at the rate of 1 mile in 20 minutes; compare the rates at which they travel.

12. One bookseller allows 2*d.* in the shilling, and another 10 per cent.; compare their respective discounts.

13. A printing machine turns out 3,500 sheets per hour; if its speed be decreased in the ratio of 7 to 4, how many sheets may be printed in a day of 12 hours?

14. If the price of coal rises in the ratio of 3 to 5, what will be the value of coal which now costs £10?

CHAPTER V.

SIMPLE PROPORTION, OR RULE OF THREE.

Simple Proportion consists of two equal ratios, and in every problem in simple proportion the *two* terms of one ratio are given together with *one* term of the other ratio, and we are required to find the other term of the second ratio. Thus *three* terms are given, and the fourth has to be found. Hence simple proportion is sometimes called the "*Rule of Three.*"

Since there can be no ratio between quantities of unlike kinds, in solving problems in simple proportion we must first search for the *two* quantities of like kind to form one ratio; the remaining quantity, with the answer of like kind, will form the second ratio. In the second place, we must arrange the ratios so that the one is equal to the other. This is what is called "*stating the sum.*"

Thirdly, we find the unknown term by the rule that the product of the extremes equals the product of the means.

We will illustrate each of these steps by an example.

1. If 15 horses cost £900, what will 25 horses cost at the same rate?

(1) Which terms form the ratios? Evidently the terms of the one ratio are 15 horses and 25 horses; and of the other £900, and the number of £ in the required answer (call the unknown term x). The ratios then are—

$$(1) \quad 15 : 25$$
$$(2) \quad 900 : x$$

(2) How shall we arrange the terms of the ratios to make the ratios equal, and so to make the correct proportion? Which is the greater, £900 or x? If 15 horses cost £900, 25 horses at the same

rate will cost more. Hence x is greater than £900. And hence we must arrange the first ratio to have its greater term first or second according as x occupies the third or fourth place. That is, if 900 : x is one ratio, 15 : 25 must be the other; and if x : 900 is one ratio, 25 : 15 must be the other.

and $900 : x = 15 : 25$
and $x : 900 = 25 : 15$

And since we may place either ratio first, we can make the following proportions :—

(1) $15 : 25 :: 900 : x$
(2) $900 : x :: 15 : 25$
(3) $x : 900 :: 25 : 15$
(4) $25 : 15 :: x : 900$

From any one of these proportions we may obtain the value of x; but it is usual in practice to place the unknown term in the fourth place, as in (1), and then to arrange the other terms accordingly.

In the given example, if x is the fourth term, then £900 must be the third term, because it is of like kind to x, and must, therefore, be associated with it to form one ratio,

viz. $900 : x$

We have now to consider which is the greater term, £900 or x, and arrange the other terms accordingly.

Thus—

$$15 : 25 :: 900 : x$$

From this proportion we have to find the value of x.

Product of extremes = Product of means.

Hence $15 \times x = 25 \times 900$

But if 15 times $x = 25 \times 900$

$x = 25 \times 900 \div 15$.

This is best arranged in the form of a fraction, thus—

$$x = \frac{25 \times 900}{15}$$

And this fraction we can reduce to its lowest terms by dividing numerator and denominator by the same number.

$$x = \frac{\overset{5}{\cancel{25}} \times \overset{300}{\cancel{900}}}{\underset{3}{\cancel{15}}} = £1{,}500$$
$$1$$

From the foregoing explanation we may deduce certain rules for solving problems in proportion.

1. *Place* x (standing for the required answer) *as the fourth term.*
2. *Place the number which is of like kind to the required answer as the third term.*

3. *Inquire which is the greater of the third and fourth terms, and then arrange the first and second terms accordingly. If the fourth term be greater than the third, the second must be greater than the first; if the fourth term be less than the third, the second must be less than the first.*

4. *Reduce* (if necessary) *the first and second terms to the same denomination.*

5. *Multiply the second and third terms together* (viz. the "means"), *and divide by the first.*

Or, form a fraction with the product of the means for the numerator, and the first term for the denominator. Reduce to lowest terms, and then find the value of the fraction.

Ex. 1. If 8 bushels of wheat be worth £2, what is the value of 3 bushels?

We see here that the terms of one ratio are 8 bushels and 3 bushels, and of the other £2 and £x. Putting x as the 4th term, and £2 as the 3rd term, we have to ask, Which is the greater? Evidently £2, because that is the price of 8 bushels, and £x is the price of 3 bushels. Hence the ratios are—

$$8 \text{ bush.} : 3 \text{ bush.}$$
$$\text{and } £2 : £x$$

And the proportion is—

$$\underset{\text{bush.}}{8} : \underset{\text{bush.}}{3} :: \underset{£}{2} : \underset{£}{x}$$

$$x = \frac{3 \times 2}{8} = \frac{6}{8} \text{ of a } £, \text{ or}$$

$$15s. \quad \textit{Ans.}$$

Note.—We see in the above example that the price is less as the number of bushels is less; that is, generally the *price* varies *directly* with the *number* of the articles. In the same way horses and their provender, men and the work they do, &c., vary directly. Problems of this sort are said to be in *direct* proportion.

Ex. 2. If 4 men can mow a field in 9 days, in how many days will 6 men do it?

Will 6 men require more or less time than 4 men? Less time.

Hence the proportion is—

$$6 : 4 :: 9 : x$$

$$x = \frac{4 \times 9}{6} = \frac{36}{6} = 6 \text{ days.}$$

Note.—In this example more men require less time, and quantities thus related are said to be in *inverse* proportion.

Ex. 3. If a garrison of 1,200 men have provisions to last six months, how long will the provisions last if the garrison be increased by 600 men?

The total number to be provisioned will be 1,800, and the provisions will last a less time.

Hence $1800 : 1200 :: 6 : x$

$$x = \frac{1200 \times 6}{1800} = 4 \text{ mo.}$$

3

Note.—In this example one term, viz. 1,800 men, has to be found before we can "state the sum." Practice alone will give facility in finding terms when not directly given. In the next example a little ingenuity is necessary to find the terms.

Ex. 4. If 4 cows *or* 6 horses can eat the produce of a field in 39 days, in how many days will 6 cows *and* 4 horses eat it up?

4 cows eat as much as 6 horses

$\therefore$ 2 „ „ 3 „

$\therefore$ 6 „ „ 9 „

$\therefore$ 6 cows and 4 horses eat as much as 13 horses; and the question now becomes, If 6 horses can eat the produce in 39 days, how long will it take 13 horses to eat it?

It will take them a shorter time.

horses horses days days

Hence $13 : 6 :: 39 : x$

3

$$x = \frac{6 \times 39}{13} = 18 \text{ days.}$$

Ex. 5. If 51 bushels of corn will feed 24 horses for 9 days, how many bushels will feed them for 15 days?

In this example we appear to have 4 terms given instead of 3, but the *number* of horses has really nothing to do with the question. We may write in the question "a certain number of horses" instead of "24 horses" with equal propriety.

days days bush. bush.

$9 : 15 :: 51 : x$

5 17

$$x = \frac{15 \times 51}{9} = 85 \text{ bushels.}$$

3

Ex. 6. If my income tax amounts to £4 16*s.* 8*d.* when the tax is 4*d.* in the £, what is my income?

In this example all our terms are money, but a close inspection will show that we still have two terms of one kind given, and one term of

another kind. The terms 4*d.* and £4 16*s.* 8*d.* each stand for tax. The £ stands for income.

Hence the second ratio is £1 : £x of income, and £x is greater than £1.

Hence the proportion is—

$$\underset{4d.}{\text{tax}} : \underset{£4\ 16s.\ 8d.}{\text{tax}} :: \underset{£1}{\text{income}} : \underset{£x}{\text{income}}$$

We must reduce the 1st and 2nd terms to the same denomination, in this case fourpenny pieces.

£4 16*s.* 8*d.* = 290 fourpenny pieces.

Hence $1 : 290 :: 1 : x$

$\therefore\ x =$ £290 income.

Ex. 7. (1) Find the interest on £500 for four years at 4 per cent. (2) In what time will £500 amount to £580 at 4 per cent.? (3) At what rate per cent. will £500 amount to £580 in four years?

This is an important group of examples, because they represent the chief problems of Simple Interest. They are worked by proportion.

(1) We must first find the interest for *one* year.

$$\underset{£100}{\text{prin.}} : \underset{£500}{\text{prin.}} :: \underset{£4}{\text{int.}} : \underset{£x}{\text{int.}}$$

$$x = \frac{4 \times 500}{100} = £20$$

£20 × 4 = £80 int. for four years.

(2) Here the whole interest is £580 − £500 = £80, and the interest for *one* year is £20.

$$\underset{20}{\text{int.}} : \underset{80}{\text{int.}} :: \underset{1}{\text{yrs.}} : \underset{x}{\text{yrs.}}$$

$$x = \frac{80}{20} = 4 \text{ years.}$$

(3) Whole interest = £80, viz. for four years.

$\therefore$ Interest for one year = £20.

$$\underset{£500}{\text{prin.}} : \underset{£100}{\text{prin.}} :: \underset{£20}{\text{int.}} : \underset{£x}{\text{int.}}$$

$$x = \frac{10\not{0} \times 2\not{0}}{5\not{0}\not{0}} = 4 \text{ per cent.}$$

Ex. 8. A is 2,640 yards in advance of B; A runs 5 miles an hour, and B 6; in what time will B overtake A?

B runs 6 miles whilst A runs 5 miles.

$\therefore$ B „ 6 yards „ A „ 5 yards.

That is, B gains 1 yd. in 6 yds. But he has to gain 2,640 yds. before he overtakes B.

Gain Gain in
yd. yds. yds. yds.
$\therefore 1 : 2{,}640 :: 6 : x$
$x = 15{,}840$ yds.
$= 9$ miles.

And B goes 9 miles in $1\frac{1}{2}$ hours.

Exercises in Simple Proportion.

1. If 15 men can reap a field in 3 days, how long will 45 men be doing it?
2. If 16 men can build a house in 156 days, how many men will do it in 32 days?
3. If 6 sacks of potatoes are sold for £4 10s., what is the worth of 17 sacks?
4. If 15 ozs. of tea can be bought for 38s., what is the price of 7 ozs.?
5. If 51 bushels of corn will feed 24 horses for 9 days, how many bushels will be required to feed 24 horses for 15 days?
6. If a servant's wages for 12 months is £18 18s., what is that for 7 months?
7. Find the cost of $21\frac{1}{3}$ doz. eggs at 16 for 1s.
8. If I lend a friend £150 for 6 months, for how long should he lend me £50 in return?
9. How far can I travel for 5 guineas, if 18 miles cost 1s. 9d.?
10. How much land can I rent for £210, if 2 acres can be rented for £3?
11. If 12 men can do a piece of work in 15 days, in what time will 5 men do it?
12. If a shopkeeper makes a profit of $3\frac{1}{2}d.$ out of every shilling, what are his profits on £47 10s. 6d.?
13. If the pence in a school amount to 16s. $1\frac{1}{2}d.$ weekly when the children pay $1\frac{1}{2}d.$ each, what will they amount to if they pay $2\frac{1}{4}d.$ each?
14. If $3\frac{1}{4}$ yards of silk cost £1 10s. $7\frac{1}{2}d.$, how many yards can be bought for £56 10s.?
15. Find the cost of $183\frac{1}{2}$ yards of cloth when $2\frac{1}{2}$ yards cost a guinea.
16. If £57 will buy 19 cwts. of sugar, what quantity may be bought for £111?
17. If 375 apples sell for 12s., how many can be bought for £1?
18. What shall I pay for 21 bushels of wheat, when a quarter can be bought for £1 17s. 6d.?

19. If I lend £313 13*s.* 4*d.* for 6 months, for how long ought £235 5*s.* to be lent me in return ?

20. If 45 quarters of wheat cost £127 13*s.* 6*d.*, find the price of 195 quarters.

21. If 8 lbs. of sugar cost 3*s.* 2½*d.*, how many lbs. may be bought for £4 17*s.* 10½*d.* ?

22. Find the price of 1 ton 8 cwts. 8½ lbs. of sugar at 7½*d.* for 1 lb. 8 ozs.

23. If 5½ tons cost £616, what is the price of 17 cwts. 3 qrs. 17 lbs. 6 ozs. ?

24. 36 tons are carried 144 miles for £10 ; for how far should 48 tons be carried for the same money ?

25. If ¾ yard of velvet cost 7*s.* 3*d.*, how many yards can be bought for £13 15*s.* 6*d.* ?

26. If 120 men finish a work in 35 days, how soon will 150 men finish it ?

27. If a tradesman with a capital of £750 gains £117 in 6 months, what will he gain in the same time if his capital is £950 ?

28. The chain for measuring land is 66 ft. long, and contains 100 links ; what is the length of a wall which measures 2,456 links ?

29. If potatoes are sold at £3 14*s.* 8*d.* per ton, what quantity can be bought for one penny ?

30. A house is let at 50 guineas a year ; what amount of rent is due for 10 weeks 3 days ?

31. A and B divide the profits of a business in the proportion of 210 to 195 ; A's share is £567 15*s.* ; find B's share.

32. How many yards of material can be bought for £20 16*s.* 9¾*d.* when 937½ yards can be bought for £111 6*s.* 6¾*d.* ?

33. If the rent of a farm of 39 acres 2 rds. 20 pls. is £148 11*s.* 10½*d.*, what is the rent of 6 acres ?

34. If 6 blacksmiths, receiving 4*s.* 8*d.* a day each, earn as much as 7 carpenters, what are the weekly (6 days) wages of 105 carpenters ?

35. The valued rents of a parish are £5,070 16*s.* 8*d.*, and on this a rate of £405 13*s.* 4*d.* is to be paid ; what will the rate be on a rent of £68 8*s.* 9*d.* ?

36. The rates on a rental of £365 7*s.* 3*d.* amount to £36 8*s.* 9*d* ; what should be the rental of an estate on which the rates are £24 5*s.* 10*d.* ?

37. If the carriage of 5 cwt. 7 lbs. for 84 miles cost £3 18*s.* 4*d.*, what will it cost to have 21 cwt. 1 qr. 14 lbs. conveyed the same distance ?

38. If 3 quarters of wheat are worth 4 quarters of barley, what should be the price of a bushel of wheat when barley is at £2 16*s.* 6*d.* per quarter ?

39. The rental of a parish is £2,000 ; what rate in the £ must be levied to produce £110 8*s.* 4*d.* ?

40. If the rent of 39 acres 2 rds. 20 pls. is £148 11*s.* 10½*d.*, what will be the rent of 42 acres 1 rd. ?

41. A man has £576 6*s.* 8*d.* per annum after paying income tax at the rate of 5*d.* in the £ ; what is his income ?

42. In a school of 960 scholars there are 3 boys to every 2 girls ; how many scholars are there of each sex ?

43. A sum of £3,362 12*s.* yields an annual income of £157 12*s.* 5¼*d.* ; what would £5,301 17*s.* 4*d.* yield at the same rate ?

44. A man, after paying an income tax of 7*d.* in the £, has £248 10*s.* 8*d.* left ; what was his gross annual income ?

45. A tradesman buys a cwt. of coffee for £5, and sells it for £6 7*s.* 6*d.*; what is his gain per cent. ?

46. A man buys eggs at 11½*d.* a doz., and sells them at 2*d.* each ; what does he gain per cent. ?

47. If a box 2 ft. square and 3 ft. deep contain 100 oranges, how many should be in one 2 ft. long, 3 ft. broad, and 7 ft. deep ?

48. A tobacconist makes a profit of £6 11*s.* 3*d.* by the sale of 4 cwt. 2 lbs. of tobacco at 1*s.* 2*d.* per lb. ; what is his gain per cent. ?

49. I bought cloth at 7*s.* 6*d.* per yard ; what must I sell it at to gain 25 per cent. ?

50. Sea-water contains 2½ per cent. of salt ; how much sea-water would be required to obtain a ton of salt ?

CHAPTER V.

COMPOUND PROPORTION.

We have seen that in Simple Proportion we have *one ratio* and *one term* of a second ratio, viz. 3 terms given in every question.

In Compound Proportion we have given *two* or *more* ratios, and one odd term—viz. 5 or 7 or 9, &c., terms.

We may solve exercises in Compound Proportion, when only two ratios and an odd term are given, by *two* applications of the rules of Simple Proportion. When three ratios and an odd term are given, three applications will be required, and so on. We will illustrate by an example.

Ex. 1. If 12 horses consume 6 quarters of oats in 20 days, how long will 20 horses be in consuming 12 quarters at the same rate ?

The terms of the ratios are—

(1) 12 horses and 20 horses.

(2) 6 qrs. of oats and 12 qrs., and the odd term is 20 days.

Let us deal in the first place with the first ratio, omitting the second altogether.

This question may be thus written: "If 12 horses consume a certain quantity of oats in 20 days, how long will it take 20 horses to consume an equal quantity?"

20 horses will take less time than 12 horses to eat an equal quantity. Hence the proportion is—

$$20 : 12 :: 20 : x$$

$$x = \frac{12 \times \cancel{20}}{\cancel{20}} = 12 \text{ days.}$$

We may now write the second part of the question thus: If 6 quarters are consumed in 12 days by a certain number of horses, how long will 12 quarters last the same number of horses. Evidently longer. Hence the second proportion is—

$$6 : 12 :: 12 : x$$

$$x = \frac{12 \times \overset{2}{\cancel{12}}}{\cancel{6}} = 24 \text{ days.}$$

All questions in Compound Proportion may be solved in a similar manner, but a shorter and more simple method is by what is termed "*compounding the ratios*," viz. placing the ratios under each other and multiplying all the antecedents together to make a new antecedent, and all the consequents together to make a new consequent. We shall then have one ratio only, besides that made up of the odd term and x, and the question may be solved at once by Simple Proportion. Thus, in the given example the ratios are—

(1)	20	:	12
(2)	6	:	12
	120	:	144

And the new compounded ratio is—

$$120 : 144$$

Hence the proportion will be—

$$120 : 144 :: 20 : x$$

$$x = \frac{\overset{12}{\cancel{144}} \times 20}{\cancel{120}} = 24 \text{ days.}$$

It is usual in practice, instead of multiplying the antecedents together, and the consequents together, to *state* the sum thus:—

			days	days
Horses .	20	:	12 }	:: 20 : x
Oats . .	6	:	12 }	

And then to make a fraction as in Simple Proportion, thus—

$$x = \frac{\overset{2}{\cancel{12}} \times \cancel{20}}{20 \times \cancel{6}} = 24 \text{ days.}$$

From the foregoing observations we may deduce the following rules for solving problems in Compound Proportion :—

(1) *Set down the ratio made up of the odd term* (viz. the one of like kind to the required answer) *and* x *standing for the answer, making the third and fourth terms.*

(2) *Take any two terms of like kind, and arrange these with reference only to the third and fourth terms.*

(3) *Take the next pair and again arrange the terms with reference only to the third and fourth terms. Deal in the same way with any other pairs of terms.*

(4) *Arrange the separate ratios under each other; then multiply the antecedents, or first terms, to form a new antecedent or first term, and all the consequents or second terms for a new consequent or second term.*

(5) *Proceed as in Simple Proportion.*

Ex. 2. If the corn of 13 horses for 63 days cost £17 6*s.* 8*d.* when corn is 4*s.* per bushel, how many horses will in 56 days consume corn to the value of £10 13*s.* 4*d.* when corn is 4*s.* 6*d.* per bushel?

In this example there are three pairs of terms, and 13 horses is the odd term.

(1) Take the days. It will take more horses to eat a certain amount of corn in 56 days than in 63 days.

(2) Take the cost. It will take less horses to eat corn to the value of £10 13*s.* 4*d.* than corn to the value of £17 6*s.* 8*d.*

(3) Take the price per bushel. You get less corn for your money at 4*s.* 6*d.* than at 4*s.*, and hence require less horses to eat it.

The statement then is—

$$\left.\begin{array}{rcl} & & 56 : 63 \\ £17\ 6s.\ 8d. & : & £10\ 13s.\ 4d. \\ & & 4s.\ 6d. : 4s. \end{array}\right\} :: 13 : x.$$

The second ratio may be written $17\frac{1}{3} : 10\frac{2}{3}$, or $\frac{52}{3} : \frac{32}{3}$, or 52 : 32; and the third ratio may be written $4\frac{1}{2} : 4$, or 9 : 8.

Thus—

$$\left.\begin{array}{r} 56 : 63 \\ 52 : 32 \\ 9 : 8 \end{array}\right\} :: 13 : x.$$

$$x = \frac{\cancel{9}\ \ 8}{\cancel{56} \times \cancel{52} \times \cancel{8} \times \cancel{13}} = 8.$$
$$\cancel{8}\ \ \cancel{4}$$

8 horses. *Ans.*

Ex. 3. If £3,000 gain £60 in 6 months, how much will £5,500 gain in 15 months at the same rate?

(1) £60, the profit, is of like kind to the required answer; hence it forms the third term.

(2) £5,500 will gain *more* than £3,000.

(3) A sum of money will gain more in 15 mo. than in 6 mo.

Hence the proportion is—

$$\left.\begin{array}{rcl} \text{£} & 3{,}000 & : \ 5{,}500 \\ \text{Months} & 6 & : \ 15 \end{array}\right\} :: \underset{\text{£}}{60} : x.$$

$$x = \frac{5500 \times \overset{5}{\cancel{15}} \times \overset{1}{\cancel{60}}}{\underset{1}{\cancel{3000}} \times \underset{1}{\cancel{6}}} = \text{£}275 \ \textit{Ans.}$$

Ex. 4. If 6 men, 9 women, and 14 boys can do a piece of work in 108 days of 8 hours each, in how many days of 9 hours each can 12 men, 4 women, and 16 boys do the same work? (The amount of work of man, woman, and boy being in the proportion of 3, 2, and 1.)

We must first find the amount of work done by the men, women, and boys in terms of one of them.

We will substitute boys for the men and women.

Thus 6 men	=18 boys.	12 men	=36 boys
9 women	=18 boys.	4 women	= 8 „
14 boys	=14 boys.	16 boys	=16 „
	50 boys.		60 boys.

(1) 60 boys will do the work in less time than 50 boys.

(2) It will take shorter time working 9 hours per day than working 8 hours.

$$\therefore \left.\begin{array}{rcl} \text{Boys} & 60 & : \ 50 \\ \text{Hours} & 9 & : \ 8 \end{array}\right\} :: \underset{\text{days}}{108} : \underset{\text{days}}{x}.$$

$$x = \frac{\overset{10}{\cancel{50}} \times 8 \times \overset{12}{\cancel{10}}}{\underset{5}{\cancel{60}} \times 9} = 80 \text{ days. } \textit{Ans.}$$

Ex. 5. A garrison of 4,500 men is provisioned for 15 weeks at the rate of 13 ozs. per day, per man; how many men must leave that the same provisions may last those who remain 27 weeks at 10 ozs. per day per man?

We cannot find *directly* how many must leave, but we can find how many the provisions will keep for 27 weeks at 10 ozs., and this num-

14. If 8 masons build 35 yds. of wall in 2 days, how many yards will 115 masons build in the same time?

15. If a family of 9 people spend £120 in 8 months, how much will serve a family of 24 people for 16 months?

16. If 6 men can do a piece of work in 30 days of 9 hours each, how many men will it take to do ten times the amount if they work 25 days of 8 hours each?

17. A man can walk 252 miles in 6 days of 9 hours each; how far can he walk at the same pace in 7 days of 8 hours each?

18. If the penny loaf weighs 7 ozs. when wheat is 6*s.* 8*d.* a bushel, what should be the price of the 4 lb. loaf when wheat is at 5*s.* 10*d.* a bushel?

19. If 8 fires consume 6 tons 8 cwt. of coal in 6 weeks, in what time will 9 fires consume 10 tons 16 cwt.?

20. If the duty on 50 packages, each weighing 128 lbs., is £33, what is the duty on 73 packages of the same material, each weighing 98 lbs.?

21. If 7 oxen can eat 3 tons of hay in a month, how long will it take 49 sheep to eat 35 tons, an ox consuming three times as much as a sheep?

22. If 8 men can dig 24 cubic yards of earth in 6 days, how many men must there be to dig 18 cubic yards in 3 days?

23. If the 4*d.* loaf weigh 72 ozs. when wheat costs 77*s.* per qr., what should a 3*d.* loaf weigh when wheat is at 81*s.* per qr.?

24. If the 4*d.* loaf weigh 24 ozs. when wheat is at 50*s.* per qr., what is the price of wheat when the 6*d.* loaf weighs 30 ozs.?

25. If £100 will pay the expenses of 5 persons for 22 weeks 6 days, how long would 12 persons be supported by £150 under similar circumstances?

26. If the cost of 750 soldiers for 12 days be £900, how long can 200 soldiers be maintained for £3,000 at the same rate?

27. If £108 gain £6 interest in 12 months, what should be the interest on £75 for 9 months?

28. If the carriage of 9¾ cwt. for a distance of 80 miles be 9*s.* 4*d.*, how many miles should 6 tons 10 cwt. be carried for £1 15*s.*?

29. If 3 men can dig a trench 105 yds. long, 4 ft. deep, and 5 ft. wide in ten days, how long will it take 5 men to dig a trench 175 yds. long, 4½ ft. deep, and 6 ft. wide?

30. If 25 men can do a piece of work in 12 days, working 8 hours per day, how many hours per day must 15 men have to work so as to perform the same work in 16 days?

31. If 6 horses eat 18 guineas' worth of hay in 6 weeks when hay is 9*d.* a stone, what is its price per stone when 15 horses eat £110 5*s.* worth in 21 weeks?

32. If 500 bricks, 4 ins. thick, are required for a wall 180 ft. long, how many bricks, 5 ins. thick, will be required for a wall 90 ft. long?

33. I borrowed £50 for 6 months when money was at 5 per cent.; how much should I lend for 10 months in return at 4 per cent.?

34. A work is performed in 53 days by 15 men or 27 boys; in what time would 27 men *and* 15 boys perform the same?

35. If 7 men working 16 days can mow a field of corn 1,320 yds. long and 880 yards wide, what will be the length of the side of a field 1,320 yds. broad which 4 men can mow in 42 days?

36. If 240 men, in 5 days of 11 hours each, can dig a trench 220 yds. long, 3 wide, and 2 deep, in how many days of 9 hours long will 24 men dig a trench 420 yds. long, 5 wide, and 3 deep?

37. If 24 oxen require 6 acres of turnips to supply them for 10 weeks, how many acres would supply 6 score sheep for 15 weeks, 3 oxen eating as much as 10 sheep?

38. If 5 men build a wall 5 ft. high and 100 yds. long in 3 days, how long will it take 7 men to build a wall 200 yds. long and $3\frac{1}{2}$ ft. high?

39. If 16 men can do a piece of work in 13 hrs. 20 mins., how many men will do one-half as much again in 5 hrs. 20 mins.?

40. A ship is provisioned for 200 men for 5 weeks, allowing each man 15 ozs. a day; at the last moment 100 men are added, and the destination of the ship is changed so that the voyage will last 4 weeks only; find the allowance per man per day.

MISCELLANEOUS EXERCISES.

1. Divide £35 between A and B in the ratio of 4 to 3.

2. If the wages of 12 men amount to £29 14*s.*, how many men ought to work for the same time for £81 13*s.* 6*d.*?

3. If 14 horses eat 56 bushels of oats in 16 days, how many might be kept on 120 bushels for 24 days?

4. How many blocks of salt, each 2 ft. long, 1 ft. broad, and 1 ft. deep, will exactly fill a warehouse 30 ft. long, 20 ft. wide, and 10 ft. high?

5. What may I spend daily out of an income of £383 5*s.* if I save £54 15*s.* in the year?

6. If the wages of 12 men for 4 days amount to £29 14*s.*, how many men should work 11 days for £81 13*s.* 6*d.*?

7. If 12 men, working 10 hours a day, can excavate a tunnel in 18 days, how many hours a day must they work to do it in 15 days?

8. If 3 ducks are worth 4 chickens, and 3 geese are worth 10 ducks, find the value of a goose when a pair of chickens is worth 4*s.* 6*d.*

9. Three men do a certain work for £3 15*s.*; one man worked 4 days, the second 5, and the third 6 days; what money ought each to receive?

10. A and B start to run a race; A gives B 100 yards start, and then gets over 5 yards whilst B travels over 4 yards; what distance will A have travelled when he overtakes B?

11. If 12 men build a wall 60 ft. long, 4 ft. thick, and 20 ft. high in 24 days, working 12 hours a day, how many men will it take to build a wall 100 ft. long, 3 ft. thick, and 12 ft. high in 18 days, working 8 hours a day?

12. Divide £4 10*s.* in 5 parts in the proportion of 5, 4, 3, 2, 1.

13. Find the cost of papering a room 16 ft. long, 12 ft. wide, and 10 ft. high with paper 1 ft. 10 ins. wide at 1½*d.* per yd. (Allow one door 6 ft. by 4 ft., and two windows, each 5 ft. by 3 ft.)

14. A schoolroom is 60 ft. long, 22 ft. wide, and 20 ft. high; allowing 160 cubic feet of air per child, what is the school accommodation?

15. If the 8*d.* loaf weighs 44 ozs. when wheat is at 45*s.* per qr., what should be the price of wheat when the 6*d.* loaf weighs 32½ ozs.?

16. A train travelling at the rate of 30 miles an hour starts from a certain station 2½ hours before another travelling at 45 miles an hour; when, and at what distance will the second overtake the first?

17. A stick 2 ft. 11 ins. high, standing perpendicularly on a level ground, throws a shadow 3 ft. 6 ins. long; what is the height of a tower, and of a flagstaff at the top of it, throwing a shadow of 126 ft., the flagstaff being one-fourth the height of the tower?

18. Find the annual income of a man who spends 12 guineas in 35 days and saves £100 a year.

19. A courtyard requires 150,000 flagstones, each 15 in. long and 8 in. broad, to cover its surface; how many flagstones will be required of 12 in. long and 5 in. broad to cover a courtyard one-quarter the size?

20. It is now 5 o'clock; when will the hour and minute hand be next at right angles to each other?

21. A garrison of 1,800 men has provisions for 6 weeks; at the end of 17 days 800 men are sent away; how much longer will the provisions last the remainder?

22. Gunpowder is composed of 33 parts nitre, 7 charcoal, and 5 sulphur; find the weight of each ingredient in 1 ton 5 cwts.

23. What weight must be subtracted from 25 lbs. 9 ozs. so that an exact number of equal parcels, each weighing 2 lbs. 2 ozs., may be made?

24. The first, third, and fourth terms of a proportion are 3 cwts. 14 lbs., £1 7*s.* 1*d.*, and £5 1*s.* 10*d.* respectively; find the second term.

25. A and B enter into business as partners; A puts in £550, and B £750; they gain £320; find the share of each.

26. A room measures 16 ft. by 21 ft., and is 11 ft. high; find the cost of papering the walls with paper 2 ft. wide at 2½*d.* per yard? (Allow 1 door, 7 ft. by 3 ft., and 2 windows, each 8 ft. by 4 ft.)

27. The cost of covering a room 20 ft. long with floor-cloth at 9*d.* per sq. ft. is £6 15*s.*; find the width of the room.

28. If the consequent be 3 acres, and the value of the ratio $\frac{31}{100}$, what is the antecedent?

29. Divide £220 among A, B, and C, in the proportion of 5, 4, and 2.

30. What weight must be added to 12 cwt. 4 lbs. 6 ozs. that it may be exactly sufficient for a number of parcels, each weighing 2 lbs. 3 ozs.?

31. A railway train is travelling at the rate of 45 miles an hour; how many revolutions does the driving-wheel (22 ft. in circumference) make in a second?

32. In a room 22 ft. by 18 ft. there is a carpet 16 ft. square; find the cost of covering the rest of the room with floor-cloth 2 ft. wide, at 5*s.* per yard.

33. Four persons, A, B, C, and D, rent a field for £88; A puts in 25 cattle, B 31, C 17, and D 15; how much should each pay?

34. If 20 men can perform a piece of work in 12 days, required the number of men who could perform another piece of work, 3 times as great, in one-fifth part of the time?

35. The ratio of A to B is as 3 to 4; of B to C as 4 to 5; of C to D as 5 to 6; find the ratio of A to D.

36. Divide £1,000 between two persons so that they may have equal sums remaining when one has bought 8 tons 5 cwts. of sugar at $5\frac{1}{2}d.$ per lb., and the other 3 tons 19 cwts. at $6\frac{1}{2}d.$ per lb.

37. Find the cost of carpeting a room 21 ft. by 30 ft. with carpet half a yard wide, at 4*s.* 6*d.* per yard.

38. A block of oak is worth £3; find the value of another block three times as long, twice as broad, and of the same thickness.

39. If $\frac{3}{5}$ of a yard of ribbon cost $3\frac{1}{2}d.$, what will be the cost of $6\frac{1}{2}$ pieces, each containing $185\frac{1}{2}$ English ells?

40. After paying 3*d.* in the £ income tax I have £429 11*s.* 3*d.* left, what is my gross income?

41. If 11 horses require as much hay as 19 cows, and 19 tons is sufficient for 38 horses for $3\frac{1}{4}$ months, what quantity will suffice for 19 cows for the same time?

42. A commences business with a capital of £600; at the end of 6 months B joins with a capital of £1,000; at the end of 12 months they have gained £550; find each man's share of the profits.

43. A garrison was victualled for 30 days; after 10 days it was reinforced by 3,000 men; it was then found that the provisions would only last 5 days; find the original number in the garrison.

44. A farmer bought a number of sheep for £97 10*s.*, at £1 17*s.* 6*d.* each; some of them died, and he sold the remainder at £2 4*s.* 6*d.* each, thereby gaining £2 12*s.* 6*d.* on the whole; how many died?

45. If the weights of equal quantities of iron and coal are as 72 to 13, what weight of coal will occupy the space of 5 tons of iron?

46. A block of wood weighs 3 cwt. ; what is the weight of a block of granite the length, breadth, and height of the block of wood as 4 to 5, 3 to 4, and 2 to 3 respectively, weights of equal blocks of granite and wood being in the proportion of 5 to 2?

47. A street 440 yds. long and 20 yds. wide is to be paved with wooden blocks 9 ins. long, 4 ins. wide, and 6 ins. deep ; the blocks are to be placed lengthways in rows across the street, with a space of 1 in. between ; find the number of blocks required, and the cost at 2*s.* 6*d.* per cubic foot of wood.

48. A box when empty weighs 33 lbs. ; when filled with water it weighs 6 cwts. 2 qrs. 25 lbs. ; when filled with sand it weighs 11 cwts. 2 qrs. 25 lbs. ; find the specific gravity of the sand. (Specific gravity= weight of any substance compared with weight of an equal bulk of water).

CHAPTER VII.

VULGAR FRACTIONS.

SECTION I.—GREATEST COMMON MEASURE.

MEASURE.—When one number is contained an exact number of times in another number, the first number is said to be a *measure* of the second, and the second is said to be a *multiple* of the first. Thus 4 is contained in 12 *three* times exactly ; hence 4 is a *measure* of 12, and 12 is a *multiple* of 4. We see from this that *factor* and *measure*, and *product* and *multiple*, are synonymous terms. 9 multiplied by 8 gives 72. Thus 9 and 8 are *factors* or *measures* of 72 ; and 72 is the *product* or *multiple* of 9 and 8.

COMMON MEASURE.—All numbers which are exactly divisible into a given number are measures of that number. Thus the measures of

(1) 12 are 2, 3, 4, 6, and 12
(2) 16 are 2, 4, 8, and 16.

We see from this that 2 and 4 each measure both 12 and 16 ; in other words, they are *common measures* of 12 and 16.

Thus 4 is a common measure of 4, 8, 12, 16, &c.
„ 5 „ „ „ „ 5, 10, 15, 20, 25, &c.
„ 6 „ „ „ „ 6, 12, 18, 24, &c.

GREATEST COMMON MEASURE.—The measures

of 27 are 3, 9, 27.
of 54 are 2, 3, 6, 9, 18, 27, 54.

And of these only three, viz. 3, 9, and 27, are measures common to

the two numbers. Of these 27 is the greatest number, and is called the Greatest Common Measure (G. C. M.) of the numbers.

Again the measures—

(1) of 9 are 3 and 9.
(2) of 18 are 3, 6, 9, and 18.
(3) of 27 are 3, 9, and 27.

Hence 9 is the G. C. M. of the *three* numbers.

Definition.—*The greatest common measure of two or more numbers is the greatest number which divides each of them exactly.*

NOTE 1.—Numbers like 5, 7, 9, 11, &c., which have no measures in common, are called *prime* numbers. All other numbers are *composite.*

NOTE 2.—Numbers which are not prime in themselves may be prime to each other, thus 15 and 32 are not prime numbers in themselves (they can be exactly divided by other numbers), but they have no measure in common except unity, therefore they are prime to each other.

Ex. 1. Find the G. C. M. of 64 and 96.

The measures of 64 are 2, 4, 8, 16, 32, 64.
„ „ „ 96 are 2, 4, 6, 8, 12, 16, 24, 32, 48, 96.
Hence 32 is the G. C. M.

Ex. 2. Find the G. C. M. of 35, 63, and 91.

Measures of 35 = 5, 7.
„ „ 63 = 3, 7, 9, &c.
„ „ 91 = 7, 13, &c.
Hence 7 is the G. C. M.

Ex. 3. Find the G. C. M. of 315 and 693.

The preceding method is convenient only for small numbers. When the numbers are so large that the factors cannot readily be detected, we adopt the following plan :—

Divide the greater of the two numbers by the less ; if there be a remainder, divide it into the former divisor ; if there be a second remainder, divide it into the preceding divisor, and so on, until there is no remainder ; the last divisor is the G. C. M.

Thus—

315)693(2
630
63)315(5
315

63 is the G. C. M.

NOTE.—The proof of this rule depends upon two axioms.

AX. 1. If one number measures another it must measure any *multiple* of it. Thus, if three is a measure of six, it must be a measure of twice 6, three times 6, and so on.

AX. 2. If a number measures two other numbers, it also measures their *sum* and *difference*.

Thus—

If 3 is contained 6 times in 18, and three is contained 4 times in 12,
By addition 3 is contained 10 times in 30, and
By subtraction 3 is contained twice in 6.

We can now *show* the truth of the rule in any given example.

EX. 4. Find the G. C. M. of 64 and 84.

$$\begin{array}{r} 64)84(1 \\ \underline{64} \\ 20)64(3 \\ \underline{60} \\ 4)20(5 \\ \underline{20} \\ \hline \end{array}$$

(1) Any number which measures 64 and 84 must measure their difference, viz. 20 (AX. 2).

(3) Any number which measures 20 must also measure 3×20 or 60 (AX. 1).

(3) Any number which measures 60 and 64 must also measure their difference, viz. 4 (AX. 2).

(4) That is, any number which measures 64 and 84 must also measure 4. But no number greater than 4 can measure 4. Hence no number greater than 4 can measure 64 and 84.

Again—

(1) 4 measures 20, for $20 = 4 \times 5$ (AX. 1); and also 60, for $60 = 20 \times 3$ (AX. 1).

(2) 4 measures 64, because $60 + 4 = 64$ (AX. 2).

(3) 4 measures 84, because $64 + 20 = 84$ (AX. 2). Therefore, since 4 measures both numbers, and no number greater than 4 can measure both, 4 is the G. C. M.

EX. 5. Find the G. C. M. of 171, 152, and 133.

We must first find the G. C. M. of any *two* of the numbers, and then the G. C. M. of this result and the remaining number.

(1) Take 171 and 152.

$$\begin{array}{r} 152)171(1 \\ \underline{152} \\ 19)152(8 \\ \underline{152} \\ \hline \end{array}$$

G. C. M. of 171 and 152 = 19.

(2) Find the G. C. M. of 19 and 133.

$$19)133(7$$
$$133$$

Hence the G. C. M. of the three numbers is 19.

Ex. 6. What is the greatest number which will divide 4,093 and 8,487, leaving a remainder of 11?

If we are to have a remainder we must subtract it from each of the numbers before finding the G. C. M.

$$4093-11=4082$$
$$8487-11=8476$$

Next we must find the G. C. M. of 4,082 and 8,476: this is 26. Hence 26 is the required number.

$$\text{For } \frac{4093}{26}=157+11$$

$$\text{And } \frac{8487}{26}=326+11$$

Ex. 7. A lobby 7 ft. 8 in. long, and 6 ft. 6 in. wide, has to be paved with square tiles; find the largest size that can be used, and the number required.

The length is 92 in., and the width 80 in. Now the tiles must be of such a size as that an even number will fill the length, and also an even number will fill the width; in other words, the size of the square tile must exactly measure 92 and 80; and the side of the largest size that can be used must be the G. C. M. of 92 in. and 80 in., viz. 4 in. That is, the tiles must be 4 in. in the side, or 16 sq. in. in the surface.

Size of lobby in sq. in. $=92\times80=7{,}360$.

$$\text{Hence } \frac{7360}{16}=460, \text{ the number of tiles required.}$$

Exercises in Greatest Common Measure.

1. Find the G. C. M. of—
   (1) 150 and 170
   (2) 588 and 785
   (3) 195 and 225
   (4) 8,008 and 9,009
   (5) 1,302 and 1,314
   (6) 10,747 and 10,681
2. What are the common measures of—
   18, 16, 34, 45, 66, 156, and 216?
3. Find the G. C. M. of—
   (1) 1,854 and 1,778
   (2) 3,816 and 3,492
   (3) 2,163 and 3,605
   (4) 6,170 and 7,404
   (5) 16, 72, and 112
   (6) 96, 144, and 720

4. What is the largest number which will exactly divide 271,469 and 30,599 ?

5. What is the G. C. M. of 62,713 and 109,242 ?

6. Find the greatest number which divides 1,617 and 1,979 without a remainder.

7. Find the greatest number which divides 4,243 and 6,569, leaving remainders of 1 and 4.

8. Find the G. C. M. of 1,200, 1,728, and 1,656.

9. What number will divide 1,577 and 14,645, leaving in each case a remainder of 4 ?

10. Find the G. C. M. of 185,955 and 76,153.

11. What is the greatest common divisor of 99,108 and 68,805 ?

12. Find the G. C. M. of 104,811, 48,825, and 68,789.

13. Find the greatest sum of money contained exactly in £1 5*s.* 10½*d.* and £2 8*s.* 4¾*d.*

14. In finding the G. C. M. of two numbers, the last remainder is 35, and the quotients are 1, 2, 1, 3 ; find the numbers.

15. What are the four smallest numbers which have 21 for their G. C. M. ?

16. Two fields, measuring respectively 24 and 28 acres, have to be divided into equal-sized plots as large as possible ; find the number of plots and the size of each.

17. In the working of an exercise to find the G. C. M. of two numbers, the remainders are 3,552 and 888, and the quotients are 2 and 4 ; what are the numbers ?

18. Find the highest common measure of 5 days 17 hours, and 17 days 3 hours.

Section II.—Least Common Multiple.

Any number which contains another number an exact number of times is said to be a *multiple* of that number.

Thus 12 is a multiple of 2, 3, 4, and 6
„ 9 „ „ „ 3
„ 21 „ „ „ 3 and 7

The multiples—

Of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, &c.
Of 4 are 4, 8, 12, 16, 20, 24, 28, &c.

Here we see that 12 and 24 are multiples of both 3 and 4 ; hence they are called *common multiples.*

We see also that 12 is the *least* of these common multiples ; hence it is called the *least common* multiple.

Definition.—*The least number which exactly contains two or more numbers is the Least Common Multiple of those numbers.*

Ex. 1. Find the L. C. M. of 3, 6, and 9.

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, &c.
„ „ 6 are 6, 12, 18, 24, &c.
„ „ 9 are 9, 18, 27, &c.

We find 18 to be the least number which contains 3, 6, and 9, each an exact number of times ; 18 is therefore the L. C. M. required.

Ex. 2. Find the L. C. M. of 5 and 7, and also of 5 and 8.

(1) The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, &c.
„ „ „ 7 are 7, 14, 21, 28, 35, &c.

Hence 35, or 7×5, is the L. C. M. In other words, the L. C. M. of *prime numbers* is their product.

(2) The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, &c.
„ „ „ 8 are 8, 16, 24, 32, 40, &c.

Hence 40, or 8×5, is the L. C. M. In other words, the L. C. M. of numbers *prime to each other* is their product.

We can find the L. C. M. by another method which in practice is more useful ; that is, by resolving each number into its prime factors.

To divide a number into its prime factors, divide by the smallest prime number which will measure it. Then divide the quotient again by the smallest prime number which measures it, and so on until the quotient cannot again be divided except by itself. The series of divisors and the last quotient are the prime factors.

Ex. 3. Resolve 504 into its prime factors.

$$\begin{array}{r} 2)504 \\ 2)252 \\ 2)126 \\ 3)63 \\ 3)21 \\ 7 \end{array}$$

Thus the prime factors of 504 are $2 \times 2 \times 2 \times 3 \times 3 \times 7$.

Ex. 5. Find the L. C. M. of 300, 135, and 72.

Resolve each number into its prime factors. Thus—

$$300 = 2 \times 2 \times 3 \times 5 \times 5$$
$$135 = 3 \times 3 \times 3 \times 5$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

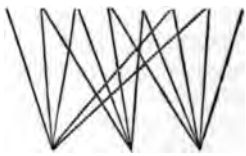
Set down all the prime factors of the first number ; then all the prime factors of the second number which do not occur in the first ;

lastly, all the prime factors of the third number which do not occur in the first and second. Thus—

$$\overbrace{2\times2\times3\times5\times5}^{\text{from first number}} \times \overbrace{3\times3}^{\text{from 2nd number}} \times \overbrace{2}^{\text{from 3rd number}}$$

On examining this series of prime numbers, it will be seen that it contains all the prime factors of each of the numbers 300, 135, and 72.

Thus— $5\times5\times3\times3\times3\times2\times2\times2=5{,}400$



300 135 72

The product, therefore, must be a common multiple of them all, and it is the *least* common multiple, because no one of the factors can be dispensed with. If we omitted one of the *fives*, the result would not contain 300 ; if we omitted one of the *threes*, the result would not contain 135 ; and if we omitted one of the *twos*, the result would not contain 72.

We may find the prime factors and take out the common factors of any numbers simultaneously by using the following method :—

(1) Place all the numbers in a line, separating them by a comma.

(2) Divide by the smallest prime factor all the numbers that are divisible ; set down the quotients, and bring down the numbers which are not divisible.

(3) Repeat this process until no other factor is common.

The continued product of the quotients and the common factors is the L. C. M.

Taking the same example, viz. to find the L. C. M. of 300, 135, and 72—

$$\begin{array}{r} 2)300,\ 135,\ 72 \\ \hline 2)150,\ 135,\ 36 \\ \hline 3)75,\ 135,\ 18 \\ \hline 3)25,\ 45,\ 6 \\ \hline 5)25,\ 15,\ 2 \\ \hline 5,\ 3,\ 2 \end{array}$$

Thus the common factors (divisors) and the quotients give us precisely the same series of factors as by the former method.

$2\times2\times3\times3\times5\times5\times3\times2=5{,}400$ is the L. C. M. of 300, 135, and 72.

Ex. 6. Find the L. C. M. of 8, 12, 15, 16, 24, 32, 36, 48.

It is evident that whatever is a multiple of 48 is a multiple of 24, and of 12 and 8; and whatever is a multiple of 32 is a multiple of 16. Hence we may at once strike out all those numbers which exactly measure others.

2	~~8~~, ~~12~~, 15, ~~16~~, ~~24~~, 32, 36, 48
2	. . 15, . . 16, 18, 24
2	. . 15, . . 8, 9, 12
2	. . 15, . . 4, 9, 6
3	. . 15, . . 2, 9, 3
	. . 5, . . 2, 3, 1

Therefore the L. C. M. $= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 2 \times 3 = 1{,}440$.

Ex. 7. A schoolmaster finds that he can arrange his boys in classes of 30, 32, or 36, and have none over; what is the smallest number he can have in the school?

This question is simply—what is the least number into which 30, 32, and 36 will divide without a remainder? or, what is the L. C. M. of 30, 32, and 36?

2	30, 32, 36
2	15, 16, 18
3	15, 8, 9
	5, 8, 3

$2 \times 2 \times 3 \times 5 \times 8 \times 3 = 1{,}440$. *Ans.*

Ex. 8. Find the least number which, when divided by either 9, 11, or 12, leaves a remainder of 5.

The L. C. M. of 9, 11, and 12 is the least number into which 9, 11, and 12 will each divide without a remainder. If, therefore, we add 5 to the L. C. M., this will give a remainder of 5 after each division.

L. C. M. of 9, 11, and $12 = 396$

$396 + 5 = 401 =$ required number.

For $\frac{401}{9} = 44 + 5$

$\frac{401}{11} = 36 + 5$

$\frac{401}{12} = 33 + 5$

Ex. 9. Find the least sum of money payable in sixpences, florins, half-crowns, crowns, sovereigns, and guineas.

Reducing all to same denomination, say of sixpences, we get—

1, 4, 5, 10, 40, 42

and the least number of sixpences into which each of these numbers will divide without a remainder is the L. C. M. of the numbers.

1, 4, 5, 10 are all contained in 40. Hence L. C. M. of 40 and 42 gives the required number, viz. 840 sixpences, or £21.

Exercises in Least Common Multiple.

1. Find by inspection the L. C. M. of—

(1) 8, 10, 25
(2) 6, 9, 30
(3) 4, 11, 20
(4) 16, 14, 2, 7
(5) 25, 35, 45
(6) 7, 8, 9, 10

2. Resolve 432, 6,930, 1,024, and 2,401 into their prime factors; thence find their L. C. M.

3. Find the L. C. M. of—

(1) 28, 17, 9, 34, 112, 36
(2) 48, 29, 58, 56, 174
(3) 3, 7, 16, 14, 63, 32
(4) 19, 17, 13
(5) 16, 18, 20, 22, 24, 26
(6) 96, 108, 192, 216, 288

4. Find the least sum of money payable in pounds or guineas.

5. Find the least number divisible by 8, 9, or 10, leaving a remainder of 3 in each case.

6. Find the least number divisible by all the numbers up to 12 inclusive.

7. Three men start on a journey at the same time; the first travels 15 miles a day, the second 20, and the third 25 miles; how far off is the first hotel where all three put up?

8. Find the least sum of money which contains 2*s.* 7½*d.*, 4*s.* 6¼*d.*, and 3*s.* 9¾*d.* an exact number of times.

9. A, B, C, D start together to walk round a field; A takes 8 minutes, B 6 minutes, C 10 minutes, and D 12 minutes to walk once round; after how many minutes walking will they all be together at the starting point?

10. Find the least sum of money which contains 5*s.* 6*d.*, 2*s.* 9*d.*, and 7*s.* 4*d.* an exact number of times.

11. Telegraph posts are fixed on each side of a straight piece of railway 1 mile long; on one side they are 100 ft. apart, and on the other 120 ft.; how many of the posts are exactly opposite to each other?

12. A number is divided by 16 and there is a remainder of 11, by 18 and there is a remainder of 13, and by 20 when there is a remainder of 15; find the number.

13. A schoolmaster counts his scholars 4 at a time, 6 at a time, and 9 at a time, and finds he has none over; but on counting them 7 at a time he finds he has 4 over; find the smallest number of scholars there can be in the school.

14. Find two numbers whose G. C. M. is 9, and L. C. M. is 1,188.

Section III.—Reduction of Fractions to Lowest Terms.

We have seen how to reduce simple fractions to their lowest terms; viz. by dividing the numerator and denominator by common factors. But suppose we have such a fraction as $\frac{1189}{1363}$ to reduce to its lowest terms. In this fraction it is not easy to determine by inspection any common factor. Here our rule for finding the greatest common measure, or factor, of two numbers becomes of service. The G. C. M. of 1,189 and 1,363 is 29. Hence we may divide numerator and denominator by 29, and get the fraction as $\frac{41}{47}$, viz. in its lowest terms.

The following facts will assist us in finding factors by inspection:—

1. The number is divisible by 2 if the figure in the *unit's* place is even.

2. The number is divisible by 4 if the *tens* and *units* be divisible by 4.

3. The number is divisible by 8 if the *hundreds*, *tens*, and *units* be divisible by 8.

4. The number is divisible by 5 if the *unit's* place is 5 or 0.

5. The number is divisible by 9 or 3 if the *sum* of the digits is divisible by 9 or 3.

6. An even number is divisible by 6 when it is divisible by 3.

7. A number which has *three* digits repeated in the same order is divisible by 7, 11, and 13.

Note.—A fraction is in its lowest terms when the numerator and denominator have no factor in common.

Exercises on Reduction of Fractions.

1. Reduce $\frac{58}{174}$, $\frac{9}{117}$, $\frac{815}{1630}$, and $\frac{281}{6917}$ to their lowest terms.

2. Express with the least possible number of figures $\frac{193}{721}$, $\frac{1899}{2205}$, $\frac{3312}{3456}$.

3. Convert the following fractions to mixed numbers:—$\frac{1984}{17}$, $\frac{3002}{216}$, $\frac{1011}{301}$.

4. Reduce to improper fractions $7\frac{131}{513}$, $133\frac{2}{5}$, $1\frac{316}{517}$.

5. Convert to improper fractions, and reduce to their lowest terms, $5\frac{84}{108}$, $19\frac{143}{187}$, $57\frac{203}{217}$.

6. Find the G. C. M., and reduce $\frac{4455}{7336}$ to lowest terms.

7. Find the G. C. M., reduce to lowest terms, and change to mixed number $\frac{125874}{117558}$.

8. Reduce $\frac{2442}{867867}$ and $\frac{1232}{321321}$ to lowest terms.

9. Is the fraction $\frac{16553}{180018}$ in its lowest terms? if not, reduce it.

10. Is the fraction $\frac{527}{721}$ in its lowest terms? if not, reduce it.

11. Reduce $\frac{140553}{108867}$ to lowest terms, and convert to a mixed number.

12. Find the G. C. M., and reduce $\frac{10}{16}$, $\frac{741}{171}$, $\frac{456}{184}$ to lowest terms.

Section IV.—Addition and Subtraction of Fractions.

Add half an apple to a quarter of an apple, and we get three-quarters; that is, $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$. But how do we arrive at this result? By considering the half as two quarters. Thus $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$. We have changed the $\frac{1}{2}$ to a fraction *with the same denominator* as the smaller fraction, by multiplying numerator and denominator by 2, and then we have added the numerators. *To add or subtract fractions reduce to the same common denominator,* and *add or subtract the numerators for the new numerator.*

Ex. 1. Add $\frac{1}{5}$, $\frac{3}{8}$, and $\frac{3}{4}$.

$$\frac{1\times 8}{5\times 8}=\frac{8}{40}\quad \frac{3\times 5}{8\times 5}=\frac{15}{40}\quad \frac{3\times 10}{4\times 10}=\frac{30}{40}$$

$$\text{And}\quad \frac{8}{40}+\frac{15}{40}+\frac{30}{40}=\frac{53}{40}=1\tfrac{13}{40}$$

It is not always easy to find the common denominator by inspection, and hence we have to employ the method of finding the L. C. M., the L. C. M. of the denominators being always the smallest common denominator.

Ex. 2. Find the value of $\frac{3}{7}+\frac{9}{14}+\frac{1}{3}+\frac{1}{15}$.

The L. C. M. of 7, 14, 3, 15 is 210. How can we convert $\frac{3}{7}$ into a fraction having 210 for its denominator? $\frac{21}{7}\frac{0}{}=30$; that is, if we multiply 7 by 30 we shall get 210; but if we multiply the denominator by 30 we must also multiply the numerator by the same number, so that the value of the fraction may not be altered, and the same method must be pursued with the other fractions. Thus—

$$\frac{210}{7}=30 \text{ and } \frac{3\times 30}{7\times 30}=\frac{90}{210}$$

$$\frac{210}{14}=15 \text{ and } \frac{9\times 15}{14\times 15}=\frac{135}{210}$$

$$\frac{210}{3}=70 \text{ and } \frac{1\times 70}{3\times 70}=\frac{70}{210}$$

$$\frac{210}{15}=14 \text{ and } \frac{1\times 14}{15\times 14}=\frac{14}{210}$$

Adding the numerators, we have $\frac{299}{210}=1\frac{89}{70}$ as the value of the fraction.

The working of this and similar examples is best shown as follows :—

L. C. M. of 7, 14, 3, 15 = 210.

$$\frac{3}{7}+\frac{9}{14}+\frac{1}{3}+\frac{1}{15}=$$

$$\frac{90+135+70+14}{210}=\frac{309}{210}=1\tfrac{33}{70}$$

Ex. 3. From $\frac{3}{7}$ subtract $\frac{1}{5}$.

L. C. M. of 7 and 5 = 35.

$$\frac{3}{7}-\frac{1}{5}=\frac{15-7}{35}=\frac{8}{35}$$

Ex. 4. Find the value of $\frac{3}{5}+\frac{1}{7}+\frac{7}{35}-\frac{19}{42}-\frac{3}{14}$.

L. C. M. of 5, 7, 35, 42, 14 = 210

$$\frac{3}{5}+\frac{1}{7}+\frac{7}{35}-\frac{19}{42}-\frac{3}{14}=$$

$$\frac{126+30+42-95-45}{210}=$$

$$\frac{198-140}{210}=\frac{58}{210}=\frac{29}{105}$$

NOTE.—When the numerator consists of numbers to be added (positive quantities), and numbers to be subtracted (negative quantities), first add all the positive quantities, and then the negative quantities, and subtract the less from the greater.

Ex. 5. Find the value of $3\frac{5}{11}+6\frac{115}{132}-4\frac{53}{66}-\frac{110}{132}$

L. C. M. of 11, 132, and 66 = 132.

We may reduce the mixed numbers to improper fractions, and proceed as in the former example, or we may find the value of the whole numbers, and then of the fractions. Thus—

(1) $6+3-4=5$

(2) $$\frac{5}{11}+\frac{115}{132}-\frac{53}{66}-\frac{110}{132}$$

$$\frac{60+115-106-110}{132}=\frac{175-216}{132}$$

But we cannot subtract $\frac{216}{132}$ from $\frac{175}{132}$, so we must borrow one whole number from the 5.

$$1=\frac{132}{132} \text{ and } \frac{175}{132}+\frac{132}{132}=\frac{307}{132}$$

We can now subtract, thus—

$$\frac{307-216}{132}=\frac{91}{132} \text{ and adding the } 4=4\frac{91}{132}=Ans.$$

By reducing fractions to a common denominator we can compare their value.

Ex. 6. Arrange $\frac{24}{35}$, $\frac{5}{7}$, and $\frac{39}{56}$ in descending order of magnitude.

L. C. M. of 35, 7, and 56 = 280.

$$\frac{24}{35}=\frac{192}{280} \quad \frac{5}{7}=\frac{200}{280} \quad \frac{39}{56}=\frac{195}{280}$$

Hence the required order is—

$$\frac{5}{7}, \frac{39}{56}, \frac{24}{35}$$

Exercises in Addition, &c.

1. Find the value of the following :—
   (1) $\frac{3}{7}+3\frac{1}{6}+19\frac{1}{21}+3\frac{11}{18}$
   (2) $4\frac{2}{3}+6\frac{1}{5}+3\frac{4}{7}+6\frac{1}{8}$
   (3) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$
   (4) $19\frac{1}{19}+27\frac{1}{27}+31\frac{1}{31}$
2. Find the value of the following :—
   (1) $19\frac{3}{7}-7\frac{2}{9}$; $23\frac{1}{4}-17\frac{3}{4}$
   (2) $\frac{7}{8}+\frac{2}{4}+\frac{1}{8}-\frac{3}{8}-\frac{5}{8}$
   (3) $7\frac{1}{2}-2\frac{3}{4}+3\frac{3}{8}-4\frac{2}{7}+5$.
   (4) $\frac{6}{7}+2\frac{1}{21}-\frac{3}{24}-2\frac{1}{7}$
3. How much greater is $\frac{1}{3}+\frac{5}{42}$ than $\frac{1}{4}+\frac{5}{40}$?
4. How much greater is 1 than $\frac{1}{2}\frac{3}{5}$?
5. After using $\frac{2}{19}$ of two parcels of tea, how much is left?
6. A woman sold $\frac{3}{5}$ of her eggs, and broke $\frac{2}{5}$ of the remainder; what part of the original number had she left?
7. What fraction must be added to $\frac{13}{18}+\frac{1}{30}$ to make 1?
8. What fraction added to $\frac{12}{125}$ will make $3\frac{1}{4}$?
9. Which is the greater, and by how much, $\frac{11}{12}$ or $\frac{13}{18}$?
10. Compare the values of $2\frac{1}{4}$ and $\frac{3}{4}$ of $3\frac{1}{6}$.
11. A can do a piece of work in $1\frac{1}{3}$ days, B in $2\frac{1}{9}$ days, and C in $1\frac{1}{8}$ days; in what time will they finish it, if all work together?
12. What number must be taken from $13\frac{8}{11}$ to leave $5\frac{3}{8}$?
13. Write in the order of magnitude $1\frac{23}{72}$, $1\frac{1}{36}$, and $1\frac{1}{3}$.
14. Take the fifth of one from the third of one, and then add one half to the remainder.
15. What is the difference between $6\frac{1}{2}$ shillings and $3\frac{7}{16}$ shillings?
16. What part of a cwt. must be added to $\frac{13}{120}$ to make it $\frac{1}{3}$?
17. Simplify $9\frac{2}{5}+12\frac{1}{6}-3\frac{5}{6}+1\frac{3}{18}-4\frac{11}{30}$.
18. Find the sum of £$3\frac{3}{5}$ $16\frac{2}{3}$*s.* and $5\frac{3}{4}$*d.*
19. Add £471 15*s.* $4\frac{1}{5}$*d.*, £6,950 17*s.* $5\frac{3}{7}$*d.*, £70 16*s.* $9\frac{3}{7}$*d.*, and £725 13*s.* $2\frac{1}{8}$*d.*
20. From £304 11*s.* $2\frac{1}{14}$*d.* subtract £120 1*s.* $1\frac{3}{4}$*d.*
21. Find the difference between 5 cwt. 3 qrs. $17\frac{14}{16}$ lbs. and 2 cwt. 3 qrs. $12\frac{19}{17}$ lbs.

22. When a man has spent $\frac{3}{5}$ of a year's income, and also $\frac{3}{4}$ of the remainder, he has £20 left; what is his income?

23. After losing a fifth, a sixth, and a seventh of my money, what portion of the whole remains?

24. After paying away $\frac{5}{16}$ of the money in my purse, I find I have £$1\frac{3}{8}$ remaining; how much had I at first?

25. If 1,000 guineas represent the value of $\frac{7}{64}$ of a ship, find the total value.

26. Find the value of $13\frac{4}{9} - 29\frac{5}{8} + 41\frac{5}{24} - 15\frac{7}{18}$.

27. Add £27 $4\frac{1}{2}$*s.*, £19 6*s.* $2\frac{5}{6}$*d.*, £5$\frac{2}{7}$, and $59\frac{1}{2}$*d.*

28. A sum of money is to be divided between A, B, and C; A receives $\frac{2}{5}$, and B $\frac{3}{7}$; what share does C receive?

29. What sum must be added to $\frac{31}{99}\frac{5}{0}$ of a guinea to make a guinea?

30. Find the value of £$\frac{1}{3} + \frac{1}{4}$ of £$\frac{1}{3} + \frac{1}{5}$ of £$\frac{1}{6}$.

31. What number is that whose third exceeds its fifth by 30?

32. A and B can do a piece of work in 12 days; when C joins them they can do it in 9 days; in what time can C do it working alone?

33. A milkman starts on his "walk" with a can filled with pure milk; after selling one-fifth he fills up with water; and after selling one-fifth again he fills up with water; what part of the mixture is milk, and what part water?

34. Divide £19 5*s.* between A, B, C, and D in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

35. What sum of money is that of which $\frac{7}{15}$ amounts to £$9\frac{1}{3}$?

36. John sold $\frac{2}{5}$ of his marbles, and lost at play $\frac{2}{7}$ of the whole number, and then had 11 left; how many had he at the first?

Section V.—Multiplication and Division of Fractions.

I. *To Multiply a Fraction by a Whole Number.*

Multiplication is a short method of performing the addition of equal quantities.

Thus $6 \times 5 = 30$ is the same as $5+5+5+5+5+5=30$.

To add fractions with a common denominator, we add the numerators. Thus—

$$\frac{5}{7}+\frac{5}{7}+\frac{5}{7}+\frac{5}{7}+\frac{5}{7}+\frac{5}{7}=\frac{30}{7}$$

but the shortest way of adding six fives is to multiply 5 by 6.

$$\text{Thus } \frac{5\times 6}{7}=\frac{30}{7}$$

Hence, to add $\frac{5}{7}$ to itself 6 times, and to multiply the numerator by 6, gives the same result, the second process being shorter than the first.

That is, *to multiply a fraction by a whole number, multiply the numerator.*

Ex. 1. Multiply $\frac{3}{4}$ by 4.

$$\frac{3\times 4}{4}=\frac{12}{4}=3 \text{ } \textit{Ans.}$$

But $\frac{3}{4\div 4}$ produces the same result, viz. 3.

Hence we may enlarge the above—and say, *To multiply a fraction by the whole number, multiply the numerator or divide the denominator.*

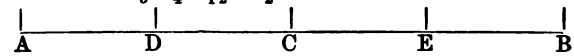
NOTE.—In the above example—

$\frac{3}{4}\times 4=3$; but $\frac{3}{4}$ *of* 4 is 3

that is, the word *of* means the same thing as the sign (×) of multiplication.

II. *To Multiply a Fraction by a Fraction.*

Ex. 2. Show that $\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}$ or $\frac{1}{2}$.



Draw a straight line, A B, and divide it into 4 equal parts on the points D, C, E.

Then A E $=\frac{3}{4}$ of the whole line A B.

And A C $=\frac{2}{3}$ of A E, viz. $\frac{2}{3}$ of $\frac{3}{4}$ of the whole line.

But A C $=\frac{1}{2}$ of the whole line.

$$\therefore \frac{2}{3} \text{ of } \frac{3}{4} \text{ or } \frac{2}{3}\times\frac{3}{4}=\frac{1}{2} \text{ or } \frac{6}{12}$$

Ex. 3. Show that $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}=\frac{6}{24}$ or $\frac{1}{4}$.

Taking the same A B divided into four equal parts—

A E$=\frac{3}{4}$ of A B

A C$=\frac{2}{3}$ of A E, viz. $\frac{2}{3}$ of $\frac{3}{4}$ of A B

A D$=\frac{1}{2}$ of A C, viz. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of A B

But A D$=\frac{1}{4}$ of A B

$$\therefore \frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4}=\frac{1}{4} \text{ or } \frac{6}{24}$$

$$\therefore \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{1}{4} \text{ or } \frac{6}{24}$$

From these examples we deduce the rule for multiplying a fraction by a fraction :—*Multiply the numerators together for a new numerator, and the denominators for a new denominator.*

Ex. 4. Find the value of $3\frac{2}{7} \times 4\frac{1}{5} \times 3\frac{2}{3}$.

When there are mixed quantities we must reduce to improper fractions. Thus—

$$3\tfrac{2}{7} \times 4\tfrac{1}{5} \times 3\tfrac{2}{3} =$$
$$\frac{23}{7} \times \frac{21}{5} \times \frac{11}{3} = \frac{5313}{105} = \frac{1771}{35} = \frac{253}{5} = 50\tfrac{3}{5}$$

In this example we have multiplied the numerators and denominators for the new fraction, and then reduced to lowest terms. It is better to perform this reduction before multiplying. Thus—

$$\frac{23 \times \cancel{21} \times 11}{\cancel{7} \times 5 \times \cancel{3}} = \frac{253}{5} = 50\tfrac{3}{5}$$

III. *To Divide a Fraction by a Whole Number.*

Ex. 5. Divide $\frac{3}{4}$ by 4.

This expression is equivalent to saying, find $\frac{1}{4}$ of $\frac{3}{4}$, which by the preceding rule we know to be $\frac{3}{16}$.

$$\text{But } \frac{3}{4 \times 4} = \frac{3}{16}$$

Hence, *to divide a fraction by a whole number, multiply the denominator.*

Ex. 6. Divide $1\frac{2}{19}$ by 3.

$$\frac{12}{19 \times 3} = \frac{12}{57} = \frac{4}{19}$$

But this result may also be obtained by dividing the numerator. Thus—

$$\frac{12 \div 3}{19} = \frac{4}{19}$$

We may now enlarge the rule :—*To divide a fraction by a whole number, multiply the denominator, or divide the numerator.*

IV. *To Divide a Fraction by a Fraction.*

Ex. 7. Show that $1\frac{2}{35} \div \frac{3}{5} = \frac{4}{7}$.

By the preceding rule, $1\frac{2}{35} \div 3 = \frac{4}{35}$; but this gives a result five times too small, because we are required to divide not by 3, but a fifth part of 3. *We must therefore multiply* by 5, and $\frac{4}{35} \times 5 = \frac{4}{7}$.

In dividing $\frac{12}{35}$ by $\frac{3}{5}$, we have divided by the numerator and multiplied by the denominator of the divisor.

Thus— $$\frac{12}{35} \div \frac{3}{5} = \frac{12}{35} \times \frac{5}{3} = \frac{60}{105} = \frac{4}{7}$$

Hence the rule, *Invert the terms of the divisor and multiply.*

This rule is better expressed thus :—*To divide by a fraction, multiply by its* RECIPROCAL.

NOTE.—When one fraction multiplied by another gives 1 as a product, one fraction is said to be the reciprocal of the other. Thus $\frac{3}{5} \times \frac{5}{3} = \frac{15}{15} = 1$. Hence $\frac{3}{5}$ is the reciprocal of $\frac{5}{3}$, and $\frac{5}{3}$ is the reciprocal of $\frac{3}{5}$.

Ex. 8. Divide $21\frac{5}{6}$ by $3\frac{7}{10}$.

First reduce to improper fractions

$$\frac{131}{6} \div \frac{37}{10} = \frac{131 \times \overset{5}{\cancel{10}}}{\underset{3}{\cancel{6}} \times 37} = \frac{655}{111} = 5\tfrac{100}{111}$$

Ex. 9. Divide $\frac{2}{3}$ of $\frac{5}{6}$ of $1\frac{1}{4}$ by $\frac{1}{3}$ of $\frac{3}{7}$ of 4.

$$\left(\frac{\cancel{2}}{3} \times \frac{5}{\underset{3}{\cancel{6}}} \times \frac{5}{4}\right) \div \left(\frac{1}{\cancel{3}} \times \frac{\cancel{3}}{7} \times \frac{4}{1}\right) =$$

$$\frac{25}{36} \div \frac{4}{7} = \frac{25 \times 7}{36 \times 4} = \frac{175}{144} = 1\tfrac{31}{144}$$

Ex. 10. Simplify $\dfrac{5\frac{5}{6} \div 9\frac{1}{4}}{3\frac{3}{8} \div 11\frac{3}{4}}$

NOTE.—This kind of fraction is sometimes called *complex*. It means that all above the line has to be divided by all below. It may be written thus—

$$(5\tfrac{5}{6} \div 9\tfrac{1}{4}) \div (3\tfrac{3}{8} \div 11\tfrac{3}{4})$$

$$\frac{5\frac{5}{6} \div 9\frac{1}{4}}{3\frac{3}{8} \div 11\frac{3}{4}} = \frac{\frac{45}{8} \div \frac{37}{4}}{\frac{27}{8} \div \frac{47}{4}} = \frac{\frac{45}{8} \times \frac{4}{37}}{\frac{27}{8} \times \frac{4}{47}}$$

$$= \left(\frac{45}{8} \times \frac{4}{37}\right) \div \left(\frac{27}{8} \times \frac{4}{47}\right)$$

$$= \frac{\overset{5}{\cancel{45}}}{\cancel{8}} \times \frac{\cancel{4}}{37} \times \frac{\cancel{8}}{\underset{3}{\cancel{27}}} \times \frac{47}{\cancel{4}} = \frac{235}{111} = 2\tfrac{13}{111}$$

Ex. 11. Simplify $\dfrac{3\frac{1}{2} \text{ of } 4\frac{1}{3} \text{ of } 5\frac{1}{4}}{6\frac{1}{5} \text{ of } 7\frac{1}{6} \text{ of } 8\frac{1}{7}} \div \left(\dfrac{14}{817} \text{ of } \dfrac{65}{93}\right)$

$$= \frac{3\frac{1}{2} \times 4\frac{1}{3} \times 5\frac{1}{4}}{6\frac{1}{5} \times 7\frac{1}{6} \times 8\frac{1}{7}} \div \left(\frac{14}{817} \times \frac{65}{93}\right)$$

$$= \frac{\frac{7}{2} \times \frac{13}{3} \times \frac{21}{4}}{\frac{31}{5} \times \frac{43}{6} \times \frac{57}{7}} \div \left(\frac{14}{817} \times \frac{65}{93}\right)$$

$$= \frac{7}{2} \times \frac{13}{3} \times \frac{21}{4} \times \frac{5}{31} \times \frac{6}{43} \times \frac{7}{57} \times \frac{817}{14} \times \frac{93}{65}$$

$$= \frac{7}{2} \times \frac{21}{4} = \frac{147}{8} = 18\frac{3}{8} \text{ } Ans.$$

Exercises in Multiplication and Division of Fractions.

1. Multiply $\frac{3}{10}$ of $\frac{2}{3}$ by $\frac{5}{7}$ of $\frac{7}{16}$.
2. Divide $\frac{3}{10}$ of $\frac{2}{3}$ by $\frac{5}{7}$ of $\frac{7}{16}$.
3. Simplify (*a*) $\dfrac{9\frac{4}{5} \div 3\frac{3}{10}}{6\frac{3}{4} \times 3\frac{1}{2}}$; (*b*) $\dfrac{9\frac{4}{5} \times 3\frac{3}{10}}{6\frac{3}{4} \div 3\frac{1}{2}}$
4. What is the value of $\frac{3}{5}$ of $\frac{5}{12}$ of $\frac{1}{4}$ of 4.
5. Find the value $(4\frac{9}{12} \text{ of } 2\frac{3}{5}) \div 3\frac{1}{6}$.
6. Simplify $2\frac{2}{15} \times 16\frac{5}{7} \div \frac{2}{3}$ of $2\frac{2}{5}$.
7. Find the value of $6\frac{2}{5} \times 2\frac{1}{19} \times \frac{5}{21} \times \frac{7}{13} \times 2\frac{3}{11} \times 1\frac{1}{4}$.
8. Simplify $\dfrac{6\frac{4}{5} \text{ of } \frac{5}{22} \text{ of } \frac{7}{10} \text{ of } 2\frac{19}{17}}{2\frac{3}{11} \text{ of } 7\frac{1}{2}}$
9. Simplify (*a*) $\dfrac{5\frac{1}{6} \div 3\frac{7}{12}}{3\frac{3}{8} \div 14\frac{1}{3}}$; (*b*) $\dfrac{\dfrac{2\frac{1}{2} \div 1\frac{1}{4}}{3\frac{1}{3} \div 2\frac{2}{3}}}{\dfrac{3\frac{3}{5}}{2\frac{1}{2}} \times \dfrac{2\frac{2}{5}}{1\frac{1}{4}}}$
10. Simplify (*a*) $\dfrac{9\frac{2}{6} \div 7\frac{2}{7}}{1\frac{1}{4}\frac{3}{5}\frac{2}{9}}$; (*b*) $\dfrac{1\frac{1}{8} \div 1\frac{1}{5}}{\frac{2}{3} \text{ of } \frac{5}{6} \div 10\frac{1}{3}}$
11. Find the value of $\dfrac{1\frac{2}{3} \text{ of } \frac{3}{5} \text{ of } 21}{23} \text{ of } \dfrac{3\frac{5}{6}}{5\frac{3}{12}}$
12. Simplify $\dfrac{\frac{2}{3} \text{ of } \frac{5}{7} \text{ of } \frac{3}{6}\frac{5}{5}}{\frac{1}{7} \text{ of } \frac{2}{3} \text{ of } \frac{8}{8}\frac{4}{1}} \div 18\frac{3}{4}$
13. Reduce to a simple form—

$$\frac{2\frac{1}{5} \times 1\frac{3}{33}}{\frac{3}{7} \times 1\frac{2}{17}} \div \frac{1\frac{2}{15} \times 1\frac{1}{19}}{\frac{5}{13} \times 1\frac{6}{17}}$$

14. Multiply $\frac{8\frac{1}{2}}{9\frac{7}{9}}$ of $\frac{3}{8}$ by $\frac{11\frac{8}{9}}{14\frac{7}{17}}$ of $\frac{13\frac{4}{11}}{23\frac{3}{8}}$

15. Reduce to its simplest form—

$$\frac{2\frac{1}{7} \text{ of } 1\frac{1}{23}}{\frac{11}{17} \times 1\frac{4}{17}} \div \frac{1\frac{2}{3} \times 1\frac{1}{10}}{\frac{3}{13} \text{ of } 1\frac{6}{17}}$$

16. A pint contains $34\frac{2}{3}$ cubic in.; how many pints are there in a cubic ft.?

17. How many $7\frac{1}{3}$-lbs. packets can be made up out of 2 cwts. 2 qrs. $9\frac{2}{3}$ lbs.? Multiply ($\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{2}{4}$) by ($\frac{1}{3}$ of $1\frac{2}{7}$ of $1\frac{1}{3}$ of $1\frac{2}{5}$), and divide the result by ($\frac{1}{3}$ of $\frac{1}{4}$ of $2\frac{2}{5}$).

18. What number is that of which 30 is $\frac{5}{7}$?

19. If one loaf takes $3\frac{5}{6}$ lbs. of flour, how many loaves can be made out of a sack weighing 2 cwts. 6 lbs.?

20. Find the value of $\dfrac{\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}}{\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}}$

21. What number multiplied by $\frac{5}{8}$ will give $2\frac{1}{2}$?

22. What number divided by $5\frac{1}{3}$ will give $3\frac{3}{8}$?

23. Of what sum of money is $3\frac{1}{2}$ gs. seven-thirteenths?

24. If a cubic ft. of ice weighs $57\frac{3}{8}$ lbs., how many cubic ft. of ice will weigh a ton?

25. Divide $\dfrac{7(1\frac{1}{2} \text{ of } \frac{6}{14})}{\frac{1}{6}(3\frac{1}{2} \text{ of } 7)}$ by $\frac{9}{14}$

26. If a yd. of lace cost £$1\frac{2}{3}\frac{9}{6}$, what will $16\frac{1}{2}\frac{1}{5}$ cost?

SECTION VI.—EXERCISES ON THE ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF FRACTIONS.

Ex. 1. Simplify $\dfrac{4\frac{3}{7} - \frac{5}{9} \text{ of } 4\frac{2}{13} + 6 \div 4\frac{2}{3}}{18\frac{90}{91} \div 8 - \frac{2}{3} \text{ of } 1\frac{6}{7}}$

$$= \frac{\frac{31}{7} - \frac{5}{9} \times \frac{\cancel{6}^{2}}{13} + \frac{\cancel{6}^{3}}{1} \times \frac{3}{\cancel{14}_{7}}}{\frac{\cancel{216}^{1}}{\cancel{91}_{1}} \times \frac{1}{\cancel{8}_{1}} - \frac{2}{3} \times \frac{13}{7}}$$

$$= \frac{\frac{31}{7} - \frac{30}{13} + \frac{9}{7}}{\frac{216}{91} - \frac{26}{21}} = \frac{\frac{403-210+117}{91}}{\frac{648-338}{273}}$$

$$= \frac{310}{91} \div \frac{310}{273} = \frac{\cancel{310}}{\cancel{91}} \times \frac{\overset{3}{\cancel{273}}}{\cancel{310}} = 3. \quad \textit{Ans.}$$

NOTE.—There are several points to be noted in connection with examples such as the above.

1. The main line—drawn thicker than the others—must be kept during the operation.
2. The student should not attempt to do too much at each step. Mistakes often arise, for instance, in reducing to an improper fraction and inverting for multiplication at one process.
3. The greatest care must be taken to distinguish factors from terms. The signs × and ÷ connect factors, the signs + and − separate terms. We must reduce all fractions connected with the sign × or ÷ to simple fractions before we can add or subtract the terms.

Ex. 2. Simplify $(\frac{7}{16} - \frac{1}{4}) \times (1\frac{1}{4} + \frac{1}{3}) \div (\frac{1}{16} + \frac{5}{64})$.

In this and similar examples the expressions inside the brackets must first be reduced to simple fractions before we can multiply or divide. The sign (×) of multiplication does not in this case connect the $\frac{1}{4}$ and $1\frac{1}{4}$ as parts of one term. Brackets indicate that the terms or factors within form a single term ; hence the necessity for reduction to a simple fraction.

$$\left(\frac{7}{16} - \frac{1}{4}\right) \times \left(1\frac{1}{4} + \frac{1}{3}\right) \div \left(\frac{1}{16} + \frac{5}{64}\right)$$

$$= \left(\frac{7}{16} - \frac{4}{16}\right) \times \left(\frac{5}{4} + \frac{1}{3}\right) \div \left(\frac{4}{64} + \frac{5}{64}\right)$$

$$= \left(\frac{7-4}{16}\right) \times \left(\frac{15+4}{12}\right) \div \left(\frac{4+5}{64}\right)$$

$$= \frac{3}{16} \times \frac{19}{12} \div \frac{9}{64} =$$

$$\frac{\cancel{3} \times 19 \times \overset{4}{\cancel{64}}}{\cancel{16} \times \cancel{12} \times 9} = \frac{19}{9} = 2\frac{1}{9}$$

Ex. 3. Simplify $\dfrac{3\frac{3}{4}+4\frac{2}{5}}{6\frac{1}{2}-1\frac{3}{5}} \div \dfrac{5\frac{3}{5}+2\frac{11}{20}}{9\frac{1}{5}-4\frac{3}{10}}$

$$\frac{3\frac{3}{4}+4\frac{2}{5}}{6\frac{1}{2}-1\frac{3}{5}} \div \frac{5\frac{3}{5}+2\frac{11}{20}}{9\frac{1}{5}-4\frac{3}{10}} =$$

$$\frac{\frac{15}{4}\div\frac{22}{5}}{\frac{13}{2}-\frac{8}{5}} \div \frac{\frac{28}{5}\times\frac{51}{20}}{\frac{46}{5}-\frac{43}{10}} =$$

$$\frac{\frac{75+88}{20}}{\frac{65-16}{10}} \div \frac{\frac{112+51}{20}}{\frac{92-43}{10}} =$$

$$\frac{\frac{163}{20}}{\frac{49}{10}} \div \frac{\frac{163}{20}}{\frac{49}{10}} = \left(\frac{163}{\cancel{20}_{2}}\times\frac{\cancel{10}^{1}}{49}\right) \div \left(\frac{163}{\cancel{20}_{2}}\times\frac{\cancel{10}^{1}}{49}\right)$$

$$= \frac{\cancel{163}^{1}}{\cancel{98}_{1}} \times \frac{\cancel{98}^{1}}{\cancel{163}_{1}} = \frac{1}{1} = 1$$

NOTE.—In this example all the numerators and denominators cancel out, or, in other words, the factors become units only. This must not be taken to mean that the numerator or denominator is reduced to nothing, for *one* taken once over is *one*. Hence the numerator is *one* and the denominator *one*, and *one* divided by *one* gives a quotient of *one*. The required answer, therefore, is 1.

Ex. 4. Simplify $\dfrac{3\frac{3}{7}+1\frac{5}{12}-4\frac{9}{14}-\frac{17}{84}}{2\frac{7}{9}-2\frac{1}{3}+3\frac{17}{18}}$

$$\frac{3\frac{3}{7}+1\frac{5}{12}-4\frac{9}{14}-\frac{17}{84}}{2\frac{7}{9}-2\frac{1}{3}+3\frac{17}{18}}$$

$$= \frac{\frac{24}{7}+\frac{17}{12}-\frac{65}{14}-\frac{17}{84}}{\frac{25}{9}-\frac{7}{3}+\frac{71}{18}}$$

$$= \frac{\frac{288+119-390-17}{84}}{\frac{50-42+71}{18}}$$

$$= \frac{\frac{407-407}{84}}{\frac{121-42}{18}} = \frac{\frac{0}{84}}{\frac{79}{18}}$$

But $\frac{0}{84}=0 \quad \therefore \quad \frac{0}{\frac{79}{18}}=0$

Hence the required answer is 0.

Ex. 5. Simplify $1+\frac{1}{1+\frac{1}{1+\frac{1}{2}}}$

In simplifying fractions similar to this, commence at the end.

$$1+\frac{1}{2}=\frac{3}{2} \text{ and } \frac{1}{\frac{3}{2}}=\frac{2}{3}$$

$$1+\frac{2}{3}=\frac{5}{3} \text{ and } \frac{1}{\frac{5}{3}}=\frac{3}{5}$$

$$1+\frac{3}{5}=\frac{8}{5}=1\frac{3}{5} \textit{ Ans.}$$

The working is best shown by steps, thus :—

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{2}}} = 1+\frac{1}{1+\frac{1}{\frac{3}{2}}} =$$

$$1+\frac{1}{1+\frac{2}{3}}=1+\frac{1}{\frac{5}{3}}=1+\frac{3}{5}$$

$$=\frac{8}{5}=1\frac{3}{5} \textit{ Ans.}$$

EXERCISES.

1. Simplify $\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{4}-\frac{1}{5}}$

2. Find the value of $(2\frac{1}{2} \times 1\frac{1}{4}) \times \frac{2}{5}$.

3. Which is the greater, and by how much, $\frac{2\frac{1}{2}+3\frac{1}{4}}{1\frac{3}{5}-\frac{2}{4}}$ or $\frac{3\frac{1}{3}+2\frac{1}{4}}{2\frac{1}{4}-\frac{1}{6}}$?

4. Find the simplest expressions for $\frac{8}{13} \times 2\frac{2}{3} \times 1\frac{1}{2\frac{1}{5}} \times 1\frac{11}{64}$ and $(3\frac{1}{1\frac{2}{3}}$ of $3\frac{1}{7}) \div (\frac{4}{5\frac{1}{7}}$ of 9).

5. Simplify $\frac{2\frac{5}{12}+8\frac{1}{2}+2\frac{2}{3}+\frac{1}{16}+14\frac{3}{4}}{1\frac{1}{12}+1\frac{1}{13}+\frac{5}{7}+\frac{33}{40}}$

6. Find the value of $(1 \times \frac{1}{2} \times \frac{1}{3}) \times (1+\frac{1}{2}+\frac{1}{3}) \times (1-\frac{1}{2}-\frac{1}{3})$.

7. To $\frac{7}{1\frac{2}{3}}$ of 30 add $\frac{5}{16}$ of $80\frac{1}{2}$, and divide the result by $4\frac{7}{8}$.

8. How many times does the sum of $6\frac{5}{7}$ and $4\frac{3}{8}$ contain their difference?

9. Simplify $(\frac{1}{4}+\frac{2}{5})-(\frac{1}{4}+\frac{3}{7})$.

10. From $11\frac{9}{10}$ take the sum of $2\frac{1}{2}$, $3\frac{2}{3}$, and $4\frac{3}{5}$.

11. Find the difference between $\frac{4}{7}$ of $\frac{2}{3}$ of $504\frac{1}{6}$ and $\frac{2}{3}$ of $\frac{5}{11}$ of 1,089. Divide the result by $\frac{2}{5}$ of $\frac{2}{11}$ of $504\frac{1}{6}$.

12. What number added to $1\frac{7}{11}+2\frac{7}{10}+3\frac{5}{2\frac{1}{2}}+\frac{9}{2\frac{1}{4}}$ will make 20?

13. Simplify $\frac{1}{7}-\frac{1}{9}-\frac{1}{11}+\frac{1}{13}$, and multiply the result by $\frac{9}{2\frac{1}{5}}$.

14. Divide the product of the sum and difference of $7\frac{1}{8}$ and $3\frac{2}{8}$ by $\frac{7}{12}$.

15. Simplify $\frac{(2+\frac{1}{2})\div(3+\frac{1}{2})}{(\frac{1}{2}-\frac{1}{3})\times(4-3\frac{3}{7})}$

16. How much does the sum of $53\frac{7}{8}$ and $12\frac{5}{9}$ exceed the sum of $9\frac{1}{5}$ and $\frac{8\frac{2}{3}1}{16}$?

17. Simplify $(2\frac{1}{3}+\frac{4}{5}$ of $3\frac{1}{3}$ of $\frac{3}{4}-2\frac{1}{2}) \div 3\frac{2}{3}$.

18. Find the value of $\frac{\frac{3}{4}\text{ of }1\frac{2}{5}-\frac{1}{2}\text{ of }\frac{1}{5}}{\frac{1}{3}\text{ of }\frac{1}{2}+\frac{1}{3}}$

19. Simplify (*a*) $\frac{16}{21}+\frac{4}{3\frac{1}{2}}-\frac{81}{10\frac{1}{2}}$ and (*b*) $\frac{5}{16}$ of $\frac{24}{7}$ of $\frac{9\frac{5}{9}}{6\frac{5}{8}}$ of $\frac{9\frac{5}{9}}{4\frac{6}{7}}$

20. Simplify $\left(\frac{2\frac{1}{3}-\frac{7}{9}+\frac{2}{7}}{1+\frac{9}{7}-\frac{1}{6}}\right) \div \left(\frac{\frac{2}{7}-\frac{2}{5}+\frac{1}{5}}{\frac{6}{9}+\frac{2}{5}-\frac{1}{6}}\right)$

21. Divide the continued product of $\frac{4}{5}$, $\frac{3}{7}$, $1\frac{3}{4}$, $\frac{8}{9}$, $2\frac{1}{4}$, and $\frac{5}{6}$ by the difference between $2\frac{1}{3}$ of 6 and $\frac{5}{9}$ of 4.

22. Multiply $\frac{7\frac{1}{6}-3\frac{1}{3}}{6\frac{1}{4}+1\frac{1}{8}}$ by $\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$ and divide the product by $\frac{21\frac{1}{2}-9\frac{2}{5}}{12\frac{1}{2}+2\frac{1}{4}}$

23. Reduce to its simplest form $\frac{5\frac{2}{7}-\frac{1}{5}\text{ of }2\div 3\frac{1}{2}}{2\frac{13}{14}-\frac{1}{2}+\frac{1}{4}-\frac{1}{5}-\frac{1}{7\frac{1}{8}}}$

24. Simplify $\frac{17}{\frac{1}{2}+\frac{1}{3}+\frac{7}{8}+\frac{5}{12}}-\frac{7\frac{1}{2}}{3\frac{1}{2}}$

25. Simplify $\frac{2\frac{1}{6}-3\frac{7}{8}+8\frac{5}{24}-5\frac{2}{3}+\frac{4}{5}}{1-\frac{1}{3}+\frac{1}{30}-\frac{1}{120}}$

26. Find the simplest form of $\frac{6\frac{5}{6}-5\frac{5}{8}+9\frac{7}{12}+8\frac{1}{3}-\frac{9}{10}}{1+\frac{1}{2}+\frac{1}{20}+\frac{1}{120}}$

27. Simplify $1+\cfrac{1}{3+\cfrac{1}{3+\frac{1}{3}}}$

28. Simplify the fraction $\frac{1\frac{1}{2}}{7}$ of $\frac{4\frac{3}{4}}{2\frac{1}{4}}+\frac{1}{2\frac{1}{4}}$ of $\frac{4\frac{1}{2}}{3\frac{1}{2}}-\frac{1\frac{8}{9}}{14}$

29. Simplify $1\frac{1}{2}+\cfrac{1\frac{1}{2}}{1\frac{1}{2}+\cfrac{1}{1\frac{1}{2}+\cfrac{1}{1\frac{1}{2}}}}$ and $\cfrac{\frac{3}{7}+\cfrac{3\frac{5}{7}}{2-\frac{2}{21}}}{1-\frac{3}{7}\text{ of }\left(\cfrac{\frac{2}{14}}{3-\frac{1}{7}}+\cfrac{2\frac{1}{2}}{1\frac{3}{4}}\right)}$

30. Simplify $\frac{\frac{1}{4}}{(\frac{1}{2}-\frac{1}{3})\times(\frac{1}{2}-\frac{1}{4})}+\frac{\frac{1}{9}}{(\frac{1}{3}-\frac{1}{4})\times(\frac{1}{3}-\frac{1}{2})}+\frac{\frac{1}{16}}{(\frac{1}{4}-\frac{1}{2})\times(\frac{1}{4}-\frac{1}{3})}$

Section VII.—Reduction of Fractions.

I. *To find the value of a fractional part of a given quantity.*

This is an application of the simple rules in fractions, and as such a few easy examples have already been given.

Ex. 1. Find the value of $\frac{3}{8}$ of 9*s.* 4*d.* + $\frac{5}{9}$ of £1 10*s.* 9*d.* + $\frac{6}{7}$ of 4*s.* 1*d.* + $\frac{2}{5}$ of $3\frac{3}{4}$*d.*

			£	*s.*	*d.*
$\frac{3}{8}$ of 9*s.* 4*d.*	$=\frac{9s.\ 4d.\times 3}{8}$	=	0	3	6
$\frac{5}{9}$ of £1 10*s.* 9*d.*	$=\frac{£1\ 10s.\ 9d.\times 5}{9}$	=	0	17	1
$\frac{6}{7}$ of 4*s.* 1*d.*	$=\frac{4s.\ 1d.\times 6}{7}$	=	0	3	6
$\frac{2}{5}$ of $3\frac{3}{4}$*d.*	$=\frac{3\frac{3}{4}d.\times 2}{5}$	=	0	0	$1\frac{1}{2}$
			£1	4	$2\frac{1}{2}$ *Ans.*

Ex. 2. Find the difference between $\frac{3}{8}$ of a mile and $\frac{3}{5}$ of a furlong.

furlongs.
$\frac{3\times 8}{8}=3$ furlongs $=120$ poles.

poles.
$\frac{3\times 40}{5}=24$ poles.

Difference $=120-24=96$ poles.

NOTE.—In this example we reduce to furlongs and poles by the ordinary method of reduction.

Ex. 3. Find the value of $\frac{1}{3\frac{3}{4}}$ cwt. $+\frac{2\frac{1}{3}}{3\frac{3}{4}}$ qrs. $-15\frac{5}{9}$ lbs., and give the answer as the fraction of a cwt.

$$\frac{1}{3\frac{3}{4}} \text{ cwt.} + \frac{2\frac{1}{3}}{3\frac{3}{4}} \text{ qrs.} - 15\frac{5}{9} \text{ lbs.}$$

$$= \frac{1}{\frac{15}{4}} \text{ cwt.} + \frac{\frac{7}{3}}{\frac{15}{4}} \text{ qrs.} - \frac{140}{9} \text{ lbs.}$$

$$= \frac{4}{15} \text{ cwt.} + \left(\frac{7}{3} \times \frac{4}{15}\right) \text{ qrs.} - \frac{140}{9} \text{ lbs.}$$

$$= \frac{4}{15} \text{ cwt.} + \left(\frac{7}{3} \times \frac{4}{15} \div 4\right) \text{ cwt.} - \left(\frac{140}{9} \div 112\right) \text{cwt.}$$

$$= \left(\frac{4}{15} + \left(\frac{7}{3} \times \frac{4}{15} \times \frac{1}{4}\right) - \frac{\cancel{140}^{\,5}}{9 \times \cancel{112}_{\,28}}\right) \text{ cwt.}$$

$$= \left(\frac{4}{15} + \frac{7}{45} - \frac{5}{36}\right) \text{ cwt.}$$

$$= \frac{48 + 28 - 25}{180} \text{ cwt.}$$

$$= \frac{51}{180} \text{ cwt.} \quad \textit{Ans.}$$

II. *To reduce one quantity to the fraction of another.*

Ex. 1. Reduce 5*s.* to the fraction of £1.

This question may be written, What fraction or even part of a £ is 5*s.*? That is, we are to take £1 as unity, and, dividing it into 20 parts (shillings), take five of these parts. Our fraction will be $\frac{5}{20}$, or $\frac{1}{4}$.

In solving this problem we have performed three operations.

(1) We reduced to same denomination.

(2) We placed the quantity to be reduced above the line, and the quantity we required the fraction of below the line, to form the fraction.

(3) We reduced the fraction to lowest terms.

It is best in practice to perform the second of these operations first.

Ex. 2. Reduce 6*s.* $9\frac{3}{4}d.$ to the fraction of 11*s.* $4\frac{1}{4}d.$

In this example 6*s.* $9\frac{3}{4}d.$ has to be reduced, and is therefore placed above the line; and we require the fraction of 11*s.* $4\frac{1}{4}d.$, and this amount is therefore placed below the line. We next reduce to same denomination, viz. farthings, and then reduce the fraction to its lowest terms.

$$\frac{6s.\ 9\frac{3}{4}d.}{11s.\ 4\frac{1}{4}d.} = \frac{\overset{\text{farthings.}}{327}}{545} = \frac{3}{5}\ \textit{Ans.}$$

Note.—G. C. M. of 327 and 545 = 109.

Ex. 3. Reduce 7*s.* $3\frac{3}{5}d.$ to the fraction of 10*s.* $5\frac{1}{7}d.$

$$\frac{7s.\ 3\frac{3}{5}d.}{10s.\ 5\frac{1}{7}d.} = \frac{87\frac{3}{5}d.}{125\frac{1}{7}d.} = \frac{\frac{438}{5}}{\frac{876}{7}} =$$

$$\frac{\overset{1}{\cancel{438}}}{\underset{5}{\cancel{875}}} \times \frac{7}{\underset{2}{\cancel{876}}} = \frac{7}{10}\ \textit{Ans.}$$

Ex. 4. Express $2\frac{1}{3}$ of $\frac{1}{7}$ of 3 acres + $\frac{1}{5}$ of $\frac{5}{6}$ of 3 roods as the fraction of $\frac{1}{3}$ of $\frac{1}{9}$ of $\frac{1}{2}$ an acre.

$$\frac{2\frac{1}{3} \text{ of } \frac{1}{7} \text{ of 3 acres} + \frac{1}{5} \text{ of } \frac{5}{6} \text{ of 3 roods}}{\frac{1}{3} \text{ of } \frac{1}{9} \text{ of } \frac{1}{2} \text{ an acre.}}$$

$$= \frac{\left(\frac{\cancel{7}}{\cancel{3}} \times \frac{1}{\cancel{7}} \times \frac{\cancel{3}}{1}\right) + \left(\frac{1}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} \times \frac{\overset{2}{\cancel{6}}}{1} \times \frac{1}{4}\right)}{\frac{1}{3} \times \frac{1}{9} \times \frac{1}{2}}$$

$$\left(\frac{1 + \frac{1}{8}}{\frac{1}{54}} = \frac{\frac{9}{8}}{\frac{1}{54}}\right) = \frac{9}{8} \times \frac{54}{1}$$

$$= \frac{486}{8} = 60\frac{6}{8} = 60\frac{3}{4}\ \textit{Ans.}$$

Exercises.

1. Reduce 2*s.* $8\frac{1}{2}d.$ to the fraction of 2*s.* 6*d.*
2. What part of £2 is $\frac{1}{5}$ of half-a-guinea?
3. What is the difference between $\frac{8}{9}$ of a guinea and $\frac{9}{10}$ of a £?

4. Reduce $£\frac{9}{10} - \frac{5}{8}$ of a guinea to the fraction of 2*s.* 6*d.*

5. Find the value of $\frac{3}{4}$ of a guinea $+ \frac{7}{8}$ of a crown $- 2\frac{1}{8}$ shillings $+ 3\frac{5}{8}$ of $4\frac{1}{2}d.$

6. Find the value of $(\frac{1}{3} + \frac{5}{8})$ of $£1 + \left(\frac{4}{7} - \frac{1}{3\frac{1}{2}}\right)$ of a guinea $+ \left(\frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{4} - \frac{1}{5}}\right)$ of 1*d.*

7. Reduce 13*s.* 4*d.* to the fraction of £2 6*s.* 8*d.*

8. What fraction is 3*s.* $7\frac{3}{4}d.$ of 18*s.* $2\frac{3}{4}d.$?

9. Which is the greater, and by how much, $\frac{3}{5}$ of $\frac{5}{7}$ of 15*s.* 9*d.*, or $(\frac{3}{7} - \frac{1}{4})$ of £1 17*s.* 4*d.* ?

10. Subtract $\frac{3}{11}$ of $2\frac{3}{4}$ cwts. from $3\frac{1}{7}$ tons.

11. Reduce $\frac{3}{10}$ of a guinea to the fraction of a halfpenny.

12. Find the difference in pence between $\frac{3}{16}$ of £1 and $\frac{1}{12}$ of half-a-crown.

13. Reduce 7 lbs. 8 ozs. to a fraction of 18 lbs.

14. Which is the greater, $\frac{1}{2\frac{1}{5}}$ of a day, or $\frac{2}{7}$ of an hour, and by how much ?

15. Express 42 lbs. as a fraction of a ton.

16. Find the difference between $\frac{3}{5}$ of $\frac{5}{8}$ of a guinea, and $\frac{4}{5}$ of $\frac{3}{8}$ of £1.

17. Express 4 hrs. as a fraction of a week.

18. Express $3\frac{5}{8}$ of £1 0*s.* $9\frac{3}{4}d.$ as the fraction of £1 10*s.* 10*d.* and of £5 9*s.* $7\frac{5}{8}d.$

19. Divide the sum of $\frac{3}{5}$ of £3 7*s.* 6*d.* and $\frac{1}{6}$ of $4\frac{1}{2}$ gs. by $10\frac{5}{7}$.

20. What part of a cwt. is $\frac{2}{7}$ of $\frac{1}{2}$ of $\frac{1}{5}$ of a ton ?

21. Express 30 sq. yds. as the fraction of a rood.

22. Subtract $\frac{1}{2}\frac{3}{0}$ of 4 sq. poles from $\frac{4}{11}$ of an acre.

23. What fraction is 5 qrs. 3 lbs. of 8 cwts. 3 qrs. 21 lbs. ?

24. Express 3 dwts. 6 grs. as a fraction of 1 lb. troy.

25. Reduce $31\frac{1}{\frac{1}{2}}$ of 3 days 7 hrs. to the fraction of 3 weeks ; and $2\frac{5}{7}$ of 14 lbs. to the fraction of 3 qrs. 10 lbs.

26. Write 26 mins. 15 secs. as the fraction of an hour.

27. Express $2\frac{2}{3}$ half-guineas as the fraction of 10*s.* $11\frac{1}{4}d.$

28. What fraction of a mile is $\frac{2}{7}$ of $\frac{3}{4}$ of a mile $+ \frac{5}{6}$ of $\frac{3}{8}$ of $2\frac{1}{2}$ furs. $+ \frac{2}{7}$ of 4 furs. $+ 18\frac{3}{4}$ poles ?

29. Express 89 galls. 1 qt. 1 pt. as a fraction of 572 gals.

30. Write 1 acre 3 roods 23 poles as a fraction of a square mile.

MISCELLANEOUS EXERCISES.

SECTION I.—(WORKED OUT.)

Ex. 1. Find the G. C. M. of $\frac{3}{7}$ of £17 17*s.* and $\frac{3}{11}$ of £37 8*s.*

$$\frac{3}{7} \text{ of } £17\ 17s. = \frac{3}{\cancel{7}} \times \frac{\overset{51}{\cancel{357}}}{1} = 153s.$$

$$\frac{3}{11} \text{ of } £37\ 8s. = \frac{3}{\cancel{11}} \times \frac{\overset{68}{\cancel{748}}}{1} = 204s.$$

G. C. M. of 153 and 204 = 51

That is, 51*s.* or £2 11*s.* is the G. C. M. required.

Ex. 2. What is the L. C. M. of $1\frac{7}{13}$ of 11*s.* $7\frac{3}{4}d.$ and $\frac{12}{19}$ of 17*s.* $0\frac{1}{4}d.$?

$$1\tfrac{7}{13} \text{ of } 11s.\ 7\tfrac{3}{4}d. = \frac{20}{13} \times \frac{559}{1} = \overset{\text{farthings.}}{860} \text{ farthings.}$$

$$\tfrac{12}{19} \text{ of } 17s.\ 0\tfrac{1}{4}d. = \frac{12}{19} \times \frac{817}{1} = \overset{\text{farthings.}}{516} \text{ farthings.}$$

$860 = 2 \times 2 \times 5 \times 43$

$516 = 2 \times 2 \times 3 \times 43$

$\therefore$ L. C. M. of 860 and 516

$= 2 \times 2 \times 5 \times 43 \times 3 = 2580$

and 2580 farthings = £2 13*s.* 9*d.*

$\therefore$ £2 13*s.* 9*d.* is the L. C. M. required.

Ex. 3. Express the ratio of $\frac{1}{2} \times (\frac{1}{3} + \frac{1}{2}) - \left(\frac{1\frac{1}{2}}{2} - \frac{1\frac{1}{2}}{3}\right)$ of £6 16*s.* $8\frac{1}{4}d.$ to $\frac{\frac{1}{2}}{\frac{1}{2} - \frac{1}{3}} + \frac{1\frac{1}{2}}{\frac{1}{2} + \frac{1}{3}} - \frac{\frac{1}{3}}{\frac{1}{4} - \frac{1}{6}}$ of £11 7*s.* $9\frac{3}{4}d.$ in its lowest terms.

Writing it first in the form of a fraction we get—

$$\frac{\frac{1}{2} \times (\frac{1}{3} + \frac{1}{2}) - \left(\frac{1\frac{1}{2}}{2} - \frac{1\frac{1}{2}}{3}\right) \text{ of } £6\ 16s.\ 8\tfrac{1}{4}.}{\frac{\frac{1}{2}}{\frac{1}{2} - \frac{1}{3}} + \frac{1\frac{1}{2}}{\frac{1}{2} + \frac{1}{3}} - \frac{\frac{1}{3}}{\frac{1}{4} - \frac{1}{6}} \text{ of } £11\ 7s.\ 9\tfrac{3}{4}d.}$$

$$= \left(\frac{1}{2} \times \frac{2+3}{6}\right) - \left\{\begin{matrix} \frac{3}{2} & \frac{3}{2} \\ \frac{2}{1} & \frac{2}{1} \end{matrix}\right\} \text{ of 6561 farthings.}$$

$$= \frac{\frac{1}{2}}{\frac{3-2}{6}} + \frac{\frac{3}{2}}{\frac{3+2}{6}} - \frac{\frac{1}{3}}{\frac{9-4}{36}} \text{ of 10935 farthings.}$$

$$= \frac{\left(\frac{1}{2} \times \frac{5}{6}\right) - \left(\frac{3}{4} - \frac{3}{6}\right) \text{ of 6561}}{\frac{1}{2} + \frac{3}{5} - \frac{1}{3} \text{ of 10935}}$$

$$= \frac{\left(\frac{1}{2} \times \frac{5}{6}\right) - \left(\frac{9-6}{12}\right) \text{ of 6561}}{\frac{6}{2} + \frac{18}{10} - \frac{36}{15} \text{ of 10935}}$$

$$= \frac{\frac{5}{12} - \frac{3}{12} \text{ of 6561}}{\frac{90+54-72}{30} \text{ of 10935}}$$

$$= \frac{\frac{1}{6} \times \frac{6561}{1}}{\frac{12}{5} \times \frac{10935}{1}} = \frac{1}{6} \times \frac{\cancel{6561}}{1} \times \frac{1}{\cancel{10935}} \times \frac{\cancel{5}}{12} = \frac{1}{24}$$

Hence the required ratio is 1 to 24.

Ex. 4. If $1\frac{9}{20}$ tons of coal cost $£2\frac{5}{12}$, what is the cost of $9\frac{3}{10}$ tons?

$9\frac{3}{10}$ tons will cost more than $1\frac{9}{20}$.

Hence the proportion is

$$\text{tons. } 1\tfrac{9}{20} : \text{tons. } 9\tfrac{3}{10} :: £2\tfrac{5}{12} : £x$$

$$x=\frac{9\frac{3}{10}\times 2\frac{5}{12}}{1\frac{9}{20}}$$

$$=\frac{\frac{93}{10}\times\frac{29}{12}}{\frac{29}{20}}$$

$$=\frac{\overset{31}{\cancel{93}}}{\cancel{10}}\times\frac{\cancel{29}}{\cancel{12}_{\cancel{6}\,2}}\times\frac{\overset{2}{\cancel{20}}}{\cancel{29}}=\frac{31}{2}=£15\ 10s.$$

Ex. 5. If 5 women and 6 boys can do a certain piece of work in $9\frac{1}{4}$ days of $8\frac{1}{2}$ hrs. each, how many days of $10\frac{1}{5}$ hrs. each will it take 3 men to do $\frac{9}{17}$ of the same work, supposing 1 man can do as much work as 2 women or 4 boys?

By the supposition 6 boys will do the same work as 3 women; hence 8 women will do the same work as 5 women and 6 boys. Also 4 men will do the same work as 8 women; hence 4 men do the same work as 5 women and 6 boys.

1. $9\frac{1}{4}$ days is the odd and therefore third term.
2. 3 men will take *more* time than 4.
3. Working $10\frac{1}{5}$ hrs. a day will take *less* time than working $8\frac{1}{2}$ hrs. a day.
4. It will take less time to do $\frac{9}{17}$ of the work than the whole work.

Hence the statement is

$$\left.\begin{matrix}3 & : & 4\\ 10\frac{1}{5} & : & 8\frac{1}{2}\\ 1 & : & \frac{9}{17}\end{matrix}\right\} :: 9\tfrac{1}{4} : x$$

$$x=\frac{4\times 8\frac{1}{2}\times\frac{9}{17}\times 9\frac{1}{4}}{3\times 10\frac{1}{5}\times 1}$$

$$x=\frac{4\times\frac{17}{2}\times\frac{9}{17}\times\frac{37}{4}}{3\times\frac{51}{5}}$$

$$x=\frac{\cancel{4}}{1}\times\frac{\cancel{17}}{\cancel{2}}\times\frac{\cancel{9}}{\cancel{17}}\times\frac{37}{\cancel{4}}\times\frac{1}{\cancel{3}}\times\frac{5}{\cancel{51}_{\cancel{3}}}=\frac{185}{34}$$

and $\frac{185}{34}=5\frac{15}{34}$ days. *Ans.*

Ex. 6. A can do a certain piece of work in 15 days, B can do it in 12 days, and C in 20 days ; if A and B work together for 6 days, how long will it take C to finish the work ?

In solving all problems of this kind, we must find the amount done by one, or more, in *one* day.

Thus if A can do the work in 15 days, he can do $\frac{1}{15}$ in 1 day ; in the same way B can do $\frac{1}{12}$ in a day, and C $\frac{1}{20}$.

If A can do $\frac{1}{15}$ and B $\frac{1}{12}$ in 1 day, they can do together $\frac{1}{15}+\frac{1}{12}=\frac{9}{60}=\frac{3}{20}$; hence in 6 days they will do $6\times\frac{3}{20}=\frac{18}{20}$. This leaves $\frac{2}{20}$ for C to do. But C does $\frac{1}{20}$ in a day, and hence can finish the work in 2 days.

Ex. 7. A reservoir is supplied by 3 pipes, which can fill it in 12, 15, and 20 days respectively ; the reservoir feeds a canal by 2 pipes, which can empty it in 3 and 5 days respectively ; if when full all the pipes are opened, how soon will the reservoir be empty ?

This problem is similar to the preceding : we must find the work of each pipe per day.

(1) Supply pipes fill $\frac{1}{12}+\frac{1}{15}+\frac{1}{20}$ a day $=\frac{12}{60}=\frac{3}{15}$ a day.

(2) Discharge pipes empty $\frac{1}{3}+\frac{1}{5}$ a day $=\frac{8}{15}$ a day.

That is, $\frac{3}{15}$ are added per day, and $\frac{8}{15}$ are taken out, viz., $\frac{5}{15}$ per day are discharged.

Hence the time to empty it is $\frac{15}{5}=3$ days.

Ex. 8. A owns $\frac{1}{4}$ of an estate, B owns $\frac{2}{9}$; the remainder, which belongs to C, is worth £2,500 more than A's share ; what is the value of the whole ?

A's share + B's share $=\frac{1}{4}+\frac{2}{9}=\frac{9+8}{36}=\frac{17}{36}$

And the whole, less $\frac{17}{36}$ viz. $\frac{36}{36}-\frac{17}{36}=\frac{19}{36}=$ C's share.

But C's share, $\frac{19}{36}$, is worth £2,500 more than A's share, $\frac{1}{4}$, or the difference between the value of C's and A's shares is £2,500, that is, $\frac{19}{36}-\frac{1}{4}=$ £2,500.

$$\therefore \frac{19-9}{36}=\frac{10}{36}=\frac{5}{18}=£2{,}500$$

$$\therefore \frac{1}{18}=\frac{2500}{5}$$

$$\therefore \text{whole}=\frac{2500}{5}\times 18=£9{,}000.\ \textit{Ans.}$$

Ex. 9. A cistern 10 ft. long and $5\frac{1}{3}$ ft. broad, has water in it to the height of 1 ft. 8 in ; if this water is allowed to run into another cistern 8 ft. 8 in. long and $4\frac{1}{2}$ ft. broad, to what height will the water fill it?

The number of cubic feet of water in the 1st cistern =

$$5\tfrac{1}{3} \times 10 \times 1\tfrac{2}{3} = \frac{16}{3} \times \frac{10}{1} \times \frac{5}{3} = \frac{800}{9}$$

The area of 2nd cistern =

$$8\tfrac{2}{3} \times 4\tfrac{1}{2} = \frac{26}{3} \times \frac{9}{2} = \frac{234}{6}$$

Hence $\frac{800}{9} \div \frac{234}{6}$ = depth of water.

$$\frac{800}{9} \div \frac{234}{6} = \frac{\overset{400}{\cancel{800}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{\underset{117}{\cancel{234}}} = \frac{800}{351}$$

$= 2\frac{98}{351}$ ft. *Ans.*

Ex. 10. Find the cost of papering a room 16 ft. 6 in. long, 13 ft. 8 in. wide, and 10 ft. 4 in. high, with paper 1 ft. 10 in. wide at $2\frac{1}{4}d.$ per yd. (allow 90 sq. ft. for doors and windows).

(16 ft. 6 in. + 13 ft. 8 in.) × 2 =
(30 ft. 2 in.) × 2 = 60 ft. 4 in.
$= 60\frac{1}{3}$ ft. $= \frac{181}{3}$ ft. = length of walls.
10 ft. 4 in. $= 10\frac{1}{3} = \frac{31}{3}$ = height.

Therefore $\frac{181}{3} \times \frac{31}{3} = \frac{5611}{9}$ area of walls.

$\frac{5611}{9} - 90$ sq. ft. $= \frac{5611}{9} - \frac{810}{9} = \frac{4801}{9}$ = area to be papered.

Hence $\frac{4801}{9} \div 1$ ft. 10 in. = length of paper required.

$$\frac{4801}{9} \div 1\tfrac{5}{6} = \frac{4801}{9} \div \frac{11}{6}$$

$$= \frac{4801}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{11} = \frac{9602}{33} \text{ ft.} = \frac{9602}{33 \times 3} \text{ yds.}$$

Therefore $\frac{9602}{33 \times 3} \times 2\frac{1}{4}d.$ = cost of paper.

$$\frac{\overset{4801}{\cancel{9602}}}{\underset{11}{\cancel{99}}} \times \frac{\overset{1}{\cancel{9}}}{\underset{2}{\cancel{4}}} = \frac{4801}{22} = 18s.\ 2\tfrac{5}{22}d.$$

Ex. 11. If a clock showed the correct time at noon yesterday, and had gained 10 mins. at 10 o'clock, correct time, in the evening, what is the correct time when the hour and minute hands are opposite to each other between 11 and 12 o'clock to-day?

(1) To find the time between 11 and 12 o'clock when the hands are opposite to each other. At 11 o'clock the minute-hand has to gain 25 mins. space; out of 12 mins. space the minute-hand gains 11.

$$\therefore\ 11 : 25 :: 12 : x$$

$$x = \frac{25 \times 12}{11} = \frac{300}{11} = 27\tfrac{3}{11}$$

That is, the hour and minute-hands are opposite to each other (between 11 and 12 o'clock) at $27\frac{3}{11}$ mins. past 11 o'clock.

(2) To find the correct time :—

Out of 10 hrs. 10 mins. 10 mins. are gained :

$\therefore$ out of 61 mins. there is a gain of 1 min.

$\therefore$ „ 1 „ „ $\frac{1}{61}$ „

From 12 o'clock yesterday to $27\frac{3}{11}$ minutes past 11 to-day = 23 hrs. $27\frac{3}{11}$ mins. $= 1407\frac{3}{11}$.

Hence out of $1407\frac{3}{11}$ mins. $\frac{1407\frac{3}{11}}{61}$ mins. are gained.

$$\frac{1407\frac{3}{11}}{61} = \frac{15480}{11 \times 61} = \frac{15480}{671} = 23\tfrac{47}{671} \text{ mins.}$$

At $27\frac{3}{11}$ mins. past 11 o'clock by the clock, the correct time is $23\frac{47}{671}$ mins. less.

$$27\tfrac{3}{11} - 23\tfrac{47}{671} = 4\tfrac{136}{671}.$$

That is, the correct time is $4\frac{136}{671}$ mins. past 11 o'clock.

NOTE.—Questions similar to the following are constantly given in examination papers :—It is now 12, or 1, or 2, &c., o'clock ; find when (1) the hands will be next together, (2) at right angles, (3) exactly opposite. They are all solved in a similar manner to (1) in the above example.

The minute-hand makes one complete revolution, viz. moves over 60 minute spaces, whilst the hour-hand moves from one figure to the next, viz. over 5 minute spaces. Hence the minute-hand gains 55 minute spaces out of every 60, or 11 out of 12.

We have then to note the position of the hands at any given time, and see what space has to be gained by the minute-hand to place the hands in any assigned position with regard to each other.

Thus at 6 o'clock the hands are opposite ; the minute-hand has to gain 30 minute spaces for the hands to be together, and 15 for them to be at right angles. Or, again, at 12·30, $32\frac{1}{2}$ minute spaces have to be gained for the hands to be together, $16\frac{1}{2}$ for them to be at right angles, and $2\frac{1}{2}$ for them to be opposite. We have thus the

3 terms of the proportion, of which 2 terms (gain) being alike, and the third term (12) being similar to the required answer.

11 will always be the 1st term, the minute spaces to be gained the 2nd term, and 12 min. the 3rd term.

EXAMPLES.—(A.) The hour and minute hands of a watch are at right angles at 3 o'clock ; when will they be next together ?

The minute-hand has to gain 15 minute spaces.

Thus $11 : 15 :: 12 : x$

$$x = \frac{12 \times 15}{11} = 16\tfrac{4}{11} \text{ mins. past 3.}$$

(B.) It is now 4 o'clock ; when will the hour and minute hands be next at right angles ?

The minute-hand has to gain 5 mins.

Hence $11 : 5 :: 12 : x$

$$x = \frac{60}{16} = 5\tfrac{5}{11} \text{ past 4 o'clock.}$$

(C.) It is now a quarter past 7 o'clock ; when will the hour and minute hands be exactly opposite to each other ?

The minute-hand is $21\frac{1}{4}$ minute spaces behind the hour-hand ; it must therefore gain this and 30 more spaces, viz., $51\frac{1}{4}$, before the hands can be opposite.

Hence $11 : 51\frac{1}{4} :: 12 : x$

$$x = \frac{51\frac{1}{4} \times 12}{11}$$

$$= \frac{205 \times 12}{11 \times 4} = \frac{615}{11} = 55\tfrac{10}{11}$$

viz., $55\frac{10}{11}$ mins. after a quarter past 7, or $70\frac{10}{11}$ mins. after 7, or $10\frac{10}{11}$ mins. after 8 o'clock.

The principle involved in the above examples can be applied to the solution of many other problems. Thus—

I. A square field measures $146\frac{2}{3}$ yds. in the side ; A and B start to walk round from opposite corners ; A walks at the rate of 4 miles an hour, and B at 3 ; where will A overtake B ?

A walks 4 miles whilst B walks 3 ; that is, A gains 1 mile or 1 yd. in 4 miles or 4 yds. As they are at opposite corners at starting, A has to gain 2 sides of square, viz. $293\frac{1}{3}$ yds.

Hence the proportion is $1 : 293\frac{1}{3} :: 4 : x$

$$x = 293\tfrac{1}{3} \times 4 = 1173\tfrac{1}{3} \text{ yds.}$$

In other words, A has to walk twice round the field, and so overtakes B at the point whence A started.

II. A hare takes 7 leaps whilst a greyhound takes 5, but the length of

the leaps is as 2 to 3 ; if the hare is 50 yds. ahead, how many yds. will it have travelled before it is overtaken by the dog ?

Suppose the length of leaps to be 2 yds. and 3 yds. (this being in the proportion of 2 to 3).

Then 7 leaps of hare = 14 yds.
and 5 leaps of dog = 15 yds.

Therefore the dog gains 1 yd. in 15 yds.

But it has to gain 50 yds.

Hence the proportion is $1 : 50 :: 15 : x$

$x = 750$ yds.

That is, the dog travels 750 yds. ; therefore the hare will have travelled $750 - 50 = 700$ yds.

MISCELLANEOUS EXERCISES.

Section II.

1. Divide £28 2*s.* 6*d.* among three persons, so that their shares may be proportional to the numbers 3, 5, and 7.

2. If a shopkeeper use a false weight of $15\frac{7}{8}$ ozs. to the pound, what will a true cwt. appear to weigh ?

3. If 24 boys, *or* 15 men, can do $\frac{3}{4}$ of a piece of work in $7\frac{1}{2}$ hrs., in what time will 10 men *and* 12 boys do the remainder ?

4. If it cost £61 18*s.* 5*d.* to keep two horses for 11 months, how long can 3 horses be kept for £59 2*s.* $1\frac{1}{2}d.$?

5. If I pay £19 13*s.* 9*d.* as income tax at 5*d.* in the £, find the amount of my income.

6. Find the value of $3\frac{11}{17} - 1\frac{30}{51}$ of 7*s.* $5\frac{1}{4}d.$

7. Simplify $\dfrac{\frac{5}{6} - \frac{1}{2} \times (\frac{2}{3} - \frac{1}{2}) \times \frac{6}{7} \times (\frac{4}{5} + \frac{1}{2})}{\frac{2}{4} \times (\frac{7}{8} - \frac{1}{2}) - \frac{1}{3} \times (\frac{1}{7} - \frac{1}{10})}$

8. The rates of a railway train and a coach are as 7 to 2. The former can travel 42 miles an hour ; how long will it take the latter to go 342 miles ?

9. In a besieged town containing 45,500 people there are provisions for three weeks ; how many must be sent away so that the provisions may last seven weeks ?

10. If a postage stamp measures 1 in. by $\frac{4}{5}$ in., find the number required to paper a room 18 ft. 9 in. long, 16 ft. 10 in. broad, and 10 ft. 6 in. high.

11. A rectangular board 7 in. broad contains 12 sq. ft. ; find its length.

12. What is the largest sum of money exactly contained in £25 17*s.* 6*d.* and £42 11*s.* 0*d.* ?

13. If 81 gallons of water will fill a cistern 4 ft. 4 in. long, 2 ft. 8 in. wide, and 1 ft. $1\frac{1}{2}$ in. deep, how many cubic inches are there in a pint ?

14. A room 21 ft. long requires 49 yards of carpet $\frac{3}{4}$ yd. wide to cover it; find the breadth of the room.

15. Express 49 yds. 2 ft. 9 in. as a fraction of a furlong.

16. A man alone can do a piece of work in 10 days, which if his son help him he can do in 6; in what time would his son working alone do the work?

17. Find the value of $\frac{8\frac{3}{5}-7\frac{3}{4}+5\frac{3}{8}-4\frac{1}{2}}{13-11\frac{9}{10}+10\frac{7}{9}-9\frac{17}{20}}$ of $\frac{2}{11}$ of £365.

18. If a garrison of 1,800 men have provisions for 12 weeks, how long will the provisions last if 600 more men are introduced, but the daily allowance reduced by $\frac{1}{3}$?

19. A certain number of yards of carpeting $\frac{3}{4}$ yd. wide will cover the floor of a room 16 ft. 8 in. long, 13 ft. 5 in. wide, except the hearth-stone, which measures 4 ft. 2 in. by 1 ft. 8 in.; find the cost at 6*s.* per yard.

20. A bankrupt's effects realise £961 15*s.* $4\frac{1}{2}$*d.*; his debts are £1,697 1*s.* 3*d.*; find the amount he can pay in the £.

21. If 10 men can do a piece of work in 24 days, how many will do another piece of work 7 times as great in $\frac{1}{5}$ part of the time?

22. What is the value of 3 qrs. of oats, if $7\frac{5}{8}$ qrs. cost £$33\frac{3}{64}$?

23. A tank is supplied by a spout which can fill it in two hours; how long will it take to fill the tank if it has a leak which would empty it in 10 hours?

24. Divide 7*s.* $1\frac{1}{2}$*d.* by $\frac{9}{11\frac{5}{7}}$, and from the quotient subtract $\frac{2}{3}\frac{9}{0}$ of 4 half-crowns.

25. Find the greatest and least of the fractions $\frac{3}{8}$, $\frac{9}{20}$, $\frac{23}{48}$, and $\frac{13}{28}$.

26. The rental of a parish is £6,720 12*s.*, and the assessment to the poor rate is £700 1*s.* 3*d.*; how much will the rate be on £63 3*s.* 4*d.*?

27. Find the value of $(12\frac{5}{6}-8\frac{3}{4}-1\frac{1}{10}+\frac{2}{5})\times 4\frac{1}{2}\div(7\frac{5}{12}-6\frac{1}{2})$.

28. Find the value of x in the proportion $\frac{3}{4} : \frac{1}{4} :: \frac{2}{7} : x$.

29. Express in pecks $\frac{3}{5}$ of 1 qr. 3 bus. 2 pks.

30. Divide 3 cwt. 3 qrs. 7 lbs. by $3\frac{1}{3}\frac{1}{7}$.

31. Find the value of $\dfrac{6\frac{5}{11}+\frac{1}{17}+3\frac{21}{22}-5\frac{19}{34}}{5\frac{1}{9}\text{ of }3}$

32. Find, by Practice, the value of $753\frac{3}{10}$ at £28 6*s.* $10\frac{1}{2}$*d.*

33. Express the ratio between 15*s.* 9*d.* and £1 6*s.* 3*d.* in its simplest form.

34. If a man travel $15\frac{13}{18}$ miles in $3\frac{1}{4}$ hours, how far can he go in $18\frac{7}{9}$ hours at the same rate?

35. A sum of £386 3*s.* 9*d.* has to be raised on a parish, of which the rental is £6,179; what is the amount of the rate in the £?

36. In a mile race A can beat B by 17 yds., or $2\frac{1}{8}$ secs.; find A's time over the course.

37. If 7 pipes can empty a reservoir running 3 hrs. a day for 15 days, how long will 5 of the pipes take to empty it running 5 hrs. a day?

38. How many tiles $4\frac{1}{2}$ in. long, and $2\frac{1}{2}$ in. broad, are required to pave a square court 12 ft. 6 in. in the side?

39. Add $\frac{9}{16}$ of 1*s.* 6*d.* to $\frac{13}{960}$ of 10*s.*, and subtract their sum from $\frac{3}{16}$ of 12*s.* 6*d.*

40. If $2\frac{1}{2}$ bus. can be bought for $\frac{2+\frac{1}{3}}{3+\frac{1}{2}}$ of £1, what should be the cost of $793\frac{1}{16}$ bus.?

41. The first line of an army consists of $\frac{1}{4}$ of the whole force, the second line of $\frac{1}{6}$, and the third line of $\frac{2}{5}$; the remainder is 1,210 men; find the numerical strength of the whole army.

42. A man and a boy working together for 9 hrs. a day can do a piece of work in 11 days; the man by himself would be 17 days about it; how long would it take the boy alone to do it?

43. Simplify $\left(\frac{15\frac{3}{4}-1\frac{4}{5}\times 1\frac{5}{6}}{\frac{1}{5}\times 23\frac{1}{3}+\frac{9}{35}\frac{1}{1}}\right)\div\frac{7\frac{2}{3}\times 7\frac{2}{5}\times 7\frac{2}{5}-27}{7\frac{2}{5}\times 7\frac{2}{5}-9}$

44. If $2\frac{10}{11}$ ozs. of tea cost $1\frac{4}{11}$*s.*, what part of a pound of tea can be bought for $2\frac{7}{8}$*s.*?

45. What fraction of £37 14*s.* $8\frac{1}{4}$*d.* is equal to $\frac{1}{11}$ of £296 9*s.* $8\frac{1}{4}$*d.*?

46. If $\frac{7}{15}$ of a guinea be taken from $\frac{3}{13}$ of $\frac{5}{8}$ of £5, what fraction of £100 will remain?

47. A man has £576 6*s.* 8*d.* a year after paying income tax at 4*d.* in the £; what is his gross income?

48. If 120 men can make an embankment $\frac{3}{4}$ mile long, 30 yds. wide, and 7 yds. high, in 42 days, how many men will it take to make an embankment 1,000 yds. long, 36 yds. wide, and 22 ft. high in 30 days?

49. A man leaves $\frac{1}{3}$ of his property to his widow, $\frac{1}{2}$ to his son, and the remainder to be equally divided between his four daughters; the son gets £330 more than either of his sisters; find the whole sum left.

50. The dimensions of a room are $29\frac{1}{2}$ ft. by $11\frac{1}{4}$ ft.; what length of carpet $\frac{3}{4}$ yd. wide will cover it, and what will be the expense at $3\frac{2}{3}$*s.* per yard?

51. Simplify $\dfrac{\frac{1}{3\frac{1}{2}}+\frac{1}{2\frac{1}{2}+1\frac{1}{4}}}{\frac{1}{3\frac{1}{2}}+\frac{1}{2\frac{1}{2}+1\frac{1}{4}}}$ of $1+\dfrac{5\frac{1}{6}+1\frac{9}{16}-10\frac{9}{16}}{2\times 2\frac{1}{4}\times 1\frac{1}{4}}$

52. If when wheat is 60*s.* a qr. the 6*d.* loaf weighs 4 lbs., how much should be paid for 25 lbs. of bread when wheat is at 40*s.* a qr.?

53. A can finish a piece of work in $2\frac{1}{2}$ days, and B in $3\frac{1}{3}$ days; if they work together, what part of the work can they finish in 1 day?

54. How many cubic feet are there in a box $4\frac{1}{2}$ ft. long, $3\frac{1}{3}$ ft. broad, *and* $7\frac{1}{7}$ *ft. deep?*

55. If a cistern can be filled by one pipe in 3 hrs., by a second in 5 hrs., and by a third in 6 hrs., in what time will it be filled by all running together?

56. Express the ratio between £282 11*s.* 5½*d.* and £362 17*s.* 3*d.* in its simplest form.

57. Simplify $\frac{1}{\frac{1}{4}+\frac{1}{1+\frac{\frac{1}{4}+1}{3-\frac{1}{4}}}}$

58. Find the cost of papering the walls of a room 22 ft. 6 in. long, 18 ft. 9 in. wide, and 11 ft. high, 99 sq. ft. not requiring to be papered, the paper being 2 ft. 9 in. wide, at 3*d.* per linear yd.

59. In a 100 yds. race, A can beat B by 10 yds.; B in the same distance can beat C by 10 yds.; by how many yards can A beat C?

60. If after selling $\frac{3}{4}$ of an estate I sell $\frac{3}{5}$ of $\frac{7}{8}$ of the remainder for $\frac{3}{5}$ of $\frac{7}{8}$ of £200$\frac{5}{8}$, what is the value of the whole estate?

61. Simplify $\frac{2\frac{1}{2}+1\frac{1}{4}-2\frac{1}{8}}{3\frac{3}{4}+2\frac{1}{3}-4\frac{1}{4}} \times \frac{\frac{9}{49}-\frac{4}{25}}{\frac{7.5}{121}-\frac{1.2}{27}} \div \frac{\frac{3}{7}+\frac{2}{5}}{\frac{5}{11}+\frac{4}{9}}$

62. I agree to pay a man £5 13*s.* 9*d.* for working at a certain rate during a certain time; he is absent $\frac{1}{7}$ of the time, and when present only works at $\frac{1}{2}$ the rate agreed upon; what should I have to pay him?

63. If 75 men can do a piece of work in 12 days of 10 hrs. each, how many men will do a piece of work twice as large in a tenth part of the time, if they work the same number of hours per day, supposing that 2 of the second set can do as much work in an hour as 3 of the first set?

64. How much paper 22½ in. wide is required to paper a room 18 ft. 9 in. long, 13 ft. 3 in. broad, and 14 ft. 6 in. high?

65. If 8 men working 6 hrs. a day for 30 days can mow a field of corn 1,320 yds. long, and 880 yds. wide, what will be the length of the side of a field 1,320 yds. broad which 4 men can mow in 42 days working 9 hrs. a day?

66. A creditor receives £173 15*s.* 6*d.* from the estate of a bankrupt paying 15*s.* 9*d.* in the £; what was the original debt?

67. How many boards 4 ft. long and 8 in. wide are required for the floor of a room 16 ft. long and 12¾ ft. wide?

68. Find what fraction 258 days 9 hrs. 19 mins. 57 secs. is of 446 days 7 hrs. 23 mins. 33 secs.

69. Three pipes, A, B, and C, in connection with a cistern are opened at 1, 2, and 3 o'clock respectively. A alone can fill it in 3 hrs., and B alone in 4 hrs., while C can empty it (when full) in 1 hr.; at what o'clock will the cistern be empty?

70. A can copy 6 pages whilst B can copy 5; B copies 15 whilst C copies 12; and C copies 4 whilst D copies 3. A, who can write 20 pages a day, receives a paper of 240 pages to copy, and after doing ⅓ of it calls in B, C, *and D to help him*; when will the work be finished?

71. A man leaves to one of his children $\frac{1}{5}$ of his property, to another £200, and the remainder, being $\frac{3}{8}$ of the property, among the rest; how much did he leave?

72. Two engines, 40 miles apart, are approaching each other at the rates of 25 and 35 miles an hour; determine the time and place of meeting.

73. If 7 needlewomen can finish a piece of work in $10\frac{5}{8}$ days of $9\frac{3}{5}$ hrs. each, how long will it take three women to do two such pieces of work if they work 10 hrs. a day?

74. A watch which loses $2\frac{1}{2}$ mins. per day is set right at 6 o'clock on Monday night; what will be the true time when the watch indicates 12 o'clock (noon) on Friday?

75. If a watch showing correct time on Tuesday at noon gains 3 mins. 4 secs. in 24 hrs., what is the correct time on the following Saturday evening when it is 10 o'clock by the watch?

76. At what times between 10 and 11 o'clock is there 5 mins. division between the hour and minute hands?

77. A watch which loses 5 secs. in 5 mins. is set right at 8 A.M.; what will be the correct time when the hands are together between 4 and 5 o'clock in the afternoon?

CHAPTER VIII.

DECIMAL FRACTIONS.

SECTION I.—INTRODUCTORY.

IN the common notation of integers, or whole numbers, each figure has not only an intrinsic value, but also a value which depends upon its *position* with regard to the other figures in any given number. Each figure to the left has *ten times* the value of the same figure on the right; and each figure to the right has *one-tenth* the value of the same figure to the left. *Ten*, or some power of ten, being the constant multiplier or divisor, our notation is called the *decimal* notation.

We may extend this decimal notation to numbers *less than* one, if we are careful to follow out the rule that the figure to the right is one-tenth of the same figure to the left.

Thus—

thousands	hundreds	tens	units.	tenths	hundredths	thousandths
5	5	5	5	5	5	5

The figure 5 next to the 5 units must be a tenth of 5 ($\frac{1}{10}$ of 5), or $\frac{5}{10}$.

The next figure 5 must be a tenth of $\frac{5}{10}$, viz. $\frac{5}{100}$.

The third figure from the units $=\frac{5}{1000}$, and so on.

All the figures to the right of the unit's place are fractions with 10, or some power of 10, for denominators; hence these fractions are called *decimal fractions.**

It is not necessary to write tenths, hundredths, thousandths, &c., over the figures, any more than it is to write units, tens, hundreds, &c., over the integers; because, if we can mark the position of the unit's place, the *position* of each figure on the right will indicate its value.

We mark the position of the unit's place by placing a point between it and the first fractional number; thus—

5555·555

We have to note then that—

The first place to the right of the point indicates *tenths*;
The second place to the right of the point indicates *hundredths*;
The third place to the right of the points indicates *thousandths*;
and so on.

We may write the above number to show its value in full; thus—

$$5000+500+50+5+\frac{5}{10}+\frac{5}{100}+\frac{5}{1000}$$

Ex. 1. Show the separate value of each figure on the following numbers:—

(1) 32·409 (2) 6·0105 (3) ·0606 (4) ·7010 (5) ·07010

(1) $32\cdot409=30+2+\frac{4}{10}+\frac{9}{1000}$

(2) $6\cdot0105=6+\frac{1}{100}+\frac{5}{10000}$

(3) $\cdot0606=\frac{6}{100}+\frac{6}{10000}$

(4) $\cdot7010=\frac{7}{10}+\frac{1}{1000}$

(5) $\cdot07010=\frac{7}{100}+\frac{1}{10000}$

From (4) it will be seen that a cipher *affixed* to a decimal fraction does not alter its value; and from (5) that a cypher *prefixed* to a decimal fraction, diminishes its value tenfold.

Ex. 2. Write the following as fractions without denominators:

$\frac{5}{10}$, $\frac{4}{100}$, $\frac{3}{1000}$, $\frac{2}{10000}$, $\frac{1}{100000}$

(1) $\frac{5}{10}=\cdot5$ (2) $\frac{4}{100}=\cdot04$ (3) $\frac{3}{1000}=\cdot003$

(4) $\frac{2}{10000}=\cdot0002$ (5) $\frac{1}{100000}=\cdot00001$

* *A decimal fraction is a fraction* having 10, or some power of 10, for its denominator.

Ex. 3. Write, in the form of vulgar fractions, the separate value of each of the figures 4, 2, and 1, in the following fractions :—

·04021, ·1042, ·02104

In ·04021 we have $\frac{4}{100}$, $\frac{2}{10000}$, and $\frac{1}{100000}$

In ·1042 we have $\frac{1}{10}$, $\frac{4}{1000}$, and $\frac{2}{10000}$

In ·02104 we have $\frac{2}{100}$, $\frac{1}{1000}$, and $\frac{4}{100000}$

Ex. 4. Compare the values of (*a*) ·7 and 7 ; (*b*) ·07 and 7 ; (*c*) 5 and ·5 ; and (*d*) 5 and ·05.

(*a*) $\cdot 7 = \frac{7}{10}$ and $7 = \frac{70}{10}$

but $\frac{7}{10} \times 10 = \frac{70}{10} = 7\cdot$

$\therefore \cdot 7 \times 10 = 7\cdot$

or, to multiply a decimal fraction by 10, move the decimal point *one* place to the *right*.

(*b*) $\cdot 07 = \frac{7}{100}$ and $7 = \frac{700}{100}$

but $\frac{7}{100} \times 100 = \frac{700}{100} = 7\cdot$

$\therefore \cdot 07 \times 100 = 7\cdot$

or, to multiply a decimal fraction by 100, move the decimal point *two* places to the *right*.

(*c*) $5 = \frac{5}{1}$ and $\cdot 5 = \frac{5}{10}$

but $\frac{5}{1} \div 10 = \frac{5}{1} \times \frac{1}{10} = \frac{5}{10} = \cdot 5$

$\therefore 5 \div 10 = \cdot 5$

or, to divide a decimal fraction by 10, move the decimal point *one* place to the *left*.

(*d*) $5 = \frac{5}{1}$ and $\cdot 05 = \frac{5}{100}$

but $\frac{5}{1} \div 100 = \frac{5}{1} \times \frac{1}{100} = \frac{5}{100} = \cdot 05$

$\therefore 5 \div 100 = \cdot 05$

or, to divide a decimal fraction by 100, move the decimal point *two* places to the *left*.

From these examples we may deduce the following rules :—

1. To multiply a decimal fraction by 10, or any power of 10, move *the decimal* point as many places to the right as there are noughts *in the multiplier* ; add ciphers if necessary.

2. To divide a decimal fraction by 10, or any power of 10, move the point as many places to the left as there are noughts in the divisor; prefix ciphers if necessary.

Exercises.

1. Express in the decimal notation eight-tenths, seven-hundredths, four-thousandths, and six ten-thousandths.

2. Show the value of each figure in ·3264 as a vulgar fraction.

3. Express the value of the figure 7, as a vulgar fraction, in each of the following decimal fractions:—·7, ·070, ·007, ·171, ·0007, ·0070007.

4. Express $\frac{5}{100}$, $\frac{5}{1000}$, $\frac{3}{10}$, $\frac{9}{10000}$, as decimal fractions.

5. $\cdot 601 = \frac{6}{10} + \frac{0}{100} + \frac{1}{1000}$, from this show that $\cdot 601 = \frac{601}{1000}$, and then express ·37, 2·47, 3·005, as vulgar fractions.

6. Write 37·5, 19·0104, and ·000076, as vulgar fractions.

7. Write as decimals three-hundredths, ninety-seven ten-thousandths, and two hundred and fifty seven hundred-thousandths.

8. Express as decimals $\frac{3029}{10000}$, $\frac{4109}{100}$, $\frac{32104}{100}$, $\frac{291064}{10000}$, $37\frac{39}{100}$, and $2\frac{309}{100000}$.

9. Express as fractions or mixed numbers, 41·42, 3·047, 42·1, 800·008, and 111·111.

10. Multiply 77 by 100, and divide by 1000.

11. Divide 320·47 separately by 10 and by 10,000.

12. Find the value of $\frac{\cdot 35 \times 100}{1000}$ as a decimal fraction.

13. Multiply 32·764 separately by 1,000 and 100,000.

14. Divide 30472·00641 separately by 1,000,000 and 10,000.

15. Multiply each of the following numbers by 10,000:—32, ·6042, 9·06427, 41·062754.

16. Divide each of the following numbers by 10,000:—·4, ·072, ·0642694, 2652741.

17. Find the value of $20 \cdot 4204 \div 1000 \times 100{,}000 \div 1001 \times 10$

18. Find the value of $\cdot 00001 \times 100{,}000$ and $\cdot 00001 \div 100{,}000$.

Section II.—Addition and Subtraction.

We have shown that decimal fractions are merely extensions of the ordinary decimal notation of integers to fractional numbers; we must now show how to extend the rules for the addition, subtraction, multiplication, *and division of integers* to decimal fractions.

Section II.—Addition and Subtraction.

Ex. 1. Add 3·459, 62·9805, 7·69, ·0076, 40·0106.

As in addition of whole numbers, our first care is to arrange like under like, tenths under tenths, hundredths under hundredths, &c.

A.

tens	units	tenths	hundredths	thousandths	ten-thousandths
	3·	4	5	9	
6	2·	9	8	0	5
	7·	6	9		
	·0	0	7	6	
4	0·	0	1	0	6
1	1 4·	1	4	7	7

B.

$$3+\frac{4}{10}+\frac{5}{100}+\frac{9}{1000}$$

$$62+\frac{9}{10}+\frac{8}{100}+\frac{0}{1000}+\frac{5}{10000}$$

$$7+\frac{6}{10}+\frac{9}{100}$$

$$+\frac{0}{10}+\frac{0}{100}+\frac{7}{1000}+\frac{6}{10000}$$

$$40+\frac{0}{10}+\frac{1}{100}+\frac{0}{1000}+\frac{6}{10000}$$

$$114+\frac{1}{10}+\frac{4}{100}+\frac{7}{1000}+\frac{7}{10000}$$

(1) Adding the ten-thousandths column, we get $\frac{17}{10000}=\frac{10}{10000}+\frac{7}{10000}$ and $\frac{10}{10000}=\frac{1}{1000}$; hence we put down $\frac{7}{10000}$, and carry $\frac{1}{1000}$

(2) Adding the thousandths column, we get (with the figure carried) $\frac{17}{10000}$ and $\frac{17}{1000}=\frac{10}{1000}+\frac{7}{1000}=\frac{1}{100}+\frac{7}{1000}$; put down $\frac{7}{1000}$ and carry $\frac{1}{100}$

(3) Adding the hundredths column, we get (with the figure carried) $\frac{24}{100}$ and $\frac{24}{100}=\frac{20}{100}+\frac{4}{100}=\frac{2}{10}+\frac{4}{100}$; put down $\frac{4}{100}$ and carry $\frac{2}{10}$

(4) Adding the tenths column, we get (with the figure carried) $\frac{21}{10}=\frac{20}{10}+\frac{1}{10}=\frac{2}{1}+\frac{1}{10}$; set down $\frac{1}{10}$ and carry 2.

(5) Complete, as in addition of integers.

On comparing the working as shown above in A and B, it will be seen that in adding we may forget all about tenths and hundredths, &c., just *as in addition of integers* we forget all about tens and hundreds, &c.

Ex. 2. From 3·142 subtract 1·261.

3·142	$3+\frac{1}{10}+\frac{4}{100}+\frac{2}{1000}$
1·261	$1+\frac{2}{10}+\frac{6}{100}+\frac{1}{1000}$
1·881	$1+\frac{8}{10}+\frac{8}{100}+\frac{1}{1000}$

(1) $\frac{1}{1000}$ from $\frac{2}{1000}$ leaves $\frac{1}{1000}$; set down $\frac{1}{1000}$

(2) $\frac{6}{100}$ from $\frac{4}{100}$ is impossible; change the $\frac{1}{10}$ to $\frac{10}{100}$, then $\frac{6}{100}$ from $\frac{14}{100}$ leaves $\frac{8}{100}$; set down $\frac{8}{100}$

(3) $\frac{2}{10}$ from $\frac{0}{10}$ impossible; change 1 from the 3 units to $\frac{10}{10}$; $\frac{2}{10}$ from $\frac{10}{10}$ leaves $\frac{8}{10}$; set down $\frac{8}{10}$

(4) 1 from 2 leaves 1; set down 1.

As in addition, we may forget all about tenths, hundredths, &c., and proceed as in subtraction of integers.

Exercises.

1. Add 42·309, 651·72, 8104·009, ·6004, and ·0004.
2. From 32·1004 subtract 12·1006.
3. Add 60·004, 741·609, ·0047, 3·2, 7·94, and 3·00004.
4. Subtract ·02426 from 3·2.
5. Add ·1, ·01, ·001, ·0001, ·00001, and ·000001.
6. From 1 subtract ·30741.
7. Find the value of $\cdot043+72\cdot64+91\cdot0479+\frac{3}{1000}+\frac{9}{100}+\frac{5}{10}$
8. From 3·024 subtract $2\frac{9}{100}$.
9. Find the value of $11641\cdot729+1381\cdot404-29\cdot6104-3104\cdot7+6\cdot4721$.
10. Take nine-thousandths from nine-hundredths.
11. Subtract three-tenths from 1.
12. Express in words the difference between 2·000002 and ·1000001.
13. Divide 2*s.* 6*d.* between two boys in the ratio of 1·6 to 1·4.
14. Divide £2 11*s.* 0*d.* among A, B, and C, in the proportion of 2·3, 3·4, and 4·5.

15. Find the sum of 3·04 lbs., 2 cwts. 1 qr. 4·64 lbs., and 4·725 lbs.

16. From 5 a. 2 r. 3·2 p. subtract 2 a. 3 r. 14·34 p.

17. From what number must 32·0159 be subtracted to leave 169·20147?

18. The smaller of two numbers is 3·017, and their sum is 6·7; find the larger number.

19. What number must be subtracted from the sum of ·05, ·5, 62·45, and 2·00104 to leave 31·0042?

20. What must be added to the difference between ·01, and ·001 to make ·1?

Section III.—Multiplication and Division of Decimal Fractions.

Ex. 1. To show that $3{\cdot}04 \times 21{\cdot}6$ is equal to 65·664 :—

$$3{\cdot}04 = 3\tfrac{4}{100} = \frac{304}{100}$$

$$21{\cdot}6 = 21\tfrac{6}{10} = \frac{216}{10}$$

$$\text{and } \frac{304}{100} \times \frac{216}{10} = \frac{65664}{1000} = 65\tfrac{664}{1000}$$

$$\text{But } \frac{664}{1000} = \frac{600}{1000} + \frac{60}{1000} + \frac{4}{1000}$$

$$= \frac{6}{10} + \frac{6}{100} + \frac{4}{1000}$$

$$= {\cdot}664$$

$$\therefore\ 3{\cdot}04 \times 21{\cdot}6 = 65{\cdot}664.$$

We note that the number of decimal places in the product is the same as the number of decimal places in the factors added together. On trial this will be found to hold in every example of multiplication of decimals.

In the above example we may obtain the same result by multiplying as with integers, and marking off as many decimal places to the right in the product as there are decimal places in the multiplier and multiplicand.

$$\begin{array}{r} 21{\cdot}6 \\ 3{\cdot}04 \\ \hline 864 \\ 6480 \\ \hline 65{\cdot}664 \end{array}$$

Hence the rule:—*To multiply decimal fractions, multiply as in whole numbers. Mark off as many places to the right of the decimal point as there are decimal places in both factors.*

Ex. 2. To show that $50{\cdot}82 \div 7{\cdot}7 = 6{\cdot}6$.

$$50{\cdot}82 = 50\tfrac{82}{100} = \frac{5082}{100}$$

$$7{\cdot}7 = 7\tfrac{7}{10} = \frac{77}{10}$$

$$\text{and } \frac{5082}{100} \div \frac{77}{10} = \frac{\cancel{5082}^{\,66}}{\cancel{100}} \times \frac{\cancel{10}}{\cancel{77}_{\,7}} = \frac{66}{10} = 6{\cdot}6$$

$$\therefore\ 50{\cdot}82 \div 7{\cdot}7 = 6{\cdot}6.$$

We note that the number of decimal places in the quotient is the number of decimal places in the dividend, less the number in the divisor. On trial this will be found to hold in all cases of division of decimals.

$$\begin{array}{r} 7{\cdot}7)50{\cdot}82(6{\cdot}6 \\ \underline{462} \\ 462 \\ \underline{462} \end{array}$$

In the above example we may obtain the same result by dividing as with integers, and marking off as many decimal places in the quotient as there are in the dividend, less the number in the divisor.

Hence the rule :—*To divide decimal fractions, divide as with whole numbers. Mark off as many decimal places in the quotient as there are in the dividend, less the number in the divisor.*

Ex. 3. Divide 64 by ·0016.

$$\begin{array}{r} ·0016)64{\cdot}0000(40{,}000 \\ \underline{64} \\ 0000 \end{array}$$

In this example we have no decimal places in the dividend; we therefore add ciphers to make as many decimal places as there are in the divisor. Then subtracting the number of decimal places in the divisor from the number in the dividend we get nothing. That is, there are no decimal places in the quotient.

Ex. 4. Divide ·0064 by 16.

$$\begin{array}{r} 16)·0064(·0004 \\ 64 \end{array}$$

In this example the number of decimal places in the dividend, less the number in the divisor, is 4. Hence we must prefix 3 ciphers to make 4 *decimal places in the quotient.*

The most simple method of dividing by decimals, however, and the one we recommend students to adopt in practice, is to arrange in the form of a vulgar fraction, and then multiply the numerator and denominator by 10, or powers of 10, until we get rid of *all* the decimal places in both divisor and dividend.

Thus, taking Ex. 3 and 4 above:—

$$(3)\ \frac{64}{\cdot0016}=\frac{640000}{16}=40{,}000$$

$$(4)\ \frac{\cdot0064}{16}=\frac{64}{160000}=\frac{8}{20000}$$

$$=\frac{4}{10000}=\cdot0004$$

Ex. 5. Divide 231302·896 by 751·96.

$$\frac{231302\cdot896\times1000}{751\cdot96\times1000}=\frac{231302896}{751960}$$

or, which is the same thing, multiply divisor and dividend by 1,000, and we get—

751960)231302896·0(307·6
2255880
5714896
5263720
4511760
4511760

In this example, after "bringing down" the whole numbers, there was a remainder, and we made one decimal place in the dividend by adding one nought. Hence the decimal place in the quotient.

Exercises.

1. Multiply—
   (1) 3·016 by ·00123.
   (2) 2146·5 by ·00126.
   (3) 600·7 by ·0000042.
   (4) ·000999 by ·000999.
   (5) 625·0142 by 3·014.
   (6) 333·333 by 222·222.
   (7) 75·6942 by 3·4251.
   (8) 654·001 by 359·01.
   (9) 32754·695 by ·300071.
   (10) 725·4 by ·06 by ·004.
   (11) 3·2 by 3·2 by 3·2 by 3·2 by 3·2.
   (12) 1·1 by 1·1 by 1·1 by 1·1 by 1·1 by 1·1.

2. Divide—
   (1) 71·55 by 13·25.
   (2) 7·5625 by 13·75.
   (3) 244·006 by 40·6.
   (4) 12·3125 by ·000625.
   (5) ·00001 by 1001.
   (6) ·000000278695 by ·000401.
   (7) 171·738 by 36·54.
   (8) ·2345 by ·0001024.
   (9) 40·30366 by ·00542.
   (10) 222·3758 by 19·27.
   (11) 1·813998 by ·007654.
   (12) ·00712738 by 3·47.

3. Find the value of $\frac{\cdot001 \times \cdot01}{101}$

4. Simplify $\frac{60\cdot4 - 2\cdot09}{9\cdot875}$

5. Simplify $\frac{2\cdot1775 \times \cdot064}{\cdot00032}$

6. Find the value of $\frac{\cdot004 \times \cdot02}{\cdot08}$

7. Find the value of $\frac{(\cdot75 \times \cdot75) - (\cdot05 \times \cdot05)}{\cdot75 - \cdot05}$

8. Find the product of the sum, and difference of ·37 and 37.

9. Simplify $\frac{1\cdot01 - \cdot2}{3\cdot5 - \cdot8} \times \frac{2 - \cdot04}{1 - \cdot11}$

10. Find the sum, difference, product, and quotient of ·151904 and ·0505.

11. Simplify $\frac{\cdot0099 \times \cdot0078}{77\cdot22} \times \frac{12\cdot5}{\cdot0125}$

12. The product of two numbers is ·0498552198, and one of them is 1·3569 ; find the other.

13. Divide the sum of 18·3 and 21·065 by the difference between 17·09867 and 25·1.

14. What number multiplied by 7·2 will give ·012 ?

15. Find the product of the sum and difference of 5·4 and 4·5.

16. Divide 304·767 into two parts so that the one is double the other.

17. Find the product of the sum and difference of ·001 and ·01.

18. There are two numbers, of which the larger is 39·61, and their sum is 65 ; give the product of the numbers.

19. *Divide the sum of* 5·005 and 2·605 by their difference.

20. What must be added to 7, ·7, ·07, and ·007 to make 8.

21. Divide ·1 by ·0005; what must be subtracted from the quotient to make it 101·01?

22. When the divisor is 3·075, the quotient ·25, and the remainder ·00011, find the dividend.

23. How many times can you subtract 3·01 from 65·23, and what is the remainder?

24. How many times must ·017 be added to itself to produce 612?

25. If one sovereign weighs 123·274 grains, how many sovereigns will weigh 21 lbs. 4 ozs. 16 dwts. 10 grs.?

26. What decimal fraction subtracted 3·5 times from 42·875 will leave no remainder?

27. Divide £4·25 among three persons in the proportion of 4·9, 5·2, and 6·9.

28. Simplify $\left(\frac{\cdot1\times\cdot01}{\cdot1-\cdot01}-\frac{\cdot1-\cdot01}{\cdot1+\cdot01}+\frac{\cdot0004}{\cdot1-\cdot00001}\right)\times\left(\frac{\cdot1+\cdot01}{\cdot02}\right)$ and give the answer as a vulgar fraction.

SECTION IV.—TO CHANGE DECIMAL FRACTIONS TO VULGAR FRACTIONS, AND VULGAR FRACTIONS TO DECIMAL FRACTIONS.

Ex. 1. Express 1·6384 as a vulgar fraction.

$$1\cdot6384=1+\frac{6}{10}+\frac{3}{100}+\frac{8}{1000}+\frac{4}{10000}$$

Reducing the fractions to a common denominator, and adding, we get—

$$\frac{6000}{10000}+\frac{300}{10000}+\frac{80}{10000}+\frac{4}{10000}=\frac{6384}{10000}$$

On examining this fraction it will be seen that the numerator consists of the decimal fraction written as a whole number, and the denominator of 1 followed by as many *noughts* as there are decimal places in the decimal to be changed.

It will be found on trial that this holds good in every case.

Hence the rule:—*To change decimal to equivalent vulgar fractions, place the figures forming the decimal as the numerator, and the figure* 1 *followed by as many noughts as there are figures in the decimal as the denominator.*

NOTE 1.—We may include the whole number which precedes the decimal point in the numerator; our equivalent vulgar fraction will then be an improper fraction.

NOTE 2.—Reduce the fraction to its lowest terms.

In the above example the equivalent vulgar fraction is $1\frac{6384}{10000}$, or $\frac{16384}{10000}=\frac{1024}{625}=1\frac{399}{625}$. (G. C. M. of 16384 and 1000 is 16.)

Ex. 2. Express 4·0140, 2·005, ·0014, 1·00005, and ·001 as vulgar fractions.

(1) $4{\cdot}0140 = \frac{4014}{1000}$ or $4\frac{14}{1000} = 4\frac{7}{500}$

(2) $2{\cdot}005 = \frac{2005}{1000}$ or $2\frac{5}{1000} = 2\frac{1}{200}$

(3) $\cdot0014 = \frac{14}{10000} = \frac{7}{5000}$

(4) $1{\cdot}00005 = \frac{100005}{100000}$ or $1\frac{5}{100000} = 1\frac{1}{20000}$

(5) $\cdot001 = \frac{1}{1000}$

Ex. 3. Express $\frac{216}{625}$ as its equivalent decimal fraction.

It is evident that if by multiplying or dividing both numerator and denominator by the same numbers we can change the denominator so as to make it consist of 1 followed by noughts, we can write the decimal fraction at once.

Thus $\frac{216 \times 8}{625 \times 8} = \frac{1728 \times 2}{5000 \times 2} = \frac{3456}{10000}$

and the decimal fraction is ·3456.

NOTE.—This method we shall find of great use in a succeeding section.

The method of division is the more common method.

A vulgar fraction has been defined to be "numerator divided by denominator," so that $\frac{216}{625}$ means $216 \div 625$. We will add decimal places and divide

$$
\begin{array}{r}
625)216{\cdot}0000(\cdot3456 \\
1875 \\
\hline
2850 \\
2500 \\
\hline
3500 \\
3125 \\
\hline
3750 \\
3750 \\
\hline
\end{array}
$$

By the ordinary rule of division of decimals, the quotient will consist of 4 decimal places.

Rule.—*To convert vulgar into equivalent decimal fractions, divide the numerator by the denominator.*

Ex. 4. Reduce to decimals $3\frac{3}{5}$, $79\frac{5}{16}$, $\frac{2\frac{1}{5}}{12\frac{8}{10}}$, $\frac{11\frac{3}{20} - 3\frac{14}{70}}{5\frac{19}{95} + 10\frac{7}{10}}$

(1) $3\frac{3}{5} = \frac{18}{5} = 3{\cdot}6$.

(2) $79\frac{5}{16}$ $\frac{5}{16} = \cdot3125$, therefore $79\frac{5}{16} = 79\cdot3125$.

(3) $\frac{2\frac{1}{5}}{12\frac{8}{10}} = \frac{\frac{11}{5}}{\frac{128}{10}} = \frac{11}{64} = \frac{55}{320} = \frac{275}{1600} = \frac{1375}{8000} = \frac{6875}{40000} = \frac{34375}{200000}$

$= \frac{171875}{1000000} = \cdot171875$

(4) $\frac{11\frac{3}{20} - 3\frac{14}{10}}{5\frac{19}{95} + 10\frac{7}{10}} = \frac{\frac{223}{20} - 3\frac{2}{10}}{5\frac{1}{5} + \frac{107}{10}} = \frac{\frac{223}{20} - \frac{32}{10}}{\frac{26}{5} + \frac{107}{10}} = \frac{\frac{223-64}{20}}{\frac{104+214}{20}} = \frac{159}{318} = \frac{1}{2} = \cdot5.$

Exercises.

1. Express as vulgar fractions in their lowest terms:—

(1) ·84.
(2) 5·03125.
(3) ·0025.
(4) 16·7014.
(5) 222·36.
(6) 1·006.
(7) 15·296.
(8) 171·129.
(9) ·00001.
(10) ·000110.
(11) ·4015.
(12) 20·0002.

2. Express as decimal fractions:—

(1) $11\frac{5}{80}$
(2) $\frac{147}{32}$
(3) $8\frac{6}{25}$
(4) $\frac{35274}{20}$
(5) $\frac{17}{128}$
(6) $\frac{3042}{78125}$
(7) $\frac{147}{96}$
(8) $9\frac{11}{64}$
(9) $13\frac{625}{31125}$
(10) $\frac{256}{1024}$
(11) $19\frac{19}{160}$
(12) $17\frac{17}{256}$

3. Find the value of $\frac{\cdot0003 \times \cdot002}{\cdot006}$, and give the result as a vulgar fraction.

4. What vulgar fraction is equivalent to $\frac{\cdot0512}{81\cdot92}$?

5. Express $\frac{\cdot5}{\cdot75 + \cdot125}$ as a vulgar fraction.

6. What decimal fraction is equivalent to $\frac{1}{\cdot25}$, and to $\frac{5}{\cdot75}$?

7. Reduce $\frac{14}{2560}$ to a decimal fraction; also $\frac{570}{\cdot 05}$

8. Give the value of $\frac{12}{25} - \frac{3}{64}$ as a decimal.

9. Simplify by decimals $3\frac{7}{16} + 9\frac{3}{8} + 12\frac{2}{5} - 17\frac{7}{10}$.

10. Simplify $\frac{61 \cdot 53861}{14 \cdot 7}$; give the result as a vulgar fraction.

11. Reduce $\frac{\cdot 00625}{\cdot 0505}$ to a vulgar fraction in lowest terms.

12. Reduce to a decimal $(\frac{1}{2} + \frac{1}{4}) - (\frac{1}{5} + \frac{1}{8})$.

13. Simplify $\frac{3 \cdot 25}{5 \cdot 5} \times \frac{31}{3 \cdot 125} \times \frac{6 \cdot 875}{\cdot 341}$

14. Reduce ·2140625 to a vulgar fraction in its lowest terms.

15. Simplify $\frac{10 \cdot 001}{\cdot 001}$, $\frac{1 \cdot 0001}{\cdot 01}$, $\frac{100 \cdot 0001}{10 \cdot 001}$, and $\frac{\cdot 0001}{\cdot 001}$

16. What are the equivalent decimal fractions to $\frac{3}{512}$, $\frac{3}{51200}$, and $\frac{3}{512000}$.

Section V.—Indeterminate Decimals.

We cannot express all vulgar fractions in equivalent decimal fractions.

Ex. 1. Express (1) $\frac{5}{16}$, (2) $\frac{1}{3}$, (3) $\frac{25}{33}$, (4) $\frac{6658}{9900}$, as decimal fractions.

(1) $\frac{5}{16} = \frac{25}{80} = \frac{125}{400} = \frac{625}{2000} = \frac{3125}{10000} = \cdot 3125$.

(2) $\frac{1}{3} = \cdot 333333 \ldots .$ &c.

(3) $\frac{25}{33} = \cdot 75757575 \ldots .$ &c.

(4) $\frac{6658}{9900} = \cdot 67252525 \ldots .$ &c.

The first fraction, $\frac{5}{16}$, can be expressed as an exact decimal fraction; such decimal fractions are called *finite* decimals.

The other fractions cannot be expressed as exact decimals; such decimals are called *indeterminate.*

When only one figure *repeats*, as in (2), the decimal is said to be *repeating.*

When two figures recur, as in (3), the decimal is said to be *pure circulating*, or *pure recurring.*

When the recurring figures are preceded by figures that do not recur, he decimal is said to be *mixed circulating*, or *mixed recurring.**

* *Repeating, circulating,* and recurring are often used as synonymous terms.

The circulating or repeating part of the decimal is called a *period*, or *repetend*.

Instead of writing the recurring figure or figures over and over again, we write the figures recurring once only, and place a dot over the first and last figure of the period ; thus—

(2) $\frac{1}{3} = \dot{3}$ (3) $\frac{25}{33} = \dot{7}\dot{5}$

(4) $\frac{6658}{9900} = 67\dot{2}\dot{5}$

NOTE 1.—Every vulgar fraction which can be converted (by multiplying or dividing numerator and denominator by the same number) into another fraction with 10 or some power of 10 for its denominator, can be expressed as an exact decimal fraction. The factors of 10 are 2 and 5 : hence every fraction whose denominator contains only powers of 2 and 5 can be converted into an equivalent decimal.

If, however, the denominator of a fraction contains any other factor than 2 or 5—such as 3, 7, 11, &c.—we cannot change it into 10, or a power of 10 ; and hence the fraction cannot be converted into an exact equivalent decimal.

If we continue the division of numerator by denominator in the latter case sufficiently far, we must get a period, or repetend ; and this period must be complete before we get as many figures in the quotient as there are units in the divisor ; for, in the division, each remainder must be less than the divisor : hence, when we have exhausted all the remainders we can possibly have (viz. one less than the number of units in the divisor), some remainder must recur, and as a cipher is always brought down, the quotient must have the same set of figures over again.

Thus, let the fraction be $\frac{5}{7}$. Here we can have 6 remainders only, and then some one must recur :—

```
7)5·000000(·714285
  49
  ---
  10
   7
  ---
  30
  28
  ---
  20
  14
  ---
  60
  56
  ---
  40
  35
  ---
   5
```

In this case no remainder recurs until we have exhausted the whole. Again, in the example $\frac{13}{14}$, we cannot have more than 13 remainders before one recurs. On trial it will be found that the first remainder recurs after the seventh division, *i.e.* $\frac{13}{14} = \cdot 9\dot{2}85714\dot{}$.

Ex. 2. Reduce $\frac{3}{7}$, $\frac{5}{9}$, $\frac{2}{11}$, and $\frac{837}{1980}$ to decimal fractions.

(1) $\frac{3}{7}$: $7)3\cdot000000\ 000000$ $\quad\therefore \frac{3}{7} = \cdot\dot{4}28571\dot{}$
$\cdot 428571\ \ 428571$ &c.

(2) $\frac{5}{9}$: $9)5\cdot000000$ $\quad\therefore \frac{5}{9} = \cdot\dot{5}$
555555 &c.

(3) $\frac{2}{11}$: $11)2\cdot00000000$ $\quad\therefore \frac{2}{11} = \cdot\dot{1}\dot{8}$
18181818 &c.

(4) $\frac{837}{1980}$: $1980)837\cdot0000(422727$
7920
4500
3960
5400
3960
14400
13860
5400
3960
14400
13860
540

$\therefore \frac{837}{1980} = \cdot 42\dot{2}\dot{7}$

Ex. 3. To show that $\cdot\dot{4} = \frac{4}{9}$. Let x stand for the vulgar fraction equivalent to $\cdot\dot{4}$. We have to show that $x = \frac{4}{9}$.

(1) $x = \cdot 44444\ldots$

and (2) $10x = 4\cdot 44444\ldots$

and, subtracting (1) from (2) we have $9x = 4$

$$\therefore x = \frac{4}{9}$$

In the same way it may be shown that all decimals consisting of one repeating figure only, when converted to a vulgar fraction, will have the repeating figure for the numerator and 9 for its denominator.

Hence the rule :—*To convert a repeating decimal to a vulgar fraction, write the repeating figure for the numerator*, and 9 for the denominator.

Ex. 4. To show that $\cdot\dot{4}1\dot{5} = \frac{415}{999}$. Let x stand for the vulgar fraction equivalent to $\cdot\dot{4}1\dot{5}$, then we have to show that $x = \frac{415}{999}$.

(1) $x = \cdot 415415415415 \ldots$

and (2) $1000\,x = 415\cdot 415415415415 \ldots$

and, subtracting the (1) from (2) we have $999\,x = 415$.

$$\therefore\ x = \frac{415}{999}$$

In the same way it may be shown that all pure circulating decimals, when converted to vulgar fractions, will have a period, or repetend, for numerator, and for denominator as many *nines* as there are figures in the repetend.

Hence the rule :—*To convert a pure circulating decimal to its equivalent vulgar fraction, write the figures in one period for the numerator, and for denominator as many* NINES *as there are digits in the period.*

Ex. 5. To show that $\cdot 41\dot{2}0\dot{7} = \frac{41166}{99900}$. Let x stand for the vulgar fraction equivalent to $\cdot 41\dot{2}0\dot{7}$, then we have to show that $x = \frac{41166}{99900}$.

(1) $x = \cdot 41207207207207 \ldots .$

(2) $100\,x = 41\cdot 207207207207 \ldots .$

(3) $100{,}000\,x = 41207\cdot 207207207207 \ldots .$

and, subtracting (2) from (3) we have—

$$\begin{array}{rcl} 100{,}000\,x & = & 41207\cdot 207207207207 \\ 100\,x & = & \quad 41\cdot 207207207207 \\ \hline 99900\,x & = & 41166 \cdots\cdots\cdots \end{array}$$

Thus $99900\,x = 41166$

$$\therefore\ x = \frac{41166}{99900}$$

In subtracting (2) from (3) it will be seen that we subtract the figures that do *not* recur from the given decimal (including only one period) and the difference forms the numerator of the fraction. It will also be seen that there are as many nines in the denominator as there are digits in the repetend, followed by as many ciphers as there are non-recurring digits. And so in the reduction of all mixed circulating decimals to vulgar fractions.

Hence the rule :—*To convert a mixed circulating decimal to its equivalent vulgar fraction, set down all the figures from the decimal point to the end of the first period, and from this subtract the figures which do not repeat. The difference will form the numerator of the fraction. For the denominator write as many nines as there are digits in the repetend, followed by as many ciphers as there are digits that do not repeat.*

Note 1.—*Addition* and Subtraction of Indeterminate Decimals may be performed with sufficient exactness by taking 3 or 4 periods, and

adding or subtracting as with finite decimals; but for Multiplication and Division the recurring decimals should first be reduced to their equivalent vulgar fractions, and, if necessary, reduced back again to decimals.

NOTE 2.—Since $\cdot\dot{9}=\frac{9}{9}=1$, whenever $\dot{9}$ occurs omit repeater and increase the preceding figure by 1.

Thus $3\cdot40\dot{9}=3\cdot41$

NOTE 3.—It is inconvenient to work with long decimal fractions, and, moreover, the figures which stand in the 5th, 6th, &c., places, represent exceedingly small values; hence they are often neglected, and we are asked to find the answer "correct to 4, 5, or 6, &c., places of decimals."

In estimating the value of a decimal correct to a certain number of places, it is necessary to look at the first figure discarded. Thus: supposing our decimal is 3·0756841, and we are asked to make it as nearly correct as possible to 4 places of decimals. The answer 3·0756 will not be so nearly correct as 3·0757, because the first discarded figure stands for $\frac{8}{100000}$, and if $\frac{2}{100000}$ were added to it, it would make $\frac{10}{100000}$, or $\frac{1}{10000}$. If, then, I write the fraction 3·0757, it will be $\frac{2}{100000}$ too much; if I write it 3·0756 it will be $\frac{8}{100000}$ too little. Hence the former is nearer the truth than the latter. If the first discarded figure is 5, the error of omitting it, or adding 1 to the preceding figure will be the same. If, therefore, the first discarded is more than 5, add 1 to the last of the digits which are retained.

Thus 5·047 true to 2 places = 5·05
·60141 „ 4 „ = ·6014
and ·00719 „ 4 „ = ·0072

Ex. 6. Express $\frac{2\frac{2}{3}\times(2\frac{1}{3}+\frac{7}{5})}{\frac{3}{4}-\frac{4}{7}}$ as a decimal.

$$\frac{2\frac{2}{3}\,(2\frac{1}{3}+\frac{7}{5})}{\frac{3}{4}-\frac{4}{7}}=\frac{\frac{8}{3}\left(\frac{56+21}{24}\right)}{\frac{21-16}{28}}=\frac{\frac{8}{3}\times\frac{77}{24}}{\frac{5}{28}}$$

$$=\frac{\overset{1}{\cancel{8}}}{\underset{3}{\cancel{3}}}\times\frac{77}{\underset{3}{\cancel{24}}}\times\frac{28}{5}=\frac{2156}{45}=47\cdot9\dot{1}.$$

Ex. 7. Simplify $\frac{(\cdot\dot{2}\dot{7}+\cdot\dot{3})\times(2\cdot\dot{1}\dot{2}-1\cdot3\dot{1}\dot{3})}{1\cdot4\dot{5}-1477\dot{2}}$

$$\frac{(\cdot\dot{2}\dot{7}+\cdot\dot{3})\times(2\cdot\dot{1}\dot{2}-1\cdot3\dot{1}\dot{3})}{1\cdot4\dot{5}-1477\dot{2}}=\frac{\left(\frac{27}{99}+\frac{3}{9}\right)\times\left(2\frac{12}{99}-1\frac{310}{990}\right)}{1\frac{45}{99}-\frac{14625}{99000}}$$

$$= \frac{\left(\frac{3}{11}+\frac{1}{3}\right) \times \left(\frac{210}{99}-\frac{130}{99}\right)}{1\frac{5}{11}-\frac{13}{88}} = \frac{\frac{9+11}{33} \times \frac{80}{99}}{\frac{128-13}{88}}$$

$$= \frac{\frac{20}{33} \times \frac{80}{99}}{\frac{115}{88}} = \frac{20}{33} \times \frac{\overset{16}{\cancel{80}}}{\underset{9}{\cancel{99}}} \times \frac{\overset{8}{\cancel{88}}}{\underset{23}{\cancel{115}}} =$$

$$= \frac{2560}{6831} = \cdot 3747621138\ ;$$

which, correct to 7 places = ·3747621.

Exercises.

1. Reduce $\frac{1}{3}$, $\frac{2}{7}$, $\frac{5}{11}$, and $\frac{8}{13}$ to decimal fractions, and mark the repetend in each case.

2. Express $3\cdot\dot{3}$, $2\cdot\dot{1}\dot{7}$, $51\cdot1\dot{4}$, and $\cdot37\dot{4}\dot{2}$ as vulgar fractions.

3. Add $5\cdot\dot{6}$, $7\cdot\dot{2}\dot{3}$, $94\cdot\dot{4}7\dot{6}$, $72\cdot\dot{9}$, $\cdot0\dot{8}3\dot{4}$, and $\cdot000\dot{7}$ correct to *four* places.

4. Express the *exact* difference between $\cdot1\dot{2}$ and $\cdot1\dot{3}$ as a vulgar fraction, and as a decimal fraction.

5. Reduce $1\cdot1736\dot{1}$ to a vulgar fraction.

6. Divide 200·1 by ·3553 correct to 5 places of decimals.

7. Find the equivalent vulgar fraction for $\cdot00\dot{1}7948\dot{7}$.

8. Reduce $4\cdot37\dot{2}$ to a vulgar fraction in its lowest terms.

9. Add together 3·3, ·3, ·03, ·003; divide the sum by $\cdot0\dot{3}\dot{6}$, and subtract $\cdot\dot{3}$ of $\cdot\dot{5}$ from the quotient.

10. What vulgar fraction is equivalent to $3\cdot71\dot{2}4\dot{3}$?

11. Find the value of $91\cdot8\dot{6}3\dot{}\div87\cdot5\dot{6}$.

12. Show (by factoring the denominators) which of the following fractions will produce finite, and which indeterminate decimals:—

$$\frac{5}{12},\ \frac{6}{32},\ \frac{19}{128},\ \frac{3}{77},\ \frac{17}{24},\ 21\tfrac{3}{32},\text{ and }\frac{512}{1024}$$

13. Express $\frac{\cdot8824}{\cdot34}$ as a vulgar fraction.

14. What vulgar fraction is equivalent to $\cdot5\dot{4}6190\dot{}$?

15. Find the difference between $63\cdot\dot{6}$ and $6\cdot3\dot{6}$.

16. Find the value of $4\cdot0\dot{3}\div\cdot1\dot{8}09523$.

17. What must be added to ·0324 to produce $3\cdot0\dot{4}7\dot{2}\dot{1}$

18. Add $2\cdot\dot{2}\dot{7}$, $15\cdot\dot{7}$, $6\cdot34\dot{0}\dot{7}$, and $91\cdot\dot{7}60\dot{1}$ correct to 5 decimal places.

19. Express $\cdot2\dot{0}012\dot{3}$ as a vulgar fraction.

20. Express $\frac{47\frac{11}{15}}{238\frac{2}{5}} \div \frac{11\frac{1}{9}}{14\frac{2}{7}}$ as a decimal fraction.

21. Add together $8\cdot\dot{9}$, $2\cdot6\dot{9}$, $5\cdot\dot{8}1\dot{9}$, $4\cdot\dot{8}77149$ without changing to vulgar fractions.

22. Change $\frac{3072}{9900}$ and $\frac{24}{9999}$ to recurring decimals without division.

23. By what number must $\cdot4485$ be divided so that the quotient may be $\cdot0069$?

24. Reduce to a decimal $\left(\frac{1\frac{3}{4}}{4\frac{1}{2}} \div \frac{2\frac{1}{3}}{1\frac{1}{4}}\right) \times (1\frac{4}{5} \text{ of } \frac{1}{2}) + 3\frac{2}{5} \text{ of } \frac{1}{11} \text{ of } \frac{3}{10000}$

25. There is a number which when multiplied by $\cdot\dot{1}$, and the product divided by $\cdot2\dot{1}$, produces $\cdot22\dot{1}$; find the number.

26. What recurring decimal is equal in value to $\cdot01\dot{2} \div \cdot00132$?

27. Reduce $\frac{481}{11111}$ to its equivalent circulating decimal.

28. Convert, without division, into circulating decimals $\frac{826131}{999900}$ and $\frac{37254}{99000}$

29. What is the value of $\frac{\cdot3\dot{7}5}{\cdot375}$?

30. A person possessing $\cdot6$ of an estate sells $\cdot3\dot{6}$ of his share; what part of the estate has he left?

31. In a town of 15,000 electors $\cdot72$ voted for one candidate, and $\cdot125$ for another; how many electors did not vote?

32. A owns $\cdot56\dot{7}$ of an estate, and B the remainder; express B's share as a vulgar fraction.

33. Simplify $\frac{2\cdot5 \text{ of } 2\cdot\dot{1}42857}{3\cdot\dot{6} \text{ of } 1\cdot3\dot{6}}$ and give the answer as a decimal fraction.

34. What decimal fraction is equivalent to $\frac{1}{3}\ (\frac{1}{5}+\frac{5}{2}) + \frac{1}{5}\ (\frac{1}{2}+\frac{2}{3}) - \frac{1}{2}\ (\frac{1}{3}+\frac{2}{5})$?

35. The population of a parish is 2,000, of which $\cdot3$ are men, $\cdot175$ women, and the rest children; find how many there are of each?

36. Reduce $\frac{3\frac{3}{4}+2\frac{5}{8}-4\frac{1}{16}}{(3\frac{1}{12} \times 2\frac{2}{5}) \div 2\frac{1}{2}}$ to a decimal fraction.

37. Find the value of $7\cdot\dot{4}07 \times 1\cdot375 \times 2\cdot\dot{4}5\dot{6}$.

38. Arrange in order of magnitude $\cdot34\dot{5}$, $\cdot345$, $\cdot34\dot{5}$, $\cdot3\dot{4}5$.

39. Simplify $\frac{\cdot3\dot{7}}{1\cdot0\dot{4}} \times \frac{\cdot052}{1\cdot\dot{8}} \div \frac{1\cdot\dot{9}}{2\cdot\dot{9}}$

Section VI.—Reduction of Decimals.

As decimals can always be changed to their equivalent vulgar fractions, in all questions of reduction of decimals we may find the equivalent vulgar fractions, and proceed as shown under reduction of vulgar fractions. In many cases, however, we may obtain the required answer by a shorter and simpler method than having recourse to vulgar fractions.

The selection of the particular method to be employed on any given example must be left to the judgment of the student. We subjoin illustrative examples.

Ex. 1. Find the value in pounds, shillings, pence, and farthings of £5·640625.

(1) We may proceed by fractions, thus—

$$£5{\cdot}640625 = £5\tfrac{640625}{1000000} = £5\tfrac{41}{64}$$

and $\frac{41}{60}$ of £1 or $\frac{41}{64}$ of $\frac{960}{1}$ farthings =

$$\frac{41}{64} \times \frac{960}{1} \overset{\text{far.}}{=} 615 = 12s.\ 9\tfrac{3}{4}d.$$

Therefore $£5\frac{41}{64} = £5\ 12s.\ 9\frac{3}{4}d.$

(2) In examples of this kind, where the decimal is long, and perhaps when written as a vulgar fraction not reducible to lower terms, it is best to proceed by the ordinary method of descending reduction. Thus: first multiply the decimal part of £1 by 20 to reduce to shillings, and mark off the decimal places; the figures to the left of the point will be *whole numbers* of shillings, and the figures to the right of the point will be a decimal fraction of one shilling. Secondly, multiply by 12 to reduce to pence, and mark off in the same way the whole number of pence from the decimal. Lastly, multiply by 4 to reduce to farthings, and mark off as before.

	£5·640625
	20
shillings	12·812500
	12
pence	9·750000
	4
farthings	3·000000

∴ £5·640625 = £5 12*s.* 9¾*d.*

Weights and measures may be worked in the same way.

Ex. 2. Find the value of 5·7890625 cwt. in cwt., qrs., lbs., and ozs.

cwts. 5·7890625
4
qrs. 3·1562500
28
12500000
3125000
lbs. 4·3750000
16
22500000
3750000
ozs. 6·0000000

Ans. 5 cwt. 3 qrs. 4 lbs. 6 ozs.

Ex. 3. Reduce £5 12*s.* 9¾*d.* to the decimal of £1; and 5 cwt. 3 qrs. 4 lbs. 6 ozs. to the decimal of a cwt.

The operation necessary in this example is the converse of that in the two preceding examples. We will work the first by reducing to a vulgar fraction, and the second by ordinary ascending reduction.

(1) $\frac{12s.\ 9\frac{3}{4}d.}{£1} = \frac{615}{960} = \frac{41}{64} = ·640625$ (farthings)

Therefore £5 12*s.* 9¾*d.* = £5·640625.

(2) We must reduce the ozs. to the next higher name, then prefix the integer, if any, of same name in the given quantity. Secondly, we must reduce to the next higher name, and prefix the integer, if any, of the same name. Proceed in a similar way to the denomination required.

5 cwts. 3 qrs. 4 lbs. 6 ozs.

16)6· ozs.
28)4·375 lbs.
4)3·15625 qrs.
5·7890625 cwts.

Ans. 5·7890625 cwts.

The second method in this case, also, is usually the shorter and *more simple.*

Ex. 4. Reduce £12 0*s.* 7½*d.* to the decimal of £60 3*s.* 1½*d.*

We can here reduce the £12 0*s.* 7½*d.* to the decimal of £1, then £60 3*s.* 1½*d.* to the decimal of £1 ; and then divide the 1st by the 2nd.

£12 0*s.* 7½*d.*		£60 3*s.* 1½*d.*	
2) 1·	half-pence.	2) 1·	half-pence.
12) 7·5	pence.	12) 1·5	pence.
20) ·625	shillings.	20) 3·125	shillings.
12·03125	£.	60·15625	£.

$$\text{And } \frac{12\cdot03125}{60\cdot15625} = \frac{1203125}{6015625}$$

6015625)1203125·0(·2
12031250

The method by vulgar fractions in this and all similar examples is much to be preferred. Thus—

$$\frac{\text{£12 0s. 7½d.}}{\text{£60 3s. 1½d.}} = \frac{5775 \text{ half-pence}}{28875 \text{ half-pence}} = \frac{231}{1155}$$

1155)231·0(·2
2310

Ex. 5. Find the value of ·1875 of 16*s.* 4*d.*

$$\text{1st method } \cdot1875 = \frac{1875}{10000} = \frac{375}{2000} = \frac{75}{400} = \frac{15}{80} = \frac{3}{16}$$

$$\text{and } \frac{3}{16} \text{ of } 16s.\ 4d. = \frac{3}{16} \times 16\tfrac{1}{3} = \frac{3}{16} \times \frac{49}{3} = \frac{49}{16} = 3s.\ 0\tfrac{3}{4}d.$$

2nd method ·1875 of 16*s.* 4*d.* = ·1875 of 196 pence = ·1875 × 196.

·1875
196
11250
16875
1875
36·7500 pence.
4
3·00

that is ·1875 of 196 pence = 3*s.* 0¾*d.* *Ans.*

The first method is preferable to the second.

Ex. 6. What decimal of a penny is ·00637 of £1? And what decimal of an acre is ·379 of a pole?

(1)
$$\begin{array}{r} \cdot00637£. \\ 20 \\ \hline \cdot12740s. \\ 12 \\ \hline 1\cdot52880d. \end{array}$$

(2)
$$\begin{array}{rl} 40)\cdot379 & \text{pole.} \\ 4)\cdot009475 & \text{rood.} \\ \cdot00236875 & \text{acre.} \end{array}$$

Ex. 7. What decimal of £3 13s. 4d. is ·41$\dot{6}$ of ·357$\dot{5}$ of £4·$\dot{7}$14285$\dot{7}$?

$$= \frac{\cdot41\dot{6} \times \cdot357\dot{5} \times 5\cdot\dot{7}14285\dot{7}}{£3\ 13s.\ 4d.}$$

$$= \frac{\frac{375}{900} \times \frac{3540}{9900} \times 5\frac{5}{7}}{3\frac{1}{3}} = \frac{\frac{5}{12} \times \frac{177}{495} \times \frac{40}{7}}{\frac{11}{3}}$$

$$= \frac{\cancel{5}}{\cancel{12}_{\cancel{4}}} \times \frac{\overset{59}{\cancel{177}}}{\underset{\cancel{99}_{33}}{\cancel{495}}} \times \frac{\overset{10}{\cancel{40}}}{7} \times \frac{\overset{1}{\cancel{3}}}{\underset{11}{\cancel{11}}} = \frac{590}{2541}$$

And $\frac{590}{2541} = \cdot2322$ nearly.

Exercises.

1. Express 1s. 5½d. as a decimal of £1.
2. Reduce 10¾d. to the decimal of 1s. 6d.
3. Find the value of ·00625 of £2 4s.
4. Express £6 13s. 2¼d. as a decimal of £3.
5. What is the value in feet of ·0253 of a mile?
6. What is the value of 1·25 of 5d. + 2·05 of 18d. + 1·5 of 3s. 4½d.?
7. What decimal fraction of £22 14s. 6d. is £12 12s. 6d.?
8. Find the value of £4·790625.
9. Find the value of ·028125 tons in lbs.
10. Find the value of 7·18 of 5 ozs. 15 dwts. 17 grs.

* The student should note that

$\frac{1}{7} = \dot{1}42857\dot{1}$ $\quad \frac{2}{7} = \dot{2}85714\dot{2}$ $\quad \frac{3}{7} = \dot{4}28571\dot{4}$ $\quad \frac{4}{7} = \dot{5}71428\dot{5}$ $\quad \frac{5}{7} = \dot{7}14285\dot{7}$ $\quad \frac{6}{7} = \dot{8}57142\dot{8}$

11. Reduce 1·0165 of an acre to square yards.

12. Find the value of £·525 + 8·425 of 3*s.* 4*d.* + ·3 of 5*s.*

13. Reduce 8 yds. 9 in. to the decimal of a mile.

14. Reduce 5·298 of 6*s.* 8*d.* to a decimal of a penny.

15. What is the difference between 22·4735 of £1, and 17·2845 of a guinea?

16. What decimal of £5 is $3\frac{8}{18}\frac{17}{48}$ of £3?

17. Express 19 lbs. 5 ozs. as a decimal of 1 cwt. 2 qrs. 25 lbs. 2 ozs.

18. Express ·000001 of a day in seconds.

19. Find the value of 11·75 of 8*s.* $0\frac{3}{4}d.$ + 7·125 of £3·675 + $1\frac{1}{16}$ of 3*d.*

20. Reduce ·0325 of £1 to the decimal of £3 6*s.* 8*d.*

21. Reduce 101 lbs. 5 ozs. to the decimal of 1 cwt.

22. Express 26 mins. 15 secs. as a decimal of an hour.

23. Find the sum of 81·7 of £3 5*s.*, 72·$\dot{3}$ of £505, and ·$\dot{2}85714\dot{2}$ of ·$\dot{3}$ of a guinea.

24. A clerk copied 62·5 instead of ·625 of £100; what was the amount of his error?

25. Find the value of 5·56875 of 6*s.* 8*d.* − ·283 of 7*s.* 6*d.*

26. Reduce 5 cwt. 1 qr. 13 lbs. 9 ozs. to the decimal of a ton.

27. What decimal fraction is £1 16*s.* 11*d.* of £33 4*s.* 6*d.*?

28. Reduce £7 16*s.* 9·254*d.* to the decimal of £1.

29. Find the value of ·0175 ton + 3·114 cwt. + ·144 qr.

30. Reduce £116 17*s.* 6*d.* to the form of a decimal of £1, and multiply the result by 1·4$\dot{5}$.

31. Reduce 118·37 ft. (1) to the decimal of a mile, (2) to the decimal of a furlong, and (3) to the decimal of a yard.

32. Add together $\frac{3}{4}$ of $\frac{5}{8}$ of £2 5*s.*, $\frac{2}{7}$ of 5 guineas, 4·3$\dot{6}$ of £1 18*s.* 6*d.* and 4·30$\dot{9}$ of £2 15*s.*, and reduce the result to the decimal of 30 guineas.

33. Reduce 3 hrs. 42 mins. 39 secs. to the decimal of a year.

34. Reduce 5 lbs. 2 ozs. to the decimal of $2\frac{1}{2}$ tons.

35. Reduce ·64 of £1 to the decimal of 1·3*d.*

36. Reduce $2\frac{1}{4}$ in. to the decimal of $2\frac{1}{8}$ miles.

37. Find the value of 2 qrs. 26 lbs. 8 ozs. × 1·4$\dot{6}$ + 2 cwts. 7 lbs. × ·$\dot{7}\dot{2}$ + 2 qrs. 16 lbs. × 1·375.

38. Find the value of ·$\dot{4}28571\dot{4}$ of 1 min. 17 secs.

39. Express 262 yds. 1 ft. 10 in. as a decimal of 1 mile, and ·36875 ozs. in dwts. and grs.

40. Reduce ·075 of a £ to the decimal of 4*s.* 6*d.*

41. What is the difference in the decimal of a lb. between ·0055 of a cwt. and ·0005 of a ton?

42. *From* what weight in the decimal of a ton can I subtract 3·78 lbs. *so as to leave* ·05 *of a* cwt.?

43. Subtract $\frac{4}{15}$ of £78 11*s.* $10\frac{1}{2}d.$ from $\cdot\dot{4}28571\dot{}$ of £51 14*s.* 3*d.*

44. If an oz. of gold is worth £3·89, what is the cost of ·04 lb?

45. Find the value of $\frac{\cdot 06812}{\cdot 131}$ of £2 0*s.* $1\frac{1}{2}d.$

46. Reduce 4 lbs. 3 ozs. 6 grs. to a decimal of 1 lb. troy.

47. Reduce $\cdot 3\dot{6}\dot{3}$ of £2 1*s.* 3*d.* to the decimal of $\cdot\dot{7}\dot{2}$ of £3·6.

48. Simplify $\frac{5}{6}$ of ·075 of £1 $+\frac{4}{7}$ of $\cdot 5\dot{3}\dot{9}$ of £1 0*s.* $7\frac{1}{2}d. + \frac{1}{\cdot\dot{2}\dot{3}}$ of $10\frac{1}{2}d.$

49. Add ·3795 of 18*s.* to ·4360 of £9·5, and express the result as a decimal of £5.

50. Find the value of $\cdot 3\dot{4}\dot{5}$ of 9*s.* 2*d.* $+\frac{5}{8}$ of ·075 of £10 $+ \cdot 0\dot{5}$ of 1·125 of £1 13*s.* 4*d.*, and express the result as a decimal of £50.

51. Find the value of 3·75 of 5*s.* 6*d.* + 5·05 of £3 1*s.* 8*d.* + $5\cdot 0\dot{7}$ of 7*s.* 6*d.* + $3\cdot 1\dot{3}\dot{5}$ of £2 1*s.* 3*d.*

52. Find the value of $5\cdot 4\dot{9}$ of $\cdot 03\dot{1}\dot{8}$ of $\cdot 047619\dot{}$ of 26 guineas.

53. Find the value of $(\frac{2}{3} \times \frac{3}{16})$ of £1 + ·23 of 4*s.* 2*d.* $+(5 - \frac{3}{2})$ of 1*d.*, and express the result as a decimal of £1.

54. Express as a decimal fraction of an acre $\frac{11}{198}$ square poles $+\frac{7}{45}$ square yards $+ 5\frac{9}{40}$ square feet.

55. Reduce 2 pls. 909 yds. 1 ft. 72 in. to the decimal of 1 acre 60 square yards.

56. Reduce 5 rds. $12\frac{1}{2}$ pls. $3\frac{1}{2}$ yds. to the decimal of an acre.

57. Out of £4·725 I paid away £2·38, and again £2·16; find the remainder, giving the answer as a decimal.

58. Find the sum of money to which, if we add ·9 of £6·875, the result will be ·8 of £13 6*s.* 8*d.*

59. Find the value of $\frac{\cdot 5\dot{4} \text{ of } 0\dot{2}\dot{7}}{\cdot 14\dot{8}}$ of one farthing.

60. Subtract ·4625 of 16 shillings from ·1136 of £8·5, and express the remainder as the decimal of £3.

MISCELLANEOUS EXERCISES.—SECTION I. (WORKED OUT).

Ex. 1. Find the G. C. M. of ·0684 and 11·172.

Multiply each number by 10,000 to get rid of all decimal places.

$$\cdot 0684 \times 10000 = 684$$

$$11\cdot 172 \times 10000 = 111720$$

The G. C. M. of the whole numbers 684 and 111,720 is 228.

This is 10,000 times too much, as we multiplied by 10,000; hence divide 228 by the previous multiplier, viz. 10,000.

$228 \div 10000 = \cdot 0228$, the required G. C. M.

Ex. 2. Find the L. C. M. of ·019, ·36, and 5·7.

Get rid of all decimal places as in the previous example.

$$\cdot 019 \times 1000 = 19$$
$$\cdot 36 \times 1000 = 360$$
$$5\cdot 7 \times 1000 = 5700$$
$$19 = 19$$
$$360 = 3 \times 3 \times 2 \times 2 \times 2 \times 5$$
$$5700 = 3 \times 19 \times 2 \times 2 \times 5 \times 5$$
$$\text{L. C. M.} = 19 \times 3 \times 3 \times 2 \times 2 \times 2 \times 5 \times 5$$
$$= 34200$$

Divide by the previous multiplier, viz. 1,000.

$34200 \div 1000 = 34{\cdot}2$, the required L. C. M.

Ex. 3. What number has the same ratio to 1 as ·004 has to ·0128?

$$x : 1 :: 004 : 0128$$
$$\therefore \quad \cdot 0128 \times x = \cdot 004$$
$$\therefore \quad x = \frac{\cdot 0040}{\cdot 0128} = \frac{40}{128} = \frac{10}{32} = \frac{5}{16}$$
$$= \cdot 3125 \; Ans.$$

Ex. 4. If 12 men can earn £21·2̇96̇ in 12·7̇ days, how much can 9 men earn in 26·5̇4̇ days at the same rate?

(1) Less men earn less money.

(2) More money will be earned in 26·5̇4̇ than in 12·7̇ days.

Hence—

$$\left.\begin{matrix} 12 & : & 9 \\ 12\cdot\dot{7} & : & 26\cdot\dot{5}\dot{4} \end{matrix}\right\} :: 21\cdot\dot{2}9\dot{6} : x$$

$$x = \frac{9 \times 26\cdot\dot{5}\dot{4} \times 21\cdot\dot{2}9\dot{6}}{12 \times 12\cdot\dot{7}}$$

$$= \frac{9 \times 26\frac{54}{99} \times 21\frac{296}{999}}{12 \times 12\frac{7}{9}}$$

$$= \frac{9 \times 26\frac{6}{11} \times 21\frac{8}{37}}{12 \times 11\frac{5}{9}}$$

$$= \frac{9 \times \frac{292}{11} \times \frac{575}{27}}{12 \times \frac{104}{9}}$$

$$= \frac{\cancel{9}^{3} \times \cancel{292}^{73} \times \cancel{575}^{5} \times \cancel{9}^{1}}{\cancel{12}^{4} \times \cancel{104} \times 11 \times \cancel{27}^{3}} = \frac{73 \times 5}{11}$$

$$= \frac{365}{11} = £33\frac{2}{11} = £33\ 3s.\ 7\cdot\dot{6}3\dot{6}d.$$

Ex. 5. Divide £17 19*s.* 4½*d.* between A, B, and C in the ratio of $\cdot\dot{0}\dot{4}$, $\cdot\dot{0}\dot{5}$, $\cdot\dot{0}\dot{6}$:

$\cdot\dot{0}\dot{4} = \frac{4}{99}$, $\cdot\dot{0}\dot{5} = \frac{5}{99}$, and $\cdot\dot{0}\dot{6} = \frac{6}{99}$

that is, we must divide in the ratio of 4 : 5 : 6.

(1) 15 : 4 :: £17 19*s.* 4½*d.* : x

x = £4 15*s.* 10*d.*

(2) 15 : 5 :: £17 19*s.* 4½*d.* : x

x = £5 19*s.* 9½*d.*

(3) 15 : 6 :: £17 19*s.* 4½*d.* : x

x = £7 3*s.* 9*d.*

Therefore—

	£	*s.*	*d.*
A's share =	4	15	10
B's „ =	5	19	9½
C's „ =	7	3	9
Total	£17	19	4½

Ex. 6. A's share of a certain sum of money is $\cdot\dot{0}\dot{9}$ of B's share, B's share is 3·08 of C's share, and C's is 1·1 of D's. C's share is £2,750 ; find the shares of A, B, and D.

C's share = 1·1 of D's share

= $\frac{11}{10}$ of D's share

Hence D's share is $\frac{10}{11}$ of C's ;

But C's = £2,750.

∴ D's = £2,750 × $\frac{10}{11}$ = £2,500.

Again— B's share = 3·08 of C's

= $3\frac{8}{100} = 3\frac{2}{25}$ of C's ;

and $3\frac{2}{25}$ of £2,750 = £8,470.

Lastly— A's share = $\cdot\dot{0}\dot{9}$ or $\frac{9}{99}$ or $\frac{1}{11}$ of B's ;

and $\frac{1}{11}$ of £8,470 = £770 ;

Hence, A's share = £770.

B's „ = £8,470.

and D's „ = £2,500.

Section II.—Miscellaneous Exercises.

1. Reduce to vulgar fractions $3\cdot4\dot{9}$, 3·49, and $3\cdot\dot{4}\dot{9}$.

2. Reduce 12 cwt. 3 qrs. 21 lbs. to the decimal of a ton.

3. Simplify $\frac{7\cdot686 \times \cdot0084}{\cdot00036}$

4. What sum of money multiplied by $\cdot\dot{8}$ and divided by $\cdot\dot{5}$ gives £6·96 ?

5. Reduce $\frac{\frac{3}{4} \times \frac{4}{5}}{\frac{3}{4} \div \frac{4}{5}}$ of a shilling to the decimal of a guinea.

6. Find the G. C. M. of 309·23 and 598·23.

7. How many times is 6·3 of 2·007 contained in ·375 of 74·679215625?

8. Find the sum, difference, product, and quotient of ·96$\dot{3}$45 and ·$\dot{3}$.

9. If 4 ozs. of tea cost ·5625 shilling, find the value of ·15625 cwt.

10. If 3 bushels cost 1·1 of a £, what will 33·4 qrs. cost at the same rate?

11. Reduce $\frac{5}{32}$ and $\frac{1}{128}$ to decimals, and find the product of their sum and difference correct to 6 decimal places.

12. Find the value of £3·89375 in pounds, shillings, and pence.

13. What number divided by 1·25 will give the product of 11 × 1·1 × ·0011?

14. Reduce 2 qrs. 3$\frac{1}{3}$ nls. to the decimal of an English ell.

15. What fraction of a £5 note is $\frac{3}{8}$ of 7*s.* 6*d.* + ·625 of 10*s.* − ·54$\dot{5}$ of 9*s.* 2*d.*

16. If 3·75 yds. cost £3·825, what will 38·5625 yds. cost at the same rate?

17. Reduce 19 cwt. 1 qr. 24 lbs. 8 ozs. to the decimal of a ton.

18. Find the value of £18 2*s.* 8*d.* × 4·6875.

19. If ·3 of an estate be worth £450·8, what is the value of ·48 of the estate?

20. The weight of a cubic inch of water is 253·17 grains, that of a cubic inch of air ·310017 grains; how many cubic inches of air are equal in weight to one cubic foot of water?

21. Add together 7 yds. 1 ft. 5·7 in., 19 yds. 2 ft. 6·95 in., 8 yds. 2 ft. 9·67 in., and 2 yds. 4·72 in., and from the sum subtract 2 ft. 2·04 in.

22. Find the L. C. M. of $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{9}{16}$, and of ·06, ·008, and ·16.

23. Find what decimal 54 is of 47$\frac{1}{4}$, and 2·07 bushels of 2·07 qrs.

24. Divide 13*s.* 3*d.* among 3 boys, in the proportion of ·3, ·4, and ·5.

25. Express 3·2175625 miles as miles, furlongs, poles, yards, feet, and inches.

26. Find the least decimal which must be added to 4·92 + 37·091 + 640·$\dot{9}$ to make the sum consist entirely of integers.

27. A man buys ·1875 of an estate, and re-sells ·1$\dot{7}$ of his purchase; what fraction has he remaining (*a*) of the estate, (*b*) of his purchase?

28. If 3·75 cwt. be carried 51·6 miles for £·875, what weight should be carried 75 miles for £2·625?

29. If £1,000 gain £35 in 10 mo., what sum will gain £36 in 11 mo.?

30. Find the G. C. M. of £27 13*s.* 5$\frac{1}{4}$*d.*, and £11 6*s.* 7$\frac{3}{4}$*d.*, and ·002048 and ·0256.

31. Find the value of $\frac{5}{8}$ of ·0375 of £1 + ·3125 of 10*s.* + $\frac{4}{7}$ of ·539 of 1*s.* 10½*d.*

32. If 40 men can reap 400·6 acres in 12·75 days, how many acres ought 30 men to reap in 3·4 days?

33. Required the cost of painting the walls of a room 18 ft. 10·25 in. long, 17 ft. 1·75 in. broad, and 12 ft. 3 in. high, at 10*d.* per sq. yd.

34. If 1½ yds. of butter weigh 1¼ lbs., how many inches of butter can be bought for 2½*d.* when butter is at 1*s.* 6*d.* per lb.?

35. Find a fourth proportional to ·004, ·052, and ·876.

36. If 4 men have a sum of money to be divided between them, so that A has ·3̇ of it, B ·25, C ·16 of it, and D the remainder, which is £28; what is the sum?

37, If ·5 of £1 buy ·4 of a gallon, how much may be bought for 6*s.* 3*d.*?

38. Express 5·6 cub. in. as a decimal of ·5 cub. ft.

39. A man having £11 14*s.* 9½*d.*, gave ·4 of it for 1·75 cwt. of beef; find the price of the beef per lb.

40. A owns ·7 of an estate, and B the remainder, and the difference in value of their shares is 1,000 gs.; what is the value of the estate?

41. The price of an oz. of coffee is ·4583 shilling; what is the value of ·0015625 of a ton?

42. Express £3,167·9 as the decimal of £3,749 6*s.* 11¾*d.* to three places.

43. Reduce 7 fur. 37 pls. 2 yds. 3 in. to the fraction of 10 m. 7 fur. 12 pls. 2½ yds. 9 in.

44. If ·25 of 1·3̇ of an oz. of gold be worth £1·3̇36̇, find the value of 6·7̇3̇ ozs.

45. What must be the length of a plot of ground if the breadth be 15¾ ft. that its area may be 46 sq. yds.?

46. Find the L. C. M. of 6·8̇1̇ and 8·7̇2̇.

47. The sum of £26 was divided amongst 4 men, A, B, C, D, in the following manner:—B's share was 3 times A's, C's was 3 times B's, and D's was as much as A's, B's, and C's together; what was the share of each?

48. If two persons can do a piece of work in 2·75 days, and one of them can do it in 5·5 days, in what time can the other do it?

49. A and B can build a boat in 18 days, and with the assistance of C they can build it in 11 days; in what time can C build it by himself?

50. What number bears the same ratio to 5·1 as 365 does to 366?

51. How many yards of carpet 2·16̇ yds. wide will cover a room 25·3̇ ft. broad and 28·8̇ ft. long?

52. Reduce $8\frac{62022}{100000}$ shillings to the decimal of half a guinea.

53. *A clock goes at the rate* of ·08 per cent. too fast for 9 months; it

is then reset and loses at the same rate for 3 months; how much will it have gained or lost at the end of a year of 365 days?

54. A pint contains $34\frac{3}{8}$ cub. in.; how many galls. of water will fill a cistern 4 ft. 4 in. long, 2 ft. 8 in. broad, and 1 ft. $1\frac{1}{2}$ in. deep?

55. If 16·125 yds. of cloth, 1·375 yd. wide, cost £2 2*s.*, what will 45 yds. ·875 yd. wide cost at the same price per sq. yd.?

56. Work the following sums decimally—(*a*) £3 7*s.* 9*d.* × 392; (*b*) £57 9*s.* 6*d.* ÷ 2299.

57. What sum of money will be the same decimal fraction of 8*s.* $10\frac{1}{2}$*d.* as 11 dwt. 8 grs. is of 17 dwts. 17 grs.?

58. A can mow a field in 10 days, and B in 12 days; if they work together what time will it take?

59. A man apportions $\cdot\dot{6}\dot{3}$ of a field in 49 allotments of equal size, and retains the remainder of the field for himself. The differences in size of an allotment and his portion of the field is 4·33125 acres; find the size of the field.

60. A watch which gains one minute per day is right at noon on the 1st of January; what will be the correct time when it points to noon on January 31st?

61. If 50 masons can build a wall 240 yds. long, ·5 yds. thick, and 3 yds. high in 18 days of $8\frac{1}{2}$ hrs. each; in how many days of 10 hrs. each will 36 men build a wall 220 yds. long, 4 yds. high, and ·45 yds. thick?

62. A person is able to travel 142·2 miles in 4·5 days of 10·164 hours each; in how many days of 8·4 hours each can he travel 505·6 miles.

63. If £40 gain £4 in 6 mo. at 20 per cent., what principal will gain £30 in 16 mo. at 10 per cent.?

64. Find the G. C. M. and the L. C. M. of 4 cwt. and 20 lbs., and 9 cwt. 3 qrs. 24 lbs.

65. A can do $\cdot\dot{3}$ of a piece of work in 2 hrs., B can do $\cdot\dot{3}$ of the remainder in 6 hrs., and C can do $\cdot\dot{3}$ of what is left in 1 hr.; in what time could A, B, and C do it together?

66. Alcohol and water being mixed in the proportion of 5·295 of the former to 6·027 of the latter, find what weight of each substance is contained in a lb. of the mixture.

67. If a piece of work can be finished in 45 days by 35 men, and if the men drop off 7 at a time at the end of every 15 days, how long will it be before the work is finished?

68. $\cdot\dot{6}$ of the number of apples in a basket exceeds $\cdot\dot{6}$ of the number by 57·4; find the number of apples.

69. A bag contains a certain number of crowns, 5 times as many shillings, and 7 times as many sixpences, and the whole sum amounts to £81; find the number of crowns, shillings, and sixpences respectively.

70. A bankrupt's estate realises £5,460, being $\frac{7}{13}$ of his debts; find the dividend in the £.

CHAPTER IX.

COMMERCIAL RULES.*

SECTION I.—PERCENTAGE.

The term per cent. (Latin, *per centum*) means so much, or so many, *for* every *hundred.*

One *hundred* is the universal standard of comparison in commercial transactions, statistical tables, &c.

Interest, Discount, Profit and Loss, Stocks, Shares, Insurance, Brokerage, Commission, &c., are all calculated at *per cent.* Statistical tables relating to populations, employments, education, &c., are all constructed at *per cent.*

Nearly all the problems in commercial rules may be solved by Proportion (Simple and Compound), and they are separated from Proportion only because they form distinctive classes of problems.

COMMISSION, BROKERAGE, INSURANCE, AND STATISTICS.

COMMISSION is the charge made at so much per cent. for buying or selling goods. Manufacturers, for instance, employ men to travel through the country to sell their wares. These men—commercial travellers—usually receive so much *commission* for every £100 worth of goods they sell.

BROKERAGE is commission paid to a broker for buying and selling stocks, shares, or merchandise.

INSURANCE is a contract to pay so much per annum, the *premium*, in return for which the insurers (generally a company) agree to bear all losses caused by death, fire, accident, shipwreck, &c. Insurers of ships and their cargoes are called *underwriters.*

EX. 1. The population of a certain town in 1871 was 171,572, which is 18 per cent. more than its population in 1861; what was the population in 1861?

Since the increase is 18 per 100, 100 in 1861 will become 118 in 1871.

Hence $118 : 100 :: 171572 : x$

$$x = \frac{17157200}{118} = 145{,}400$$

or, 118 in 1871 represents 100 in 1861.

$\therefore$ 1 „ „ $\frac{100}{118}$ „

$\therefore$ 171572 in 1871 represents $\frac{100 \times 171572}{118}$ in 1861.

$= 145{,}400$ *Ans.*

* In nearly all the exercises in Commercial Arithmetic the practice of merchants is followed and the answer obtained to the nearest penny. The halfpenny or any greater fraction is charged as another penny.

Ex. 2. An architect charged $\frac{1}{4}$ per cent. for plans and specifications, and $1\frac{3}{8}$ per cent. for superintending the erection of a building costing £40,000; how much was he paid?

Total of architect's charges $= 1\frac{3}{8} + \frac{1}{4} = 1\frac{5}{8}$.

That is, for every £100 he has £$1\frac{5}{8}$.

$$\therefore \quad \text{„} \quad £1 \quad \text{„} \quad \frac{\frac{13}{8}}{100} = \frac{13}{800}$$

$$\therefore \quad \text{„ } £40{,}000 \quad \text{„} \quad \frac{13 \times \cancel{40000}^{50}}{\cancel{800}}$$

$$= £650. \; Ans.$$

Ex. 3. My agent has bought 3,300 pieces of cloth at £1 15*s.* per piece, and I allow him 3 per cent. commission; how much money must I remit to pay both the cost of the cloth and the commission?

$$\text{Cost of cloth} = 3300 \times 1\tfrac{3}{4} = £5775.$$

$$\text{Commission on } £100 = £3$$

$$\therefore \quad \text{„} \quad \text{„} \quad £1 = \frac{3}{100}$$

$$\therefore \quad \text{„} \quad \text{„} \quad £5775 = \frac{3 \times 5775}{100}$$

$$= \frac{17325}{100} = £173{\cdot}25 = £173\ 5s.$$

Hence total cost $= £5775 + £173\ 5s. = £5948\ 5s.$

Ex. 4. An agent charged £75 for collecting rents amounting to £1,125; what was the rate of commission?

£1,125 gives £75 commission.

$$£1 \quad \text{„} \quad £\frac{75}{1125} \quad \text{„}$$

$$\therefore \quad £100 \quad \text{„} \quad £\frac{75 \times 100}{1125} \quad \text{„}$$

$$\frac{75 \times 100}{1125} = 6\tfrac{2}{3} \; Ans.$$

Ex. 5. An account stands at £14 12*s.* 11*d.* after deducting $7\frac{1}{2}$ per cent.; what was the original amount?

After deducting $7\frac{1}{2}$ per cent. an original debt of £100 would stand at £$92\frac{1}{2}$.

That is, £$92\frac{1}{2}$ gives £100 as original account.

$$\therefore \quad £1 \quad \text{„} \quad \frac{100}{92\frac{1}{2}} \quad \text{„} \quad \text{„}$$

$$\therefore \quad £14\ 12s.\ 11d. \text{ gives } \frac{£14\ 12s.\ 11d. \times 100}{92\frac{1}{2}} = \frac{200}{185} \text{ of } £14\ 12\ 11$$

$$= \frac{40}{37} \text{ of } £14\ 12s.\ 11d.$$

$$= £15\ 16s.\ 8d.\ \textit{Ans.}$$

Ex. 6. A merchant shipped a cargo of wheat from Odessa to London, and to cover the value of both cargo and premium he took out a policy for £4,200, at $3\frac{1}{2}$ per cent. ; what was the value of the wheat ?

Since the premium is $£3\frac{1}{2}$ per cent. out of every £100 insured, the value of wheat insured is $100 - 3\frac{1}{2} = £96\frac{1}{2}$.

That is, out of £100 insured, value of wheat $= £96\frac{1}{2}$

$$\therefore \quad \text{„} \quad £1 \quad \text{„} \quad \text{„} \quad \text{„} \quad \frac{96\frac{1}{2}}{100}$$

$$\therefore \quad \text{„} \quad 4{,}200 \quad \text{„} \quad \text{„} \quad \text{„} \quad \frac{96\frac{1}{2} \times 4200}{100}$$

$$= 96\tfrac{1}{2} \times 42 = £4{,}053.\ \textit{Ans.}$$

Exercises.

1. How much per cent. is $2\frac{1}{2}d.$ in the shilling, and also 2*s.* 9*d.* in the £ ?

2. How much per cent. is 6*s.* in the £ ? What per cent. of $2\frac{1}{2}$ is $\frac{3}{4}$?

3. Deduct 12 per cent. from £230 ; what sum remains ?

4. What must be paid yearly on an insurance for £9,626 11*s.* 3*d.* at £2 12*s.* per cent. ?

5. Find the amount of commission on £63,943 6*s.* 8*d.* at $\frac{1}{8}$ per cent. ?

6. In a town containing 10,000 electors, ·35 voted for one candidate, and ·45 for the other ; find the percentage of voters who abstained from voting.

7. In a town the population of which in 1868 was 5,600, it was found in 1878 to have increased 12 per cent. ; find the population in 1878.

8. Out of an army of 25,000 men 15 per cent. are invalided ; required the effective force ?

9. If, after deducting $15\frac{1}{2}$ per cent. from my money, I have £304 4*s.* left, how much money had I at first ?

10. Out of 400 sacks of wheat, barley, and oats, 35 per cent. are wheat and barley, and the rest oats. Of the wheat and barley 35 per cent. are wheat ; how many sacks are there of each sort ?

11. The population of a town in 1861 was 34,600, and in 1871 was 38,614 ; what was the increase per cent. ?

12. A miller uses for a stone weight one that weighs only $13\frac{1}{2}$ lbs. ; *what does he gain per cent.* by his dishonesty ?

13. If out of a school of 150 children 7 per cent. are up to Standard VI., and 11 per cent. are up to Standard V.; how many children are there below those standards?

14. What sum should be insured at £2 2*s.* 6*d.* per cent. on goods worth £783, that the owner may recover in case of loss the value of both goods and premium?

15. Thames water contains ·032 per cent. of organic matter; find the weight of organic matter in 30,000 tons of the water.

16. Three sacks contain equal quantities of corn, and a mixture is made by taking 50 per cent. from the first sack, 30 per cent. from the second, and 70 per cent. from the third; find what percentage of the whole quantity of corn is taken.

17. Out of an army of 44,600 men, 5,352 men were killed, and 11,596 wounded in a campaign; find the percentage of killed, of wounded, and of the total placed *hors de combat.*

18. An account stands at £16 14*s.* 9*d.* after 2½ per cent. has been deducted; what was the original amount?

19. If the number of children in the three departments (boys', girls', and infants') of a school be in the proportion of 3, 4, and 5, and the percentage of absentees 20, 30, and 36; find the percentage of absentees in the whole school.

20. Gunpowder is composed of 33 parts of nitre, 7 of charcoal, and 5 of sulphur; find its percentage composition.

21. Find the *percentage* of each ingredient, and the *quantity* of each ingredient, in 11½ cwt. of adulterated coffee, each pound of which contains 3¾ ozs. of chicory.

22. The population of a certain city in 1861 was 64,000, which is 80 per cent. of the population in 1871; what was the population in 1871?

Section II.—Interest, Discount, Present Value.

Interest is the money paid for the loan of money.

The Principal is the amount lent.

The Rate per Cent is the interest allowed per annum on every £100.

The Amount is the Interest added to the Principal.

When principal and interest only are considered, the problem is solved by simple proportion. When the principal and time are both considered in relation to the interest, the problem is solved by compound proportion.

Ex. 1. (*Without reference to time.*)

(*a*) Find the interest on £450 at 4 per cent.

(*b*) *From* what principal did £18 arise at 4 per cent. per annum?

(*c*) *At what* rate per cent. per annum will £450 amount to £486?

(*a*) £100 gives £4 interest.

$\therefore$ £1 „ £$\frac{4}{100}$ „

$\therefore$ £450 „ $\frac{4 \times 450}{100}$ „

$= £18$. *Ans.*

Or, prin. 100 : prin. 450 :: int. 4 : int. x

$$x = \frac{450 \times 4}{100} = £18. \text{ } Ans.$$

(*b*) £4 interest gives £100 as principal.

$\therefore$ £1 „ „ $\frac{100}{4}$ „ „

$\therefore$ £18 „ „ $\frac{100 \times 18}{4}$ „ „

$= 18 \times 25 = £450$. *Ans.*

Or, int. 4 : int. 18 :: prin. 100 : prin. x

$$x = \frac{100 \times 18}{4} = £450. \text{ } Ans.$$

(*c*) £450 amounts to £468.

Hence £450 gives an interest of £468 − 450 = £18.

Then £450 gives £18 as interest.

$\therefore$ £1 „ $\frac{18}{450}$ „

$\therefore$ 100 „ $\frac{18 \times 100}{450}$ „

$$\frac{\overset{2}{\cancel{18}} \times \overset{2\cancel{0}}{\cancel{100}}}{\underset{5\cancel{0}}{\cancel{450}}} = £4. \text{ } Ans.$$

Or, prin. 450 : prin. 100 :: int. 18 : int. x

$$x = \frac{100 \times 18}{450} = £4. \text{ } Ans.$$

Ex. 2. (*Including the condition of time.*)

(*a*) Find the simple interest on £500 4*s*. 2*d*. for 5½ years at 6 per cent. per annum.

(*b*) From what principal did £165 1*s*. 4½*d*. arise in 5½ years at 6 per cent. per annum?

(*c*) At what rate per cent. would £500 4*s*. 2*d*. amount to £665 5*s*. 6½*d*. in 5½ years?

(*d*) In what time will £500 4*s*. 2*d*. amount to £665 5*s*. 6½*d*. at 6 per cent. per annum?

(*a*) There are two ratios here to be considered :—

(1) If £6 be the interest on £100, what is the interest o £500 4*s.* 2*d.* ?

principal principal int. int.
100 : £500 4*s.* 2*d.* :: 6 : *x*

(2) If £6 be the interest for a year, what is the interest for 5 years ?

year years int. int.
1 : $5\frac{1}{2}$ :: 6 : *x*

And compounding these ratios—

$$\left.\begin{array}{ll} \text{Principal} & 100 : £500\ 4s.\ 2d. \\ \text{Time} & 1 : 5\frac{1}{2} \end{array}\right\} :: 6 : x$$

$$x = \frac{£500\ 4s.\ 2d. \times 5\frac{1}{2} \times 6}{100}.$$

The most simple method, and the one generally adopted, of findin the value of x from this fraction is as follows :—

£	*s.*	*d.*
500	4	2
		$5\frac{1}{2}$
2501	0	10
250	2	1
2751	2	11
		6
165·06	17	6
	20	
1·37		
	12	
4·50		
	4	
2·00		

Int. = £165 1*s.* $4\frac{1}{2}$*d.*

We have divided by 100, by marking off two figures to the right.

NOTE.—From this example we may deduce the ordinary rule for finding the simple interest : *Multiply the principal by the rate per cent., and this product by the number of years, and divide by* 100.

(*b*) We have here also two questions to consider :—

(1) If £100 is the principal from which £6 arises, what is the principal from which £165 1*s.* $4\frac{1}{2}$*d.* interest arises ?

interest interest prin. prin.
6 : £165 1*s.* $4\frac{1}{2}$*d.* :: 100 : *x*

(2) If £100 is the principal from which a certain interest arises in 1 year, what is the principal from which the same interest will arise in $5\frac{1}{2}$ years?

$$\overset{\text{years}}{5\frac{1}{2}} : \overset{\text{year}}{1} :: \overset{\text{prin.}}{£100} : \overset{\text{prin.}}{x}$$

And compounding these two ratios—

$$\left.\begin{array}{lll}\text{Interest} & . & £6 : £165\ 1s.\ 4\frac{1}{2}d. \\ \text{Time} & . & 5\frac{1}{2} : 1\end{array}\right\} :: \overset{\text{prin.}}{100} : \overset{\text{prin.}}{x}$$

$$x = \frac{£165\ 1\ 4\frac{1}{2} \times 100}{6 \times 5\frac{1}{2}}$$

$$x = \frac{£165\ 1\ 4\frac{1}{2} \times 200}{66} = £500\ 4s.\ 2d.\ \textit{Ans.}$$

(*c*) Here the interest amounts to £665 5*s.* $6\frac{1}{2}d.$ − £500 4*s.* 2*d.* = £165 1*s.* $4\frac{1}{2}d.$

(1) If £500 4*s.* 2*d.* give £165 1*s.* $4\frac{1}{2}d.$ as interest, what will £100 give?

$$\overset{\text{prin.}}{£500\ 4s.\ 2d.} : \overset{\text{prin.}}{100} :: \overset{\text{int.}}{£165\ 1s.\ 4\frac{1}{2}d.} : \overset{\text{int.}}{x}$$

(2) If £165 1*s.* $4\frac{1}{2}d.$ be the interest for $5\frac{1}{2}$ years, what is the interest for 1 year?

$$\overset{\text{yrs.}}{5\frac{1}{2}} : \overset{\text{yr.}}{1} :: \overset{\text{int.}}{£165\ 1s.\ 4\frac{1}{2}d.} : \overset{\text{int.}}{x}$$

Compounding these ratios, we get—

$$\left.\begin{array}{ll}£500\ 4s.\ 2d. & : £100 \\ 5\frac{1}{2} & : 1\end{array}\right\} :: £165\ 1s.\ 4\frac{1}{2}d. : x$$

$$x = \frac{100 \times £165\ 1s.\ 4\frac{1}{2}d.}{5\frac{1}{2} \times £500\ 4\ 2} = 6.$$

6 per cent. *Ans.*

(*d*) The interest on £500 4*s.* 2*d.* is £165 1*s.* $4\frac{1}{2}d.$ for the required time.

And the interest on £500 4*s.* 2*d.* for 1 year at 6 per cent. is £30 0*s.* 3*d.*

Hence $\frac{£165\ 1\ 4\frac{1}{2}}{£30\ 0\ 3} = 5\frac{1}{2}$ is the required time.

The preceding examples include all the typical problems in simple interest; of course, they can be as easily solved by the *unity method*, or *first principles*, but we recommend the following *formula* as the *key* to the solution, not only of the problems as given above, but also those of Discount and Present Value.

Ex. 3. Find the simple interest on £278 6*s.* 8*d.* for 12 years at 5 per cent.

By proportion—

$$\left.\begin{array}{lll} \text{Principal} & . & 100 : 278\frac{1}{3} \\ \text{Time} & . . & 1 : 12 \end{array}\right\} :: \underset{\text{int.}}{5} : \underset{\text{int.}}{x}$$

$$= \frac{12 \times 278\frac{1}{3} \times 5}{100} = £167$$

In this equation let P stand for the principal, r the rate per cent., and n the number of years.

$$\text{Then } x = \frac{P \times n \times r}{100} = \text{Interest.}$$

But the amount is the interest added to the principal. Let M stand for the amount.

$$\text{Then } M = P + \frac{P \times n \times r}{100}$$

By means of this formula we can solve all problems in simple interest, discount, and present value.

The problems in simple interest are illustrated by the following example:—

Ex. 4. (1) Find the amount of £500 at 5 per cent. in 5 years.
(2) Find what principal will amount to £625 in 5 years at 5 per cent.
(3) Find at what rate per cent. £500 will amount to £625 in 5 years.
(4) In what time will £500 amount to £625 at 5 per cent.?

(1) $M = P + \frac{P \times n \times r}{100}$

$= 500 + \frac{500 \times 5 \times 5}{100}$

$= 500 + 125 = £625.$ *Ans.*

(2) $M = P + \frac{P \times n \times r}{100}$

$625 = P + \frac{P \times 5 \times 5}{100}$

$= \frac{100P + (P \times 5 \times 5)}{100} = \frac{125P}{100}$

$625 = \frac{125}{100}P$

Multiplying both sides by 100, we get—

$125P = 62500$

Dividing both sides by 125, we get—

$P = \frac{62500}{125} = £500.$ *Ans.*

$$(3)\quad M=P+\frac{P\times n\times r}{100}$$

$$625=500+\frac{500\times 5\times r}{100}$$

Subtracting 500 from each side—

$$625-500=\frac{500\times 5\times r}{100}$$

$$\therefore\quad 125=\frac{2500\times r}{100}$$

$$\therefore\quad 12500=2500\times r$$

$$\text{and}\quad \frac{12500}{2500}=r=5.\ \textit{Ans.}$$

$$(4)\quad M=P+\frac{P\times n\times r}{100}$$

$$625=500+\frac{500\times 5\times n}{100}$$

$$\therefore\quad 625-500=\frac{2500\times n}{100}$$

$$\therefore\quad 12500=2500\times n$$

$$\text{and}\quad \frac{12500}{2500}=n=5.\ \textit{Ans.}$$

Discount and Present Value.—I have agreed to pay a man 100 guineas at the end of a year from the present time; but he wishes to receive payment now. If money be reckoned worth 5 per cent. per annum, what ought I to pay him cash?

Clearly it would be unfair to pay £105 now, because the man could put this out to interest at 5 per cent., and at the end of the year, when the 100 guineas would have become due, he would be worth £110 5*s.* (int. on £105 at 5 per cent. = £5 5*s.*) I must, then, pay him just that amount which, put out to interest at 5 per cent., would make £105 at the end of the year. This sum is £100, for £100 at 5 per cent. amounts to £105. I must then deduct £5 from the £105 for present payment. The £5 deducted is called the *discount;* the £100 is called the *present value.*

NOTE.—In practice the true discount as shown above is never calculated. The banker and the tradesman deduct simple interest, and call it discount, instead of the true discount.

EX. 5. Find the present value and discount of £224 16*s.* 3*d.* due 7 months hence at 3¾ per cent. per annum.

£100 in 7 months, at 3¾ per cent., will amount to £102 3*s.* 8*d.* That *is, the present value of* £102 3*s.* 8*d.* due 7 months hence at 3¾ per cent.

is £100; from this it is easy to find by simple proportion the present value of £224 16*s.* 3*d.*

$$\text{Thus}\quad \underset{20}{\text{£102 3}s.\ 9d.} : \underset{20}{\text{£224 16}s.\ 3d.} :: \underset{\text{pres. value}}{\text{£100}} : \underset{\text{pres. value}}{x}$$

$$\text{or}\quad 102\tfrac{3}{16} : 224\tfrac{3}{16} :: 100 : x$$

$$\text{or}\quad \frac{1635}{16}\,x = \frac{3597 \times 100}{16}$$

$$\text{or}\quad x = \frac{\overset{11}{\cancel{3597}} \times \overset{20}{\cancel{100}} \times \cancel{16}}{\cancel{16} \times \underset{\cancel{5}}{\cancel{1635}}}$$

$$\text{and}\quad x = £220.$$

The discount is the amount less the present value, viz. £224 16*s.* 3*d.* − £220 = £4 16*s.* 3*d.*

The discount may be found independently, thus:—

$$\underset{\text{amount}}{\text{£102 3}s.\ 9d.} : \underset{\text{amount}}{\text{£224 16}s.\ 3d.} :: \underset{\text{discount}}{\text{£2 3}s.\ 9d.} : \underset{\text{discount}}{x}$$

$$\text{or}\quad 102\tfrac{3}{16} : 224\tfrac{3}{16} :: 2\tfrac{3}{16} : x$$

$$\frac{1635}{16}\,x = \frac{3597}{16} \times \frac{35}{16}$$

$$\text{and}\quad x = \frac{\overset{11}{\cancel{3597}} \times \overset{7}{\cancel{35}} \times \cancel{16}}{\underset{\cancel{5}}{\cancel{1635}} \times 16 \times \cancel{16}}$$

$$\text{and}\quad x = \frac{77}{16} = £4\ 16s.\ 3d.$$

We may write the preceding example thus*:—Find what principal will amount to £224 16*s.* 3*d.* in 7 months at $3\frac{3}{4}$ per cent. per annum.

This is one of the cases under simple interest, and may be solved by the formula $M = P + \frac{P \times n \times r}{100}$.

We have simply given another name, the principal required to be found, viz. present value, and hence in our formula P stands for *principal*, or *present value.*

$$\text{Hence}\quad M = P + \frac{P \times n \times r}{100}$$

$$£224\ 16s.\ 3d.\ \text{or}\ \frac{3597}{16} = P + \frac{P \times \frac{7}{12} \times \frac{15}{4}}{100}$$

* *Special attention is called to this method of treating present value.*

$$= P + \frac{P \times 16^{35}}{100}$$

$$= P + P \times \frac{35}{1600}$$

$$= \frac{1600 \times P}{1600} + \frac{35 \times P}{1600}$$

$$= \frac{1635}{1600} \times P = \frac{327}{320} \times P$$

That is $\frac{327}{320} \times P = \frac{3597}{16}$

and $P = \frac{\overset{11}{\cancel{3597}}}{\underset{1}{\cancel{16}}} \times \frac{\overset{20}{\cancel{320}}}{\underset{1}{\cancel{327}}}$

$= £220$ the present value.

The discount is the amount less the present value. In this case £224 16*s.* 3*d.* − £220 = £4 16*s.* 3*d.*.

Ex. 6. In what time will £25,400 amount to £31,591 5*s.* at $3\frac{3}{4}$ per cent. per annum?

Let M stand for amount, P for principal, r rate per cent, n the number of years.

Then interest $= \frac{P \times n \times r}{100}$; and amount = principal + interest.

Hence $M = P + \frac{P \times n \times r}{100}$

$$£31591\ 5s. = £25400 + \frac{25400 \times 3\frac{3}{4} \times n}{100}.$$

Subtract £25400 from each side—

$$£6191\tfrac{1}{4} = \frac{\overset{1270}{\cancel{6350}} \times \overset{3}{\cancel{15}}}{\underset{4}{\cancel{100}}} \times n = \frac{3810}{4} \times n$$

$$\frac{24765}{4} = \frac{3810 \times n}{4}$$

Multiply both sides by 4—

$$24765 = 3810 \times n$$

Divide both sides by 3810—

$$n = \frac{24765}{3810} = 6\tfrac{1}{2} \text{ years.}$$

Ex. 7. Find the present value of £1,746 14*s.* 2*d.* due 9 months hence at 4 per cent. per annum.

$$M = P + \frac{P \times n \times r}{100}$$

$$£1746\ 14s.\ 2d. = P + \frac{P \times \frac{3}{4} \times \frac{4}{1}}{100} = P + \frac{P \times 3}{100}$$

$$= \frac{100P + 3P}{100} = \frac{103}{100}P$$

Multiply each side by 100, and divide by 103—

$$P = £1746\ 14s.\ 2d. \times 100 \div 103 = £1695\ 16s.\ 8d.\ \textit{Ans.}$$

EXERCISES.

1. What is the simple interest on £185 for 4 years at 4 per cent. per annum?

2. Find the simple interest on £270 for 4½ years at 5 per cent. per annum.

3. Find the amount of £3,460 at 3½ per cent. in 8 months.

4. Find the simple interest on £16,904 7*s.* 6*d.* at 3½ per cent.

5. Find the amount of £35 16*s.* 5*d.* at 4 per cent. in 3½ years.

6. What is the simple interest on £570 11*s.* at 4½ per cent. for 1¼ years?

7. Find the simple interest on £10,400 for 4 years 16 weeks at 4 per cent. per annum.

8. Find the simple interest and amount of £1,248 12*s.* for 5 years at 3½ per cent. per annum.

9. Find the simple interest on £417 7*s.* 9*d.* for 1 year 10 months at 4⅜ per cent. per annum.

10. Find the amount of £560 in 2½ years at 4½ per cent. simple interest.

11. Find the simple interest on £348 10*s.* for 7 years at 4½ per cent.

12. Find the simple interest on £183 6*s.* 8*d.* for 1 year 146 days at 4¼ per cent.

13. At what rate per cent. per annum must £365 be lent for 95 days to amount to £369 15*s.*?

14. At what rate per cent. will £250 amount to £285 in 3½ years?

15. In what time will £375 amount to £440 12*s.* 3*d.* at 3⅓ per cent?

16. What is the rate of interest when £92 10*s.* gains £29 12*s.* in 4 years simple interest?

17. What principal, lent out at 3½ per cent., will amount to £1,884 19*s.* 10½*d.* in 5 years?

18. In what time will £299 10*s.* amount to £389 7*s.* at 5 per cent. simple interest?

19. If I lend a friend £1,250 at 4 per cent. simple interest, and tell

him to keep it till principal and interest amount to £1,666 13*s.* 4*d.*, how long will he have it?

20. At what rate per cent., simple interest, will £1,248 12*s.* amount in $2\frac{1}{2}$ years to £1,443 13*s.* $10\frac{1}{2}$*d.*?

21. Find the simple interest on £1,277 10*s.* from the 31st of March till 4th of July of the same year at 8 per cent. per annum.

22. At what rate per cent., simple interest, will £112 10*s.* amount to £128 5*s.* in 4 years?

23. What sum of money will amount to £307 16*s.* in 3 years at $4\frac{2}{3}$ per cent.?

24. Find the simple interest on 10 guineas for 117 days at $3\frac{1}{2}$ per cent.

25. What sum of money must I lend to receive 14*s.* 6*d.* as interest for $91\frac{1}{4}$ days at $3\frac{1}{2}$ per cent.?

26. If I lend £150 for 8 days and receive 17*s.* 6*d.* as interest, what is the rate per cent.?

27. At what rate per cent. must a guinea be lent for 15 days to produce 2*s.* 6*d.* as interest?

28. At what rate per cent. shall I be paying if I give 3*s.* 6*d.* for the loan of £5 5*s.* for 3 weeks?

29. What is the difference between the bankers' and the true discount on £525 13*s.* 4*d.* due 6 months hence at $3\frac{1}{2}$ per cent.?

30. At what rate per cent. will £1,650 amount to £1,784 1*s.* 3*d.* in $2\frac{1}{2}$ years?

31. How long will it take £3,125 to amount to £3,681 12*s.* $9\frac{3}{4}$*d.* at $3\frac{3}{4}$ per cent.?

32. Find the present value of and discount on £149 0*s.* 5*d.* due 11 months hence at 4 per cent.

33. What principal will produce £591 12*s.* 4*d.* as interest in 4 years at $2\frac{1}{2}$ per cent.?

34. At what rate per cent. will £91 13*s.* 4*d.* amount to £117 10*s.* $7\frac{1}{2}$*d.* in $4\frac{1}{2}$ years?

35. Find the present value of £455 12*s.* 6*d.* due 4 years hence at $5\frac{3}{8}$ per cent.

36. At what rate per cent. simple interest will £3,745 amount to £3,932 5*s.* in 2 years?

37. What is the principal to be placed at interest to produce, at the rate of $4\frac{1}{4}$ per cent., in 16 months the sum of £39 12*s.* 6*d.*?

38. Find the present value of £1,842 15*s.* due 3 months hence at 5 per cent.

39. How long will it take £3,745 to amount to £3,932 5*s.* at $2\frac{1}{2}$ per cent. simple interest?

40. If £2,000 gain £17 10*s.* in $2\frac{1}{2}$ months, what sum will gain £18 in $5\frac{1}{2}$ months at the same rate?

41. Find the present value of £2,587 18*s.* 9*d.* due 5 years hence at $3\frac{1}{2}$ per cent.

42. What time will it take to double £2,220 at $3\frac{3}{4}$ per cent. simple interest?

43. A sum of £1,928 17*s.* is due $1\frac{1}{4}$ years hence; what amount should be accepted as present payment, reckoning $3\frac{1}{2}$ per cent. as the value of money?

44. In what time will £5 amount to £6 at $3\frac{1}{2}$ per cent. per annum?

45. The sum of £533 6*s.* 8*d.* is received for £560 due 2 years hence; at what rate is the discount calculated?

46. At what rate per cent. will the discount on £1,336 11*s.* 3*d.* due at the end of $3\frac{1}{2}$ years amount to £199 1*s.* 3*d.*?

47. Find the present value of £9,393 18*s.* $4\frac{1}{2}$*d.* due $5\frac{1}{2}$ years hence at $3\frac{1}{2}$ per cent.

48. What amount of capital, put out at $3\frac{1}{2}$ per cent. per annum, simple interest, will produce £14 interest in $2\frac{1}{8}$ years?

49. Which investment will bring in the larger sum, and by how much, at the end of 7 years—£925 at £5·825 per cent. per annum, or £2,500 at $2\frac{1}{8}$ per cent. per annum, both at simple interest?

50. What is the difference between the true discount and the interest on a debt of £325 which ought to have been paid 9 months ago, reckoning the value of money at $7\frac{1}{2}$ per cent.?

51. What is the bank discount on a bill for £55, payable in 90 days at 8 per cent.? What is the true discount?

52. A tradesman sells goods, either for cash—allowing 5 per cent. discount—or at 6 months' credit. If I buy £50 worth of goods and pay half now and half at the end of 6 months, what will the goods cost me?

53. What principal must be invested at 3 per cent. to produce £2,837 10*s.* interest in 4 years?

54. What cash payment will discharge a debt of £50 payable in 6 months, and £50 payable in 12 months, reckoning the value of money to be 5 per cent.?

Section III.—Compound Interest.

When the interest is added to the principal at the end of a certain period of time to form a new principal, and then at the end of another equal interval of time the interest on the new principal is added to the new principal to form again a second new principal, and so on, we calculate by Compound Interest.

Thus £100 put out to compound interest for 3 years at 5 per cent. per annum, will amount to £115 15*s.* 3*d.*; for—

(1) At the end of the *first* year we add the £5 interest to the £100 principal; and this gives a new principal of £105 with which to begin the second year.

(2) At the end of the *second* year we add the interest on £105, viz. *£5 5s., to* the principal £105 to form a new principal of £110 5*s.* *with which* to begin the third year.

(3) At the end of the *third* year we add the interest on £110 5*s.*, viz. £5 10*s.* 3*d.*, which gives £115 15*s.* 3*d.* as the amount of £100 for three years at 5 per cent. compound interest.

It will be seen from this example that to find the compound interest on a given sum, for a given time, at a given rate per cent., we have simply to find the simple interest at the end of each period, and add to the last principal.

Interest may be reckoned at any interval of time.

Ex. 1. Find the compound interest and amount of £500 in 4 years at 5 per cent per annum.

1st year. { Interest at end of 1st year $= \frac{500 \times 5}{100} = £25$
Amount „ „ $= £500 + £25 = £525$

2nd year. { Interest at end of 2nd year $= \frac{525 \times 5}{100} = £26\frac{1}{4}$
Amount „ „ $= £525 + 26\frac{1}{4} = £551\frac{1}{4}$

3rd year. { Interest at end of 3rd year $= \frac{551\frac{1}{4} \times 5}{100} = £27\frac{9}{16}$
Amount „ „ $= £551\frac{1}{4} + 27\frac{9}{16} = 578\frac{13}{16}$

4th year. { Interest at end of 4th year $= \frac{578\frac{13}{16} \times 5}{100} = £28\frac{301}{320}$
Amount „ „ $= £578\frac{13}{16} + 28\frac{301}{320}$

And $\frac{13}{16}$ of £1 = 16*s.* 3*d.*

„ $\frac{301}{320}$ of £1 = 18*s.* $9\frac{3}{4}d.$

Therefore the amount = £578 16*s.* 3*d.* + £28 18*s.* $9\frac{3}{4}d.$ = £607 15*s.* $0\frac{3}{4}d.$

And the difference between this amount and the given principal is the compound interest, viz. £607 15*s.* $0\frac{3}{4}d.$ − £500 = £107 15*s.* $0\frac{3}{4}d.$

By using decimal fractions, and by so arranging the products that the decimal points come under each other, we can shorten the work of compound interest. Thus, taking the same example—

£500
5
25·00 = Int. at end of 1st year } 1st year.
525·00 = Amt. „ „ }
5
26·25 = Int. „ 2nd year } 2nd year.
551·25 = Amt. „ „ }
5
27·5625 = Int. „ 3rd year } 3rd year.
578·8125 = Amt. „ „ }
5
28·940625 carried over.

28·940625 = Int. „ 4th year } 4th year.
607·753125 = Amt. „ „ }

20
15·062500
12
·7500
4
3·00

	£	s.	d.
Amt. =	607	15	$0\frac{3}{4}$
Prin. =	500	0	0
Int. =	107	15	$0\frac{3}{4}$

In this example we multiply the principal, £500, by 5, and divide by 100, viz. mark off 2 decimal places to the right, to find the interest. This gives £25, which added to the principal £500 gives £525 as the amount at the end of the 1st year. This becomes the new principal for the 2nd year.

Again, multiply the new principal by 5, and divide by 100, to find the interest at the end of the 2nd year. This gives £26·25, which, added to the principal at the end of the 1st year, viz. £525, gives £551·25 for the amount at the end of the second year. The same process is repeated for the 3rd and 4th years.

Ex. 2. What will £125 amount to in 2 yrs. 6 mo. at 4 per cent. compound interest, payable every 6 months?

4 per cent. per annum is 2 per cent. for 6 months.

£
125
2
2·50 = Int. at end of 1st 6 mo. }
127·5 = Amt. „ „ }
2
2·550 = Int. „ 2nd 6 mo. }
130·05 = Amt. „ „ }
2
2·6010 = Int. „ 3rd 6 mo. }
132·651 = Amt. „ „ }
2
2·65302 = Int. „ 4th 6 mo. }
135·30402 = Amt. „ „ }
2
2·7060804 = Int. „ 5th 6 mo. }
138·0101 = Amt. „ „ }
20
·202
12
2·424 Amount = £138 0s. 2·424d.

Note.—All decimal places after 5 may be discarded.

Ex. 3. What principal will amount to £367 10*s.* in 2 years, at 5 per cent. compound interest?

In problems where the amount is given to find the principal, we must find the actual amount of some fixed sum, say £100, for the given time at the given rate per cent., and from this, by simple proportion, find the required principal.

100
5
5·00 = Int. at the end of 1st year
105 = Amt. „ „
5
5·25 = Int. „ 2nd year
110·25 = Amt. „ „

The problem now is—If £100 as principal produce 110·25, what principal will produce £367 10*s.*?

amount amount prin. prin.
Hence 110·25 : 367·5 :: 100 : x

$$x = \frac{367{\cdot}5 \times 100}{110{\cdot}25} = \frac{36750 \times 100}{11025}$$

$$= \frac{3000}{9} = £333\ 6s.\ 8d.\ \textit{Ans.}$$

Ex. 4. Find the amount of £1,200 for 2 yrs. 5 mos. at 4 per cent. per annum compound interest.

1200
4
48·00 = Int. at end of 1st year
1248.00 = Amt. „ „
4
49·92 = Int. „ 2nd year
1297·92 = Amt. „ „
4
51·9168 = Int. „ 3rd year
5
12) 259·5840
21·632 = Int. for 5 mos. of 3rd year.
1297·92 = Amt. at end of 2nd year.
£1319·552 = Amt. at end of 2 yrs. and 5 mos.

NOTE.—To obtain the 5 months' interest, the 3rd year's interest is found, and $\frac{5}{12}$ of this is taken.

EXERCISES.

1. What will £200 amount to in 3 years at 4 per cent. per annum, compound interest?

2. Find the compound interest on £1,000 for $2\frac{1}{2}$ years at 5 per cent. per annum.

3. Find the amount of £460 for $1\frac{1}{4}$ years at $4\frac{1}{2}$ per cent. per annum, compound interest.

4. What is the compound interest on 100 guineas for $\frac{3}{4}$ year at $2\frac{1}{4}$ per cent. per annum?

5. Find the amount of £690 for 3 years at $4\frac{1}{2}$ per cent. compound interest.

6. Find the compound interest on £3,260 10*s.* for 3 years at 6 per cent. per annum.

7. What is the amount of £150 for 3 years at 4 per cent. per annum, compound interest?

8. Find the compound interest on £150 for 3 years at 4 per cent. per annum.

9. What is the amount of £200 for 3 years at 6 per cent. per annum, compound interest?

10. Find the amount of £800 for 3 years at £3 15*s.* per cent. per annum, compound interest.

11. Find the difference between the amount of £250 on 2 years at $3\frac{1}{2}$ per cent. per annum, at simple and at compound interest.

12. Find the compound interest on £4,000 for $2\frac{1}{2}$ years at 5 per cent. per annum.

13. What is the amount of £207 15*s.* for 2 years at $2\frac{1}{4}$ per cent. per annum, compound interest?

14. What will £240 amount to in one year at $2\frac{1}{2}$ per cent. per annum, compound interest, payable half-yearly?

15. Compare the simple and compound interest on £81 6*s.* 8*d.* for 3 years at £4 10*s.* per cent.

16. Find the difference between the simple and compound interest on £1,000 for 3 years at 5 per cent. per annum.

17. Find the amount of £49,000 for $3\frac{1}{4}$ years at 7 per cent. per annum.

18. What is the compound interest on £400 for 4 years at 4 per cent. per annum?

19. What will £750 amount to in $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent., compound interest, payable half-yearly?

20. Compare the simple and compound interest on £415 10*s.* for 2 years at $2\frac{1}{4}$ per cent. per annum.

21. If the sum of £1,200 be put out at 10 per cent. per annum, *compound interest*, paid half-yearly, what will it amount to in $1\frac{1}{2}$ years?

22. What is the compound interest on £5 for 5 years at 5 per cent. per annum?

23. What will 1,000 guineas amount to in 2 years at 4 per cent. per annum, compound interest, reckoned quarterly?

24. Find the compound interest on £250 for 1 year at 6 per cent. interest, payable every 4 months.

25. What sum of money will amount to £699 13*s.* 2·4*d.* in 2 years, reckoning compound interest for one year at 3½ per cent., and for the other at 4 per cent. per annum?

26. What sum lent at compound interest will amount to £3,500 in 3 years at 5 per cent.?

27. What is the difference between the simple and compound interest on £50 at 3½ per cent. for 9 months, interest reckoned quarterly?

28. What sum put out at compound interest will amount to £400 in 2¼ years at 4 per cent. per annum?

29. By how much will the compound interest on £1,500 at 4 per cent. per annum for 2 years be increased by the interest being calculated half-yearly instead of annually?

30. What sum lent at 5 per cent. compound interest will amount to £150 in 1½ years, interest reckoned every 6 months?

31. Find the present value of £91 17*s.* 6*d.* due in 2 years at 5 per cent. compound interest.

SECTION IV.—PROFIT AND LOSS.

Profit and Loss treats of the gain or loss on mercantile transactions, and is generally calculated on each £100. Problems in Profit and Loss usually involve simple proportion only: the chief point the student has to note is, whether the gain or loss is calculated on the selling price or on the cost price.

NOTE.—Selling price = cost price + gain, or selling price = cost price − loss.

The following typical examples must be carefully studied:—

EX. 1. A dealer bought a horse for £75, and sold it for £82 10*s.*; find the gain per cent.

Total gain by selling at £82 10*s.* = £82 10*s.* − £75 = £7 10*s.*

That is, £75 gives a gain of £7 10*s.* = 7½

Hence £1 „ „ $\frac{7\frac{1}{2}}{75}$

Also £100 „ „ $\frac{7\frac{1}{2} \times 100}{75}$

$$= \frac{\overset{20}{\cancel{15}} \times \cancel{100}}{2 \times \underset{15}{\cancel{75}}} = 10;$$ viz. 10 per cent.

Ex. 2. A linendraper bought long-cloth at 5*s.* per doz. yards, and sold it to gain 22½ per cent. ; find the selling price per doz. yards.

Suppose £100 worth are bought : the gain on this will be £22½.

That is, £100 worth sells for £122½

$$\therefore \quad £1 \quad \text{„} \quad \text{„} \quad \frac{245}{200}=\frac{49}{40}$$

$$\therefore \quad 5s. \quad \text{„} \quad \text{„} \quad £\frac{49}{160}=6s.\ 1\tfrac{1}{2}d.$$

That is, the selling price is 6*s.* 1½*d.*

Ex. 3. A grocer sold sugar for 2 guineas per cwt., thereby gaining 50 per cent. ; find the cost price per lb.

£2 2*s.* per cwt. = 4½*d.* per lb.

If £100 worth was bought, to gain 50 per cent. the selling price must be £150.

That is, a selling price of—

$$\therefore \quad £150 \text{ gives a cost price of } £100$$

$$\therefore \quad £1 \quad \text{„} \quad \text{„} \quad \frac{10}{15}=\frac{2}{3}$$

$$\therefore \quad 4\tfrac{1}{2}d. \quad \text{„} \quad \text{„} \quad \frac{2}{3} \text{ of } 4\tfrac{1}{2}d.=3d.$$

Cost price = 3*d.* a lb.

Ex. 4. A dealer bought a horse for £82 10*s.*, and sold it for £75 ; find the loss per cent.

On £82 10*s.* the loss is £7½.

$$\therefore \quad £1 \quad \text{„} \quad \text{„} \quad \frac{7\frac{1}{2}}{82\frac{1}{2}}=\frac{\frac{15}{2}}{\frac{165}{2}}=\frac{15}{165}=\frac{1}{11}$$

$$\text{and } \therefore \quad £100 \quad \text{„} \quad \text{„} \quad \frac{1}{11}\times 100=\frac{100}{11}=£9\tfrac{1}{11}$$

That is, the loss per cent. £9$\frac{1}{11}$

Ex. 5. A grocer bought coffee at 1*s.* 8*d.* per lb., and sold it to lose 5 per cent. ; find the selling price.

If £100 worth be bought, and then sold to lose 5 per cent., the selling price will be £95.

That is, £100 cost price gives £95 selling price

$$\therefore \quad £1 \quad \text{„} \quad \text{„} \quad \frac{95}{100}=\frac{19}{20} \quad \text{„}$$

$$\therefore \quad 20d. \quad \text{„} \quad \text{„} \quad \frac{19\times 20}{20}=19d.=1s.\ 7d.$$

That is, the selling price is 1*s.* 7*d.*

Ex. 6. Sold a pianoforte for £42 10*s.*, and thereby lost 15 per cent.; find the cost price.

If the pianoforte had cost £100, and had been sold at 15 per cent. loss, the selling price would have been £85.

That is, a selling price of £85 gives a cost price of £100

$\therefore$ „ „ 1 „ „ $\frac{100}{85}$

$\therefore$ „ „ $42\frac{1}{2}$ „ „ $\frac{100 \times 42\frac{1}{2}}{85}$

$$= \frac{100 \times \cancel{85}}{\cancel{85} \times 2} = £50$$

That is, the cost price is £50.

Ex. 7. If a tradesman by selling an article for 8*s.* 3*d.* loses $17\frac{1}{2}$ per cent., what should he have sold it for to gain 40 per cent.?

(*a*) To find the cost price.

$$(100 - 17\tfrac{1}{2} = 82\tfrac{1}{2})$$

At a loss of $17\frac{1}{2}$ per cent., £100 worth of goods is sold for $82\frac{1}{2}$.

That is—

Selling price of $82\frac{1}{2}$ gives cost price £100

$\therefore$ „ 1 „ $\frac{100}{82\frac{1}{2}} = \frac{200}{165} = \frac{40}{33}$

$\therefore$ „ 99*d.* „ $= \frac{40 \times 99}{33} = 120$

That is, the cost price is 10*s.*

(*b*) To find selling price to gain 40 per cent.

100 cost price gives selling price of £140.

1 „ „ $\frac{140}{100} = \frac{7}{5}$

10*s.* „ „ $\frac{7 \times 10s.}{5} = 14s.$

Selling price = 14*s.*

Ex. 8. A merchant marked a piece of cloth at 25 per cent. above the cost price, and then sold it for 20 per cent. less than the marked price; did he gain or lose, and how much?

Suppose £100 to be the cost price;

Then £125 is the marked price.

We have thus to find the selling price, viz. 20 per. cent. off £125.

20 per cent. off = $\frac{1}{5}$ off; and $\frac{1}{5}$ of £125 = £25; and £125 − £25 = £100.

Hence, the selling price is £100, and there is neither loss nor gain.

Ex. 9. What must I ask for an article which costs £4, that I may deduct 20 per cent. from the asking price, and still make a profit of 20 per cent.?

Firstly—What is the actual selling price? 20 per cent. profit on the cost price.

20 per cent. profit on £4 gives a selling price of $£\frac{24}{5}$. (See Ex. 2.)

Secondly—If my selling price is to be $£\frac{24}{5}$, what sum must I add so that I may deduct 20 per cent?

Deduct 20 per cent. £100 marked price gives £80 selling price.

That is—

£80 selling price must be marked £100

$\therefore$ 1 „ „ $\frac{100}{80}=\frac{5}{4}$

$\therefore$ $\frac{24}{5}$ „ „ $\frac{5}{4}\times\frac{24}{5}=£6.$

Ex. 10. A dealer bought goods for £500, and gained 5 per cent. by selling to another, who on his part lost 5 per cent. by selling to a third; what did the third dealer pay for the goods?

1st, £500 at a gain of £5 per cent. = £525

2nd, £525 at a loss of 5 per cent. =

£100 cost gives £95 selling.

$\therefore$ £1 „ $\frac{95}{100}$ „

$\therefore$ £525 „ $\frac{95\times525}{100}$ selling.

$$\frac{\overset{19}{\cancel{95}}\times\overset{105}{\cancel{525}}}{\underset{\underset{4}{\cancel{20}}}{\cancel{100}}}=\frac{1995}{4}=£498\ 15s.\ \textit{Ans.}$$

Exercises.

1. A farmer paid £4,500 for a farm, and sold it for £5,400; what did he gain per cent.?

2. Bought a horse for £75, and sold it to gain 10 per cent.; find the selling price.

3. A watch which cost £4 was sold for £4 10*s.*; find the gain per cent.

4. Bought sugar at £1 12*s.* 8*d.* per cwt.; at what price per lb. must it be sold to gain $7\frac{1}{2}$ per cent.?

5. *A cargo of wheat costing £5,500 being damaged, was sold for £4,400; find the loss per cent.*

6. If by selling tea at 3*s.* 2*d.* per lb. a grocer loses 5 per cent., what did it cost him per pound?

7. What do I gain or lose per cent. by selling butter at 8½*d.* a lb. which cost £5 5*s.* per cwt.?

8. By selling an article for 19*s.* 3*d.* 12 per cent. was gained; find the cost price.

9. An article which cost 7*s.* 6*d.* is sold for 9*s.* 9½*d.*; find the gain per cent.

10. At what price must an article be bought that 15 per cent. may be gained by selling it at £46?

11. By selling coffee at 1*s.* 3*d.* per lb. a grocer gains 35 per cent.; what does his profit amount to on a sale of 4 cwts. 11 lbs.?

12. By selling goods at £26 17*s.* 6*d.* I gain 25 per cent.; how much shall I gain per cent. by selling at £28 13*s.* 4*d.*?

13. A grocer purchases 84 lbs. of tea for £12 5*s.*; at what price per lb. must he sell it to gain 15 per cent.?

14. A dealer sold goods for £773 10*s.* 3½*d.*, and thereby made a profit of 3¼ per cent.; find the prime cost.

15. A grocer sold 84 lbs. of tea for £14 14*s.* thereby making a profit of 20 per cent.; what did he pay for the tea per lb.?

16. A dealer sold a herd of cows for £1,800, and lost 16⅔ per cent.; what was the cost of the herd?

17. A grocer makes a profit of £6 11*s.* 3*d.* by the sale of 4 cwts. 2 lbs. of coffee at 1*s.* 2*d.* per lb.; what is his gain per cent.?

18. Wine is sold at 35*s.* per dozen; 12½ per cent. of the cost price is lost; find the prime cost.

19. By selling a house for £4,162 10*s.* a man lost 7½ per cent.; how much would he have gained or lost per cent. if he had sold it for 4,500 guineas?

20. An article is sold for 12*s.* at a loss of 4 per cent.; at what price must it be sold to gain 4 per cent. on the outlay?

21. A merchant sold goods at 12½ per cent. profit, and thereby realised £1,000; find the cost price of the goods.

22. A merchant bought 500 yards of cloth for £166 13*s.* 4*d.*; for how much a yard must he sell it to gain 25 per cent.?

23. If 5½ per cent. be gained by selling goods at £5 5*s.* 6*d.* per cwt., what will be the gain per cent. by selling at 1*s.* 3*d.* a lb.?

24. A tradesman allows 20 per cent. discount, and adjusts his prices accordingly: what must he ask for an article worth £30?

25. By selling cheese at £3 13*s.* 6*d.* a cwt. a grocer realised a profit of 22½ per cent.; find the cost price per cwt.

26. Sold a horse for £80, and lost 20 per cent.; what selling price would have given a gain of 20 per cent.?

27. By selling oranges at 8 for $6\frac{3}{4}d.$ I gain 10 per cent.; find the price per doz. to gain 20 per cent.

28. By selling goods for £817 19*s.* a person lost 9 per cent. on his outlay; find at what price he should have sold them to gain $10\frac{1}{2}$ per cent.

29. If eggs be bought at 5 for 3*d.*; at what price must they be sold to gain 20 per cent.?

30. A man sold a carriage for £90, thereby gaining 50 per cent.; what per cent. would he have gained if he had sold it for £75?

31. Two-thirds of the price received for an article is equal to three-fourths of its cost; what is the gain per cent.?

32. A dealer sold 8 sheep for £24, and thereby lost 25 per cent.; if he had sold them for £34 would he have gained or lost, and what per cent.?

33. A merchant sells goods at retail at 30 per cent. above the cost price; and at wholesale at 12 per cent. less than the retail price; what is his gain per cent. on goods sold at wholesale?

34. Sold two pair of carriage horses for £240 a pair, and gained 20 per cent. on one pair, and lost 20 per cent. on the other; did I gain or lose by the transaction?

35. A horse is sold for 80 guineas at a loss of 10 per cent. on the outlay; find the selling price to gain 10 per cent. on the outlay.

36. If I buy 10 horses for £300, and 12 cows for £120, and resell the former at £35 each, and the latter at £12 each, what profit do I make per cent. on my outlay? For what must I sell the whole to gain 30 per cent.?

37. A merchant sells goods to a customer to the amount of £3,450, by which he would have gained 15 per cent.; before payment is made the customer becomes bankrupt, and pays 11*s.* 7*d.* in the £; how much does the merchant lose?

38. A grocer buys coffee at £8 10*s.* per cwt., and chicory at £2 10*s.*, and mixes in the proportion of 5 chicory to 7 coffee; at what price per lb. must he sell the mixture to gain $16\frac{2}{3}$ per cent. on his outlay?

39. A grocer having 1,161 cwt. of sugar in store, buys 567 cwt. more at £1 19*s.* $10\frac{1}{2}d.$ per cwt.; he mixes the two, and sells at 5*d.* per lb., thus gaining 12 per cent. on the whole; find the cost price per cwt. of the sugar in store.

40. A grocer mixes 3 cwt. of tea which costs 16 guineas per cwt. with one cwt. which costs £19 12*s.*; at what rate per lb. must he sell the mixture to gain 4 per cent. on his outlay?

41. A and B commenced business with certain capitals: A lost $33\frac{1}{3}$ *per cent. of his* capital, and B gained 50 per cent. on his capital, and *then each had* £600; with what amount of capital did each commence?

42. By selling a watch for 20 guineas 25 per cent. is gained; what would have been the gain per cent. had it been sold for 18 guineas?

Section V.—Stocks and Shares.

Stock is capital, or money, loaned to a government or to a trading company on condition of receiving certain interest. Within the past two hundred years the British Government have borrowed from private capitalists sums amounting to nearly eight hundred millions sterling. This sum constitutes the National Debt.

A national debt is brought about in this way:—Being in want of a certain sum of money, the Government *issue a loan*, viz. give notice that they wish to borrow a certain sum.

Suppose the amount required to be £6,000,000. This will constitute 60,000 £100 shares or £100 stock.

Persons wishing to invest money for the sake of interest may purchase this stock. If the interest offered is high and the security good, then £100, or even more, may be paid for £100 stock. If the interest be ever so high, but the security uncertain, the £100 stock will fetch much less than £100 cash. In fact, the price of the stock will depend on rate of interest, the kind of security offered, and the abundance or scarcity of money.

If the interest be, say, 4 per cent., the purchaser receives an acknowledgment which will entitle him to half-yearly dividends from the Government at the rate of £4 per annum for every £100 stock, until the Government chooses to repay the principal. Government stocks are called the Funds.

Stock may be bought and sold like any other commodity. From a variety of causes the price is continually varying. When the price of £100 stock is £100 cash, the stock is said to be at *par*; when the price is more than £100 cash, the stock is said to be at *a premium* and when the price is less than £100 cash, the stock is said to be at *a discount*. We must carefully distinguish between stock and sterling money. Shares in railway, banking, or other public companies are also bought and sold in the share market in the same way as stock.

Purchases or sales of stock are usually made through brokers, who charge $\frac{1}{8}$, or 2*s.* 6*d.*, for every £100 stock transferred from one holder to another, both to seller and buyer. Thus, if I buy £100 stock at $99\frac{5}{8}$, I shall have to pay the broker $99\frac{6}{8}$; or, if I sell £100 stock at $94\frac{3}{8}$, I shall receive $94\frac{2}{8}$, or $94\frac{1}{4}$.

Problems in stock are to be worked by proportion; they present little difficulty if the student only bears in mind that stock is not money.

Note.—*Consols.*—Most of the large sums of money borrowed by the Government at various times on different terms were, in 1751, consolidated *or brought* together into one scheme bearing interest at *3 per cent.*

Ex. 1. (*a*) What is the cost of £1,000 stock in the $3\frac{1}{2}$ per cents. at $95\frac{1}{8}$? (*b*) How much $3\frac{1}{2}$ per cent. stock at $95\frac{1}{8}$ can I buy for £$951\frac{1}{4}$?

(*a*) £100 stock is bought for £$95\frac{1}{8}$

$\therefore$ £1 " " " $\frac{95\frac{1}{8}}{100}$

$\therefore$ £1000 " " " $\frac{95\frac{1}{8} \times \overset{10}{\cancel{1000}}}{\underset{}{\cancel{100}}} =$ £$951\frac{1}{4}$. *Ans.*

(*b*) £$95\frac{1}{8}$ purchases £100 stock.

$\therefore$ £1 " $\frac{100}{95\frac{1}{8}}$ "

$\therefore$ £$951\frac{1}{4}$ " $\frac{951\frac{1}{4} \times 100}{95\frac{1}{8}}$ "

$$\frac{951\frac{1}{4} \times 100}{95\frac{1}{8}} = \frac{\overset{5}{\cancel{3805}} \times 100 \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{761}}} = \text{£}1000. \ \textit{Ans.}$$

Ex. 2. What income is derived from investing £1,227 in the $3\frac{1}{2}$ per cents. at $102\frac{1}{8}$? (Brokerage $\frac{1}{8}$.)

As the brokerage is $\frac{1}{8}$, this must be added to the price; that is, the price is $102\frac{2}{8}$, or $102\frac{1}{4}$.

£$102\frac{1}{4}$ purchases £100 stock

$\therefore$ £1 " $\frac{100 \times 4}{409}$ "

$\therefore$ £1227 " $\frac{100 \times 4 \times \overset{3}{\cancel{1227}}}{\underset{}{\cancel{409}}}$ " = £1200 stock

And £1200 stock at $3\frac{1}{2}$ per cent. yields $\frac{1200 \times 3\frac{1}{2}}{100}$ interest

$$\frac{1200 \times 3\frac{1}{2}}{100} = \text{£}42 \text{ income. } \textit{Ans.}$$

Ex. 3. Which is the better investment—the $3\frac{1}{4}$ per cents. at $97\frac{1}{2}$, or the $3\frac{1}{2}$ per cents. at 98?

In the former—

£$97\frac{1}{2}$ gives £$3\frac{1}{4}$ interest

$$\therefore \quad \text{£}1 \quad " \quad \frac{3\frac{1}{4}}{97\frac{1}{2}} = \frac{4}{\underset{2}{\cancel{195}}} = \frac{\overset{13}{\cancel{26}}}{780} = \frac{1}{30} \text{ interest.}$$

In the latter—

£98 gives £$3\frac{1}{2}$ interest.

$\therefore$ £1 „ $\frac{3\frac{1}{2}}{98} = \frac{7}{196} = \frac{1}{28}$ interest.

That is, in the $3\frac{1}{4}$ per cents. £1 produces £$\frac{1}{30}$ interest; and in the $3\frac{1}{2}$ per cents. £1 produces £$\frac{1}{28}$ interest. That is, the latter is the better investment.

Ex. 4. What is the price of 4 per cent. stock when a person gets the same interest as from 5 per cent. stock at £105?

The question resolves itself into—If £5 interest is produced by £105 principal, what principal will £4 produce?

£5 interest gives £105 principal.

£1 „ „ $\frac{105}{5}$ „

£4 „ „ $\frac{105 \times 4}{5} = £84$. *Ans.*

Ex. 5. What interest per cent. is derived from investing money in the $4\frac{1}{4}$ per cents. at 92?

£92 gives an interest of £$4\frac{1}{4}$

$\therefore$ £1 „ „ $\frac{4\frac{1}{4}}{92}$

$\therefore$ £100 „ „ $\frac{4\frac{1}{4} \times 100}{92}$

$\frac{4\frac{1}{4} \times 100}{92} = \frac{17 \times 100}{92 \times 4} = £4\ 12s.\ 4\frac{6}{23}d.$ *Ans.*

Ex. 6. What must be the market value of 3 per cent. stock, so that after deducting an income-tax of 6*d.* in the £ it may yield $3\frac{1}{2}$ per cent. interest?

(*a*) For 19*s.* 6*d.* nett I must receive £1

$\therefore$ „ 1*s.* „ „ „ $\frac{1}{19\frac{1}{2}}$

$\therefore$ „ 70*s.* „ „ „ $\frac{1 \times 70 \times 2}{39} = \frac{140}{39}$

(*b*) The question now is—What must be the market value of the 3 per cent. stock so that it may yield $£\frac{140}{39}$ per cent. interest?

$£\frac{140}{39}$ is the interest on £100

$\therefore$ £1 „ „ $\frac{100 \times 39}{140}$

$\therefore$ £3 „ „ $\frac{\overset{5}{\cancel{100}} \times 39 \times 3}{\underset{7}{\cancel{140}}}$

$$= \frac{585}{7} = 83\tfrac{4}{7}. \quad \textit{Ans.}$$

Ex. 7. A person derives an income of £126 13*s.* 4*d.* from money invested in the 3 per cent. Consols. He sells out when they are at 93, and buys Russian $3\frac{1}{2}$ per cents. at 95. By how much is his income increased by the transfer?

(*a*) To find the amount of stock in the 3 per cents.—

£3 interest represents £100 stock

$\therefore$ £1 „ „ $\frac{100}{3}$ „

$\therefore$ £$126\frac{2}{3}$ „ „ $\frac{100 \times 380}{9} = \frac{38000}{9}$ stock

(*b*) To change $\frac{38000}{9}$ stock in the 3 per cents. at 93 to the $3\frac{1}{2}$ per cents. at 95—

In the 3 per cents.—

(1) £100 stock gives £93

£1 „ „ $\frac{93}{100}$

£$\frac{38000}{9}$ „ „ $\frac{\overset{31}{\cancel{93}} \times \overset{380}{\cancel{38000}}}{\underset{3}{\cancel{9}} \times \cancel{100}} = £11780.$

(2) In the $3\frac{1}{2}$ per cents.—

£95 will purchase £100 stock

$\therefore$ £1 „ „ $\frac{100}{95}$ „

and £11780 „ „ $\frac{100 \times 11780}{3 \times 95}$ stock

$= \frac{\overset{20}{\cancel{100}} \times \overset{620}{\cancel{11780}}}{3 \times \underset{19}{\cancel{95}}} = \frac{12400}{3}$ = stock in the $3\frac{1}{2}$ per cents. at 95.

Interest on $\frac{12400}{3}$ stock at $3\frac{1}{2}$ $= \frac{\frac{12400}{3} \times 3\frac{1}{2}}{100} = \frac{62}{\cancel{12400}} \times \frac{7}{\cancel{2}} \times \frac{1}{\cancel{100}} =$

$\frac{434}{3} =$ £144 13*s.* 4*d.*, viz. income on the second investment; hence the increase = £144 13*s.* 4*d.* − £126 13*s.* 4*d.* = £18. *Ans.*

NOTE.—We may transfer from one stock to another in one operation by inverse proportion. Thus, in the given example (*b*)—

$$95 : 93 :: \frac{\cancel{88}000}{\cancel{9}} : x$$

$$x = \frac{\cancel{93}\cancel{8}000 \times \cancel{93}}{\cancel{95} \times \cancel{9}} = \frac{12400}{3}$$

Ex. 8. A capitalist sells out £5,000 stock from the 3 per cents. at 88, and invests the proceeds in railway shares at £137½, thereby increasing his income by one-third; what do the railway shares pay per cent.?

(*a*) To find the amount of stock when transferred—

cash		cash		stock		stock
$137\frac{1}{2}$	:	88	::	5000	:	x
		8 200				

$$x = \frac{\cancel{88} \times \cancel{5000} \times 2}{\cancel{275}} = 3200$$

$\cancel{25}$

Income from 3 per cents. $= \frac{5000 \times 3}{100} =$ £150, and income from railway shares = £150 + ⅓ of £150 = £200. That is—

£3200 stock gives 200 as interest

£1 „ „ $\frac{200}{3200} = \frac{1}{16}$ „

£100 „ „ $\frac{100}{16} = \frac{25}{4} = 6\frac{1}{4}$ „

That is, the railway shares pay $6\frac{1}{4}$ per cent.

EXERCISES.

1. How much 4 per cent. stock at 96 can be bought for £1000?

2. Certain £100 railway shares are at $10\frac{1}{2}$ per cent. premium; what shall I pay for 50? If at $5\frac{1}{2}$ per cent. discount, what shall I pay for 50?

3. A man bought 52 shares of railway stock at 127, and sold 36 shares at 135, and the rest at $137\frac{1}{2}$; how much did he gain?

4. If £1,000 be expended in the purchase of $3\frac{1}{2}$ per cent. stock at 92, what annual *income will it* produce?

5. The $3\frac{1}{2}$ per cent. stocks are at 91 ; required the current rate of interest.

6. How much will £750 stock cost in the $3\frac{1}{2}$ per cents. at 96 ?

7. How much 3 per cent. stock at $92\frac{3}{8}$ can I buy for £742 ?

8. A sum of £3,000 is invested in 3 per cent. Consols at $95\frac{1}{8}$; what annual income does it produce ?

9. Which is the better investment—4 per cents. at 102, or $3\frac{1}{2}$ per cents. at 96 ?

10. A gentleman invests £15,000 in the 3 per cents. at 88, and then transfers it to the 4 per cents. at 98 ; find the alteration in income.

11. A person invests £1,271 in the 3 per cents. at 92 ; how much will he gain by selling out at $94\frac{1}{8}$?

12. How much stock can be purchased for £3,116 5*s.* in the $3\frac{1}{4}$ per cents. standing at $103\frac{1}{8}$? What income will it yield ?

13. If I buy £1,400 stock in the New Three per Cents. at $92\frac{3}{8}$, and pay $\frac{1}{8}$ per cent. brokerage, what does it cost me ?

14. What is the income derived from investing £630 7*s.* 6*d.* in the India 4 per cents at $102\frac{1}{2}$?

15. How much must be invested in the 3 per cents. at $90\frac{5}{8}$ to produce an income of £165 12*s.* 6*d.* a year ?

16. How much stock must I sell out of the 3 per cents. at 95 to produce £16,200 ?

17. Which is the more profitable, the 3 per cents. at $87\frac{1}{2}$, or Bank Shares at $130\frac{1}{4}$, which pay £5 per £100 share ?

18. Find the yearly income derived from investing £6,040 in the 3 per cent. Consols at $94\frac{3}{8}$.

19. What interest per cent. is derived from investing money in the $3\frac{1}{2}$ per cents. at 84 ?

20. If a man receives 4 per cent. interest on his capital by investing in the $3\frac{1}{2}$ per cents., what is the price of the stock ?

21. A person transfers £1,000 stock from the 4 per cents. at 90 to the 3 per cents. at 70 ; find the change in his income.

22. If I buy £1,300 stock in the New Three per Cents. at $92\frac{3}{8}$, and pay $\frac{1}{8}$ brokerage, what will be the annual interest on my outlay ?

23. A government borrows £8,333,325, and agrees to pay £5 per annum for every £92 10*s.* borrowed ; find the total amount of interest to be paid annually.

24. A man buys £650 stock in the 3 per cents. at 90, and also invests £400 in the 4 per cents. at 100 ; find the total sum expended, and the annual income secured.

25. Bought stock at 120 and sold at 124 ; what *per cent.* of the investment did I gain ?

26. How much stock in the 3 per cents. must I buy to produce an income of £1,000 a year? What will it cost at 93 per cent.?

27. If the 3 per cent. Consols be at 93⅛, what sum of money must be expended in the purchase of stock to produce an income of £552 a year?

28. How much money must I expend in the purchase of 3 per cent. Consols at 93⅛ to produce an income of £600 a year?

29. How shall I affect my income by selling out from the 1¾ per cents. at 47¼, and investing in the 3⅜ per cents. at 99?

30. What sum must I invest in the 4 per cents. at 95⅛ to produce an income of £150 a year free of income tax at 2*d.* in the £?

31. A man invests £1,365 in the 3 per cents. at 91; he sells out £1,000 stock at 93½, and the remainder when fallen to 85; find the gain or loss.

32. Find the yearly income from investing £3,960 in railway stock standing at 103, and paying 5¼ per cent., brokerage being ⅛ per cent.

33. What is the price of 4 per cent. stock when a person gets the same interest as from 4½ per cent. at 90?

34. A person buys £1,000 3 per cent. stock at 96¼ and sells out at 88½; what does he lose thereby? If he re-invests his money at 4 per cent., find the alteration in his income.

35. A gentleman bought 4 per cent. stock at 87½, and after receiving a half-year's dividend on it, sold the stock at 85; what did he lose per cent. by these transactions?

36. A person transfers his 5 per cent. stock at 115 to another at 126½; what rate of interest should this latter pay that his income may be unaltered?

37. The proceeds of the sale of 3½ per cent. stock at 93⅛ amounted to £6,844 13*s.* 9*d*; find the amount of stock sold, and the annual income (before sale) after deducting 5*d.* in the £ for income tax.

38. Shares in a certain railway pay a dividend of £3 15*s.* per annum; how much must I give for them to get 6 per cent. for my money?

39. A person invests £1,000 in the 3 per cents. at 92, and pays a broker ½ per cent. on the stock purchased; after three years he sells at 95, and again pays ½ per cent. brokerage; what amount of interest did he receive, and what did he gain on the whole?

40. The sum of £1,245 is invested in the 4 per cents. at 84; how much money must be invested in the 3½ per cents. at 98 to produce the same income?

41. The sum of £1,037 10*s.* is invested in the 3 per cents. at 83, and when this fund has risen 1 per cent. is transferred to the 4 per cents. at 96; find the alteration of income.

42. The price of £100 Bank stock which pays 6½ per cent. is 185⅝. £6,250 of this stock is transferred to the 3 per cent. Consols at 92⅞; find the alteration in income.

43. What is the difference in income from investing £22,910 in the India 5 per cents. at £114, or in the New 3 per cents. at 90¾?

44. What must be the price of Consols in order that after deducting an income tax of 5*d.* in the £ there may remain a clear return of 3½ per cent. on the cost?

45. When a 3¼ per cent. stock is at 93, find what price 4½ per cent. stock must be at in order that an investment may be made with equal advantage in either?

46. If I withdraw my fortune (£3,000) from the 4½ per cent. stock at par, and re-invest in 4½ per cent. stock at 96, the transfer expenses (£9 4*s.* $7\frac{5}{13}$*d.*) being paid out of my capital, what is the effect on my income?

47. A's fortune, if increased to 6⅜ per cent., would be equal to ⅓ of the selling value of B's, which yields £576 a year in the 3 per cents., standing at 92⅛; what is A's fortune?

48. What sum must be invested in the 3 per cents. at 98½ to produce an annual income equal to the whole interest of £500 for 14 years at 2½ per cent.?

49. Certain £50 railway shares cost me £55, and yield a dividend of £6 a year each; what is the amount of my yearly income for every £100 invested? What percentage of the income that I might have had did I forego by not investing when the shares stood at their original price?

Section VI.*—Equation of Payments, Fellowship or Partnership, and Exchange.

I. Equation of Payments.—A owes B £100 due in three months, and also £100 due in six months. Now if A wishes to pay the two sums at one time, it is clear that the just time of payment will be between the third and sixth months, for the three months' bill will run a little longer than three months, and the six months' bill will run less than that time. We find the correct time by the rule called "Equation of Payments."

Ex. 1. Find the equated time of the following bills—

(*a*) £101 5*s.* due 3 months hence at 5 per cent.
(*b*) £102 10*s.* „ 6 „ „ „ 5 „
(*c*) £103 15*s.* „ 9 „ „ „ 5 „

† The present value of the £101 5*s.* = 100
„ „ „ „ £102 10*s.* = 100
„ „ „ „ £103 15*s.* = 100
„ „ „ „ £307 10*s.* = 300

* This section may be omitted by female pupil teachers.
† See page 139 for the method of finding the present value.

Now, in what time will £300 amount to £307 10*s.* at 5 per cent.?

$$\text{By formula, } M = P + \frac{P \times n \times r}{100}$$

$$\therefore \text{£}307\ 10s. = \text{£}300 + \frac{\overset{3}{\cancel{300}} \times n \times 5}{\underset{1}{\cancel{100}}}$$

$$\therefore \text{£}307\ 10s. - \text{£}300 = 15n$$

$$7\tfrac{1}{2} = 15n$$

$$n = \frac{7\frac{1}{2}}{15} = \tfrac{1}{2} \text{ or six months}$$

We have, in solving the above problem, reckoned true discount; in practice, when such questions occur, a mere average is struck; thus—

$$\text{(prin.)} \quad \frac{\overset{\text{prin. time}}{101\frac{1}{4} \times 3} + \overset{\text{prin. time}}{102\frac{1}{2} \times 6} + \overset{\text{prin. time}}{103\frac{3}{4} \times 9}}{101\frac{1}{4} + 102\frac{1}{2} + 103\frac{3}{4}} = \frac{303\frac{3}{4} + 615 + 933\frac{3}{4}}{307\frac{1}{2}}$$

$$= \frac{1852\frac{1}{2}}{307\frac{1}{2}} = \frac{3705}{615} = 6\tfrac{1}{41}$$

The reason for the above process is this—

The interest on	£101¼	for	3	months	=	interest on	£303¾	for	1	month
„ „ „	£102½	„	6	„	=	„	£615	„	1	„
„ „ „	£103¾	„	9	„	=	„	£933¾	„	1	„
	£307½						£1,852½			

That is, the interest on the three bills is equal to the interest on £1,852½ for *one* month.

The question now becomes—If the bills have to be paid in one sum, £307½, how long should it be held before the interest on it will amount to the same as the interest on £1,852½ for one month?

$$\text{Hence } 307\tfrac{1}{2} : 1852\tfrac{1}{2} :: 1 : x$$

$$\text{and } x = \frac{1852\frac{1}{2}}{307\frac{1}{2}} = 6\tfrac{1}{41}$$

NOTE.—It will be seen from the above explanations that in practice *interest* is reckoned instead of *discount*. In any examples which follow, unless otherwise specified, use the rule adopted in practice.

II. PARTNERSHIP.—A partnership is an association of two or more persons for the transaction of business.

A partnership association is called a *Company*, or *Firm*, or *House*, and the persons associated together are called partners.

The property or money invested in the business by the partners is called the *Capital*, *Joint-Stock*, or *Stock-in-Trade*.

Two persons, A and B, enter into partnership for the purpose of business. A advances £5,000 as capital, and B £6,000. Now, if the profits for the first year amount to £1,000, it is plain that this sum should not be equally divided between A and B; but that B having invested the

larger capital should have the larger share of profit. In fact, each should have a *proportionate* share of the gain according to the amount of his capital invested in the business.

Or, again, suppose A commences business alone with £5,000, but that at the end of six months B joins with a capital of £5,000, it is clear that B having the same amount of capital in the business as A, but only for half the time, should have at the end of twelve months only half the profits.

Problems which have to do with determining the amount of gain or loss of each individual in the joint-stock company are said to be problems in Partnership or Fellowship. Simple proportion plays the chief part in their solution.

Ex. 1. A, B, and C enter into partnership for 2 years; A puts in £3,600, B £2,400, and C £2,000, and their net profits are £3,000; what was each partner's share?

The entire capital was £8,000, and the entire profits £3,000.

That is, £8000 gained £3000

$\therefore$ £1 „ $\frac{3}{8}$

$\therefore$ £3600 „ $\frac{3}{8} \times 3600 = £1350$ = A's share of the profits

and £2400 „ $\frac{3}{8} \times 2400 =$ £900 = B's „ „ „

and £2000 „ $\frac{3}{8} \times 2000 =$ £750 = C's „ „ „

Ex. 2. A commences business with a capital of £550; after four months B joins with a capital of £1,050; and after another four months C joins with a capital of £1,000. At the end of the second year the business has brought a gain of £570; what ought each partner to receive?

A's capital is £550 for two years, and this is equivalent to £1,100 for one year; B's capital of £1,050 for one year and eight months is equivalent to £1,050 $\times 1\frac{2}{3}$ for one year; C's capital of £1,000 for one year and four months is equivalent to £1,000 $\times 1\frac{1}{3}$ for 1 year.

That is, the whole capital for one year $= \left\{ \begin{array}{ll} & £ \\ 550 \times 2 & = 1100 \\ 1050 \times 1\frac{2}{3} & = 1750 \\ 1000 \times 1\frac{1}{3} & = 1333\frac{1}{3} \end{array} \right\} = £4183\frac{1}{3}$

The question is now similar to that in Ex. 1. £4,183$\frac{1}{3}$ gains £570; what will £1,100, £1,750, and £1,333$\frac{1}{3}$ gain respectively?

£4183$\frac{1}{3}$ gains 570

$\therefore$ £1 „ $\frac{570}{4183\frac{1}{3}} = \frac{171\emptyset}{1255\emptyset}$

$\therefore$ £1100 „ $\frac{171 \times 1100}{1255} = £149\frac{221}{251}$ = A's share

And £1750 „ $\frac{171 \times 1750}{1255} = £238\frac{112}{251}$ = B's „

And £1333$\frac{1}{3}$ „ $\frac{171 \times 1333\frac{1}{3}}{1255} = £181\frac{169}{251}$ = C's „

Ex. 3. A and B entered into a partnership for three years. A invested £3,500, B £4,500. At the end of the first year they each took out £1,000, and C was received as partner with a capital of £2,500. At the end of the third year they dissolved partnership, dividing £5,000 as net profits; what was each partner's share?

In the business—

A has	£3500	for 1	year, and	£2500	for 2	years, or	£8500	for 1	year
B	„ £4500	„ 1	„ „	£3500	„ 2	„ „	£11500	„ 1	„
C	„ £2500	for 2 years, or					£5000	„ 1	„
							£25000		

£25000 gains £5000

£1 „ $\frac{5}{25}=\frac{1}{5}$

∴ £8500 „ $\frac{1}{5}$ of £8500 = £1700 = A's share

And £11500 „ $\frac{1}{5}$ of £11500 = £2300 = B's „

And £5000 „ $\frac{1}{5}$ of £5000 = £1000 = C's „

III. Exchange.—As trade is carried on between different countries, and as different countries use different coins containing different weights of pure metal, and consequently differing in value, it becomes necessary to be able to find the equivalent values of money expressed in the coins of different countries.

Ex. 1. How many francs must be transmitted from Paris to Berlin to discharge a debt of 420 thalers, a thaler being equivalent to 3*s.*, and 24 francs to £1?

$$420 \text{ thalers} = 3 \times 420 \text{ shillings}$$

$$= \frac{3 \times 420}{20} = £63$$

And £1 = 24 francs

∴ £63 = 63 × 24 = 1512 francs

That is, 1512 francs must be transmitted to discharge a debt of 420 thalers.

Ex. 2. When the exchange between London and Paris is £1 for 25 francs; between Paris and New York 102 francs for 21 dollars, and between New York and St. Petersburg $10\frac{1}{2}$ dollars for 23 roubles, how much is a rouble worth in English money?

25 francs = £1

∴ 1 „ = £$\frac{1}{25}$

And 102 „ = £$\frac{102}{25}$

But 21 dollars = 102 francs

$\therefore$ 21 „ $= £\frac{102}{25}$

$\therefore$ 1 „ $= £\frac{102}{25 \times 21}$

And $10\frac{1}{2}$ „ $= £\frac{102 \times 21}{25 \times 21 \times 2} = \frac{51}{25}$

$\therefore$ 23 roubles $= £\frac{51}{25}$

And 1 „ $= £\frac{51}{25 \times 23}$

$= 1s.\ 9\frac{53}{115}d.$

Exercises.

1. A bankrupt surrenders his property, worth £366, to three creditors, to whom he owes respectively £450, £560, and £670; divide the property fairly among them.

2. Three persons rent a field for £60 10*s.*; A puts in 5 sheep for $4\frac{1}{2}$ months, B 8 sheep for 5 months, and C 9 sheep for $6\frac{1}{2}$ months; what must each pay of the rent?

3. A merchant commenced business with £3,500; at the end of 6 months he was joined by a partner, who brought £2,000, and at the end of 12 months by a second partner, who introduced £1,000 into the business; at the end of the third year the profits had accumulated to £2,600; find the share of each.

4. If 1,185 parts of copper be added to 715 parts of tin and 100 parts of zinc, what is the percentage of each metal in the compound?

5. If 12 cows be worth 5 horses, and 11 horses be worth 120 sheep, and 5 sheep be worth 3 pigs, how many pigs are worth 11 cows?

6. A owes B £300, of which £200 is due in 3 months and £100 in 6 months; when will the payment of £300 equitably discharge the debt?

7. If 5 lbs. of tea be worth $7\frac{3}{4}$ lbs. of coffee, and 3 lbs. of coffee be worth 24 lbs. of rice, and 3 lbs. of rice be worth $4\frac{1}{2}d.$, what is the price of a lb. of tea?

8. A owes B £1,200, of which £300 is due in 4 months, £400 in 6 months, and the remainder in 12 months; what is the equated time for the payment of the whole?

9. What French money is equivalent to £350 15*s.* exchange, at 25 fr. 20 c. per £1? (1 fr. = 100 c.)

10. A merchant imports a cargo of wood from Russia valued at *5,400 roubles*; he may order the sum to be drawn directly at 37*d.* per *rouble, or on* Holland at 1 florin 85 cents per rouble, and from Holland

to London at 11 florins 95 cents per £; find the cheaper method. (1 florin = 100 cents.)

11. A merchant bought £500 worth of goods on 4 months' credit, £300 worth on 6 months' credit, and £400 at 8 months' credit; what is the equated time for the payment of the whole?

12. A and B entered into partnership for 4 years, A putting in £6,000 and B £8,000; at the close of the second year A took out £2,000 and B put in £2,000, and at the close of the fourth year they divided £8,890 as net profits; what was the share of each?

13. What is the equated time for the payment of £18 15*s.* due now, £21 5*s.* due in 30 days, £25 due in 60 days, and £30 due in 75 days?

14. Change 370 thalers 15 silver groschen 10 pfennings to English money. (360 pfennings = 30 silver groschen = 1 thaler = 2*s.* 10$\frac{3}{4}$*d.*)

15. A, B, and C, having a joint capital of £10,000, buy with it a ship and cargo, the latter being worth $\frac{2}{3}$ of the former; the ship is lost, but the cargo is saved; what amount per cent. on his capital does each partner lose?

16. If the dollar be current for 4*s.* 2*d.*, the franc for 9$\frac{3}{4}$*d.*, and the rupee for 1*s.* 10$\frac{1}{2}$*d.*, what is the *average* income of 5 persons, one of whom receives 5,000 dollars, two receive 20,000 francs, and two receive 10,000 rupees each?

17. A certain pudding consists of 3 parts of raisins at 6*d.* a lb., 4 parts of suet at 8*d.* a lb., and 2 parts of flour at 3*d.* a lb., and it costs altogether 7*s.*; what was the cost of each ingredient?

18. A, B, and C entered into business as partners, each putting in £5,000 as capital; at the end of 2 years A took out £1,000, B £2,000, and C £3,000, and at the end of the fourth year they closed the business with a loss of £3,600; what was the loss of each?

Section VII.—Miscellaneous Exercises on Commercial Rules.

1. Find the simple interest on £850 for 3 months at 2$\frac{1}{4}$ per cent. per annum.

2. Find the premium on a cargo valued at £126,315, at 8*s.* 8*d.* per cent.

3. A lady pays £140 for board and £87 10*s.* for other expenses, and puts by 35 per cent. of her income; what is her income?

4. Apples are bought at 5 for 2*d.* and sold at 2 for 1*d.*; find the gain per cent.

5. A bankrupt pays 13*s.* 7*d.* in the £; find the payment on a debt of £4,572.

6. A grocer bought 17 lbs. of tea at 4*s.* and 25 lbs. at 4*s.* 8*d.*, and sold the mixture at 5*s.* 4*d.* a lb.; find the gain per cent.

7. What is the simple interest on £640 for 7$\frac{1}{2}$ years at 4 per cent.?

8. The amount collected from a poor-rate assessed on £2,044 2*s.* 8*d.* was £51 2*s.* 0$\frac{1}{4}$*d.*; what was the rate in the £?

9. What will £500 amount to in $9\frac{3}{4}$ years at $4\frac{1}{2}$ per cent. simple interest?

10. If 113 galls. of spirit be diluted with 37 galls. of water, find the percentage of spirit and water.

11. If a capital of £250 gain £23 in 4 months, what will a capital of £1,200 gain in 3 months?

12. What is the amount of £7,506 in $7\frac{1}{2}$ years at $4\frac{3}{4}$ per cent. simple interest?

13. If the interest on £540 10*s.* be £21 14*s.* for 1 year, what principal will yield the same interest in $2\frac{1}{2}$ years?

14. Find the simple interest on £212 10*s.* 4*d.* for $2\frac{3}{4}$ years at $2\frac{1}{2}$ per cent.

15. Cheese bought at 50*s.* per cwt. is sold at $6\frac{1}{2}d.$ a lb.; what is the gain per cent.?

16. In what time will £926 15*s.* amount to £962 2*s.* $8\frac{1}{2}d.$ at 7 per cent. simple interest?

17. At what rate per cent. will £739 14*s.* $9\frac{1}{2}d.$ amount to £813 14*s.* $3\frac{1}{4}d.$ in 4 years?

18. What principal put out to simple interest for 5 years at $3\frac{1}{2}$ per cent. will amount to £1,000?

19. Find the discount on £150 10*s.* due 10 months hence at $3\frac{3}{4}$ per cent. per annum.

20. What is the price of stock when a broker can purchase £6,000 stock for £5,000?

21. In what time will £250 amount to £500 at 5 per cent. per annum, simple interest?

22. At what rate per cent. simple interest will 1*s.* become 5*s.* in $33\frac{1}{3}$ years?

23. By selling an article for 19*s.* I lose 5 per cent.; what was the cost price?

24. What amount of capital put out at $3\frac{1}{2}$ per cent. per annum, simple interest, will produce £14 interest in $3\frac{1}{3}$ years?

25. How many lbs. of tea at 2*s.* 1*d.* must be mixed with 42 lbs. at 1*s.* 8*d.*, so that the mixture may sell for 1*s.* 10*d.* per lb.?

26. A butcher sold beef at a profit of 2*d.* a lb., thus gaining 20 per cent.; find the cost price per lb.

27. Sold flour at 4 lbs. for 9*d.*, and thereby gained $12\frac{1}{2}$ per cent.; what did the flour cost me per lb.?

28. In what time will £445 amount to £486 3*s.* 3*d.* at 2 per cent.?

29. At what rate per cent. will £1,250 amount to £1,962 10*s.* in 6 years?

30. An estate with a rental of £7,890 is sold for £351,600; in order that it may yield the purchaser $2\frac{3}{4}$ per cent. on his outlay, how much per cent. must the rent be raised?

31. Find the present value of £682 18*s.* 9½*d.* due 3 years 9 months hence, at 4½ per cent.

32. In what time will £2,175 amount to £2,338 2*s.* 6*d.* at 2½ per cent. simple interest?

33. I bought coffee at £6 15*s.* 6*d.* per cwt.; what must be the selling price to gain 7½ per cent.?

34. Find the present worth of £1,336 11*s.* 3*d.*, due at the end of 3½ years, at 5 per cent.

35. A sum of money is borrowed at 5 per cent. simple interest; in 7 years it amounts to £810; what is the sum borrowed?

36. Oranges are bought at 18 for a shilling and sold at 3*d.* a doz. more than they cost; what is the gain per cent.?

37. I bought £1,500 stock at 90; at what price must I sell it to gain £75?

38. A grocer bought tea at 3*s.* 4*d.* a lb.; at what price must he sell it to gain 15 per cent.?

39. What must be the price of the 4 per cents. so that they may yield an income of 4½ per cent. on the money invested?

40. What sum of money must a person invest in the 3½ per cents. at 95⅜ to get £156 a year?

41. What sum will amount to £798 12*s.* in 3 years at 10 per cent. compound interest?

42. What present payment will discharge a debt of £75 due 1 year 5 months hence at 4 per cent.?

43. By selling goods for £5 14*s.* a dealer gained 8 per cent.; find the cost price.

44. How much must a man invest in the 3 per cents. at 10 per cent discount to receive a dividend of £500 a year?

45. By selling candles at 7*d.* a lb. a grocer gained 8 per cent. on the cost price; at what price per lb. must they be sold so that there shall be neither gain nor loss?

46. What is the present value of £1,296 due 6 months hence at £10 13*s.* 4*d.* per cent.?

47. An ironmonger bought nails at 18*s.* 6*d.* per cwt. and sold them at 3*d.* a lb.; find the gain or loss per cent.

48. If I lend a friend £640, and he returns me £690 8*s.* at the end of 2½ years, at what rate per cent. has the interest been calculated?

49. If 5 oxen are worth 24 sheep, and 4 sheep are worth £13, what are 55 oxen worth?

50. A bankrupt owes 3 creditors, A, B, and C, £290, £556, and £620 respectively; his assets are worth £412 6*s.* 3*d.*; what ought each creditor to receive?

51. What is the compound interest on £200 for 3 years at 5 per cent. per annum?

52. A house which cost £1,500 lets for £65 a year, the outgoings for insurance and other expenses amount to $1\frac{1}{4}$ per cent. on the cost price; what rate of interest does the house pay?

53. In what time will a sum of money double itself at $3\frac{1}{2}$ per cent. simple interest?

54. A man sold a horse for £50, and by so doing lost $33\frac{1}{3}$ per cent.; what did he give for it?

55. If a collector who is paid 5 per cent. commission receives £17 10*s.* for his trouble, how much money has he collected?

56. Find the course of exchange per milree between Lisbon and London when 2,268 milrees are drawn at Lisbon for an English bill of £472 10*s.*

57. Bacon is sold at 15 per cent. profit; if the price per lb. be raised 1*d.* the profit is 20 per cent.; find the cost price.

58. If a broker be allowed $3\frac{3}{4}$ per cent., what may he demand when he sells goods to the amount of £875 5*s.* 10*d.*?

59. At what rate per cent. will £1,016 amount to £1,250 in 5 years 5 months?

60. I paid £30 for a watch, by which transaction the watchmaker gained 5 per cent.; what was the amount of his profit?

61. A grocer bought 5 cwt. of sugar at £2 2*s.* per cwt. and 5 cwt. at £1 10*s.* per cwt.; at what price per lb. must he sell the mixture to gain $3\frac{19}{7}$ per cent.?

62. What ought to be the value of £150 in the $3\frac{1}{2}$ per cents. when the 3 per cents. are at $85\frac{5}{7}$?

63. If the discount on a bill of £172 be £12, simple interest, being calculated at 5 per cent., how long has the bill to run?

64. A man buys goods at the rate of 10*s.* per cwt.; find the gain or loss per cent. in selling 5 tons 4 cwt. 2 qrs. for £57 9*s.* 6*d.*?

65. By selling a horse for £24 a dealer loses 5 per cent. on his outlay; what would he have gained per cent. by selling it for £32?

66. A capitalist invests £5,187 10*s.* in the 3 per cents. at 83; he transfers $\frac{2}{5}$ to the 4 per cents. at 96; find the alteration in his income.

67. At what price are the $3\frac{1}{2}$ per cents. when a man buys stock producing £17 10*s.* a year for £401 13*s.* 4*d.*?

68. A company with a capital of £125,000 declare a dividend of 5 per cent. with £3,500 surplus; what are the nett earnings of the company?

69. If by selling a horse for £96 I lose 20 per cent., what must I have sold it for to gain 5 per cent.?

70. How many lbs. of tea at 2*s.* 8*d.* must a dealer mix with 495 lbs. at 2*s.* 9*d.* so that he may realise a profit of 10 per cent. by selling at 3*s.* a lb.?

71. A broker bought stock at 4 per cent. discount and sold it at 5 per cent. premium, and thereby gained £450; how many shares did he buy?

72. A, B, and C are partners in a business; A's capital is twice B's and three times C's, and the profits are £4,675; find the share of each.

73. By selling goods for £116 17*s.* I lose 5 per cent.; what should I gain or lose by selling at £132 4*s.*?

74. At what rate per cent. will a person receive interest who invests his capital in the 3 per cents. at 89⅞? (Commission ⅛ per cent.)

75. A tradesman, by selling an article for 8*s.* 3*d.*, loses 17½ per cent.; what should he have sold it for to gain 40 per cent.?

76. Find the selling price of the 3 per cent. Consols when an outlay of £3,020 produces a yearly income of £96.

77. By selling cheese at £3 13*s.* 6*d.* per cwt. a cheesemonger made a profit of 22½ per cent.; what did it cost him per cwt.?

78. I gained 33 per cent. by selling lace at 13*s.* 6*d.* a yd.; what shall I gain per cent. by selling at 12*s.* a yd.?

79. A tea dealer buys tea at 2*s.* 3*d.* and at 3*s.* 6*d.*; in what proportion must he mix them to sell at 4*s.* a lb., and thereby realise 20 per cent.?

80. A merchant sold cloth at 7*s.* 8*d.* a yd., which gave 6 per cent. profit; he gained £33 6*s.* 8*d.*; how many yds. did he sell?

81. By selling goods at 4*s.* I lost 7 per cent.; what do I lose by selling at 3*s.* 9*d.*?

82. A wholesale merchant sells to the retailer at a profit of 20 per cent.; the retailer sells at a profit of 30 per cent.; what part of the price paid by the consumer is profit?

83. A grocer buys tea at 2*s.* 6*d.* and 3*s.* and mixes it in the proportion of 3 to 2; what will he gain per cent. by selling at 2*s.* 9*d.*?

84. How much money must a person invest in the 3 per cent. stock at 95 to get the same income as another person who has invested £10,000 in the 3½ per cent. stock at 98½?

85. A merchant insured a vessel and cargo worth £35,000 at 7¾ per cent., so as, in case of loss, to recover the value of ship, cargo, and insurance; for what sum did he insure?

86. A man has £500 stock-in-trade, and borrows £200 at 6¼ per cent. to increase it; at what price must he retail his goods to make 8½ per cent. on the whole, after deducting £150 for working expenses?

87. If when I sell cloth for 10*s.* a yard I lose 5 per cent., how much shall I gain or lose per cent. by selling at 12*s.* 6*d.* a yard?

88. A grocer bought a chest of tea containing 90 lbs. at 2*s.* a lb., another containing 40 lbs. at 2*s.* 6*d.*, and a third containing 20 lbs. at 4*s.*; he mixed the whole and sold at 3*s.*; find the gain per cent. on the cost price.

89. Three cowkeepers hire a pasture for £35 7*s.* 6*d.* for 26 weeks; A puts 7 cows in for 13 weeks, B 14 cows for 9 weeks, and C 3 cows for 22 *weeks; how should* they divide the rent?

90. What ought to be the price of £100 Bank stock which pays a dividend of 10½ per cent. in order that it may pay 4 per cent. on the money invested?

91. At what rate should money be invested in the 3½ per cents. to obtain an interest of 4 per cent.?

92. A, B, and C join their capitals, which are in the proportion of ½, ⅓, and ¼; at the end of 4 months A withdrew half of his capital, and at the end of 9 months more they divide their profits, which amount to £284; what ought each to receive?

93. Sold £20,700 of the 3 per cents. at 92½, and invested the proceeds in Canada 5 per cents. at 103½; find the increase of income.

94. A farmer gave for 3 horses a bill of £156 due 8 months hence at 4½ per cent., and sold them at once for £180; what was his gain per cent.?

95. Which is the better investment, the 3 per cents. at 90¼, or railway shares at £114 paying a half-yearly dividend of £2 5s.?

96. If I save £10 yearly and invest it in the 3 per cents., and always put in the interest with the savings for 4 years, what sum shall I have at the end of that time?

97. A ship is valued at £14,720; what sum should be insured at 8 per cent. by a person who owns $\frac{1}{16}$ of the ship, so that in case of loss the insurer may recover both the value of his share of the ship and the premium paid?

98. At what price must cloth be marked which cost 5s. a yard, that a tailor may deduct 10 per cent. from the marked price and still make a profit of 12 per cent.?

99. A sold a house to B at 8 per cent. gain on the cost price, and B sold it to C for £972, thereby gaining 8 per cent.; find the cost price to A.

100. A capitalist speculated with £1,000 and gained 10 per cent.; he speculated again and lost 10 per cent.; he speculated a third time and gained 10 per cent., and then a fourth time and lost 10 per cent.; what was he worth then?

CHAPTER X.

MISCELLANEOUS EXERCISES.

1. A sum of £16 is divided so that each recipient has half-a-crown, a florin, a sixpence, and a fourpenny piece; how many persons were there?

2. Find the difference between ·0231 of a guinea and ·19 of half-a-crown.

3. If 3·75 yds. cost £3,825, how much will 38·5625 yds. cost?

4. A can do a piece of work in 10 days which A and B, working together, can do in 7 days ; how long would B take to do it alone ?

5. If you buy at 15*s.* 2*d.* a yd. and sell at 16*s.* 6*d.*, what do you gain per cent. on the outlay ?

6. Find the present worth of £1,405 due half a year hence at $4\frac{1}{2}$ per cent. per annum.

7. Reduce $\frac{1}{3}$ of $2\frac{1}{5}$ of a £ to the decimal of £5 10*s.*

8. What is the dividend on £1,709 10*s.* at 13*s.* 8*d.* in the £ ?

9. What is the simple interest on £4,805 for 4 yrs. at $5\frac{3}{4}$ per cent. per annum ?

10. Multiply ·00471 by ·004, and divide ·0269069292 by ·00732.

11. After paying income tax at the rate of 5*d.* in the £ a man has £636 9*s.* 2*d.* remaining ; what is his gross income ?

12. Find the value of $1\frac{1}{32}$ of £3 4*s.* 6*d.* + $7\frac{1}{8}$ of $3\frac{1}{2}$ guin. + $\frac{3}{16}d.$

13. If 1 ton cost £$1\frac{2}{3}\frac{8}{6}$, what will $16\frac{1}{2}\frac{1}{5}$ tons cost ?

14. How many times can £1,139 be subtracted from £99,093 ?

15. If a tradesman with a capital of £1,000 gains £90 in 7 months, how long will he be in gaining £20 5*s.* with a capital of £315 ?

16. Divide 72·1068325 by ·0725 and ·001 by ·000001.

17. At what rate per cent. simple interest will £225 amount to £256 10*s.* in 4 years ?

18. If a man walking at the rate of 4 miles an hour goes a certain distance in 2 hrs. 15 mins., in what time could he run the distance at the rate of $7\frac{3}{4}$ miles an hour ?

19. If $\frac{3}{5}$ ton amount to £1 18*s.* $7\frac{1}{2}d.$, what part of a ton will amount to £1 1*s.* $5\frac{1}{2}d.$?

20. After payment of $\frac{1}{2}$ of a debt, it is found that $\frac{5}{8}$ of what is still due is £1 ; what was the debt ?

21. The sum of two numbers is $9\frac{5}{8}$, and their difference is $4\frac{5}{8}$; find the ratio of the numbers.

22. Write as decimals $37_{\frac{17}{10000}}$ and $10^{\frac{5061}{100}}$.

23. If A and B, working together, can do a piece of work in 10 days, and A by himself can do it in 18 days, what time will B take to do it ?

24. A fruiterer lost $33\frac{1}{3}$ per cent. of a lot of apples, and sold the remainder at a gain of 50 per cent. ; what per cent. did he gain or lose ?

25. A tank is supplied by a spout which can fill it in 2 hrs. ; how long will it take to fill the tank if it has a leak which would empty it in 10 hrs. ?

26. A grocer bought 600 lbs. of sugar for £10 and sold it at $4\frac{1}{2}d.$ a lb. ; what profit per cent. did he realise on the cost price ?

27. The proprietor of $3\frac{1}{8}$ shares of a mine receives every year £41 2*s.* 11*d.* as dividend ; what income is derived from holding $15\frac{1}{4}$ of such shares ?

28. What fraction is as much greater than $\frac{4}{5}$ as $\frac{2}{3}$ is less ?

29. How much money must be invested in the 3 per cents. at 96 to produce an annual income of £400 ?

30. If 40 men can reap 400·5 acres in 12·75 days, how many acres ought 30 men to reap in 34 days ?

31. Divide £64 among A, B, and C, so that A may have 3 times as much as B, and C may have $\frac{1}{3}$ of what A and B have together.

32. A horse cost £90, and $\frac{3}{10}$ of the price of the horse equals $\frac{5}{8}$ of 3 times the cost of the saddle ; what did the saddle cost ?

33. What will it cost to pave a rectangular yard 20 ft. by 28 ft. at 2*s.* 6*d.* a sq. yd. ?

34. $1\frac{3}{4}$ yds. of silk cost 10*s.* 6*d.*. Express the price of $75\frac{5}{6}$ yds. as the fraction of £28 14*s.*

35. If a parcel of goods bought for £18 be sold 4 months after for £25, what is the gain per cent. per annum ?

36. A sold B a horse for $\frac{1}{8}$ more than it cost him, and B sold it for £80, thus losing $\frac{1}{6}$ of what it cost him ; how much did A pay for the horse ?

37. A garrison of 2,000 men is supplied with provisions for 20 days ; if 1,200 men leave, how long will the provisions last the remainder ?

38. I bought £128 5*s.* worth of goods and kept them on hand for 6 months, when money was worth 8 per cent. ; I then sold at a nett gain of 6 per cent. ; for how much were they sold ?

39. Simplify $\frac{1}{3\frac{1}{5}} - \frac{2\frac{1}{4}}{9} + \frac{3\frac{5}{8}}{2} + \frac{\frac{4}{7}}{4\frac{4}{7}}$

40. In what time will £75 10*s.* 5*d.* amount to £89 2*s.* $3\frac{1}{2}$*d.* at 4 per cent. simple interest ?

41. A piece of cloth, when measured with a "yard measure" which is ·6 of an inch too short, appears to measure $88\frac{1}{2}$ yds. long ; what is its true length ?

42. The produce of a field of wheat yielding 4 qrs. per acre is worth £148 when wheat is at £3 a qr. ; what will the produce of another field twice as large be worth when the yield is 5 qrs. per acre and wheat is at 50*s.* a qr. ?

43. $\frac{2}{3}$ of A's money is $\frac{3}{4}$ of B's, and $\frac{3}{4}$ of B's is $\frac{2}{3}$ of C's, which is £81; how much have A and B each ?

44. In a company of 90 persons there are 4 more men than women, and 10 more children than men and women together ; how many of each in the company ?

45. A advances £1,500 for 9 months, and B £1,200 for 6 months ; what share of a gain of £1,150 belongs to each ?

46. A house is rated at $\frac{9}{10}$ of its rent, and the rate at 5*d.* in the £ amounts to 18*s.* 9*d.* ; what is the rent ?

47. A post is $\frac{1}{3}$ in clay, $\frac{1}{4}$ in gravel, $\frac{1}{6}$ in water, and 15 ft. above water; *find its length.*

48. A man loaned £800 for 2 years 6 months, and received £90 interest; what was the rate per cent.?

49. A can mow an acre of grass in $\frac{3}{4}$ of a day, and B in $\frac{2}{5}$ of a day; in what time could A and B do it working together?

50. Find the number of lbs. in $3\frac{1}{7}$ of $5\frac{1}{2}$ tons + $4\frac{7}{16}$ of 62 cwt. + $1\frac{1}{7}$ of 513 qrs.

51. In 1861 three towns had populations of 17,341, 15,540, and 18,760 respectively; in 1871 it was found that the population of the first had decreased 17 per cent., that of the second had increased by 19 per cent., while the population of the three had increased by 5,000; find the increase or decrease per cent. in the population of the third town.

52. A stationer buys 45 reams of paper at 13*s.* 9*d.* per ream, and 73 reams at 12*s.* 10*d.*; at what average price per quire must he retail it to make a gain of 30 per cent.?

53. What number of bricks 9 in. by 4 in. will be required to pave a yard 13 ft. 6 in. by 11 ft. 4 in.?

54. Find the value of $\frac{2}{3} \div 1\frac{5}{7} - \frac{5}{8} \div 3\frac{2}{11}$.

55. Which is the greater income tax, 5*d.* in the £ or $2\frac{1}{4}$ per cent.?

56. Express £4 17*s.* $6\frac{1}{2}d.$ as a *vulgar* fraction of a £ and as a *decimal*.

57. Find the value of $\cdot\dot{2}3\dot{4}$ of £20.

58. Reduce to its simplest form the expression $\frac{1}{3} + 6\frac{1}{2} + \frac{3\frac{4}{5}}{7} - \frac{1}{3} \times \frac{2}{5}$.

59. If 7 men earn £10 7*s.* 6*d.* in $10\frac{1}{2}$ days, what sum will 28 men earn in $31\frac{1}{2}$ days?

60. An estate was divided between 2 brothers and a sister; the elder brother received $\frac{3}{8}$ of the estate, the younger $\frac{3}{10}$, and the sister the remainder, which was £450 less than the elder brother received; what was the value of the estate, and what was each brother's share?

61. A bankrupt's debts amount to £1,700, his assets to £900 15*s.*; after paying costs, his creditors receive 5*s.* 9*d.* in the £; find the amount of the costs.

62. If wheat, having been at £2 10*s.* 4*d.* per quarter, rises 6·25 per cent., what is then its price?

63. Find the value of ·5*s.* + ·7 crowns + £·125.

64. Reduce 3 cwts. 2 qrs. 31 lbs. to the fraction of a ton.

65. Simplify $2\frac{1}{3} \times 1\frac{5}{8}$ of $1\frac{2}{13} \times 3\frac{1}{4}$ of $1\frac{5}{11}$.

66. Simplify $\dfrac{\cdot002 \times 1\cdot75 \div \cdot00007}{1\frac{2}{3} \div \frac{1}{3}}$

67. If a tradesman buys an article at 5*s.* a lb. and retails at 6*d.* an oz., what is the gain on an outlay of £6, and what is the gain per cent.?

68. If I expend £11,677 10*s.* in the $3\frac{1}{2}$ per cents. at $86\frac{1}{2}$, what will be my half-yearly dividend?

69. If £50 principal gain £7 10*s.* interest in 9 months, what principal will gain £68 in a year and a quarter?

70. Multiply £30 14*s.* $6\frac{1}{2}d.$ by $\frac{4}{23}$.

71. If 2 men can reap $2\frac{1}{2}$ acres in $2\frac{3}{4}$ days, how long will 11 men take to reap 15 acres?

72. A wine merchant mixes 5 galls. of wine at 32*s.* per doz., 4 at 45*s.*, and 1 at 55*s.*; how much will the mixture be worth per doz. (2 galls. = 1 doz.)?

73. A man has £579 13*s.* 4*d.* per annum after paying income tax at 5*d.* in the £; what is his income?

74. A grocer, by selling sugar at $5\frac{1}{4}d.$ a lb., loses 12 per cent.; at what price per cwt. must he sell it to gain 8 per cent.?

75. A walks 11 miles in 4 hrs., and B 8 miles in 3 hrs.; they start at the same time, but at opposite points, to walk a distance of 65 miles; when will they meet?

76. Find the value of $\frac{3\frac{1}{3}+\frac{2}{5}}{5\frac{3}{4}-\frac{7}{8}} \times \frac{6\frac{1}{4}-3}{3\frac{1}{3}-\frac{2}{5}} \times \frac{1}{3-\frac{1}{3\frac{1}{3}}}$

77. Required the number of days in which 15 horses will consume 75 bush., it being given that 6 horses consume 60 bush. in 50 days.

78. Find the compound interest on £3,045 in 3 years, at 4 per cent. per annum.

79. How many lbs. of tea at $\frac{1}{6}$ of a shilling an oz. should be given in exchange for 17 pieces of cloth, each containing 5 yds. 3 qrs. 2 nls. at 11·52*s.* per yd.?

80. Divide $\cdot 3\dot{7}4\dot{1}$ by $37\cdot\dot{7}\dot{8}$ to 4 places.

81. Find the difference between the simple and compound interest on £432 for 4 years at 5 per cent.?

82. If 15 men can do a certain piece of work in $12\frac{3}{8}$ days of $9\frac{1}{2}$ hrs., how many days of 11 hrs. each will 9 men require to do $4\frac{1}{4}$ times as much work?

83. What will be the cost of a carpet 2 ft. wide at 6*s.* 9*d.* a yd. for a room 25 ft. 4 in. long and 21 ft. 4 in. wide.?

84. How much money will be required to purchase £8,620 10*s.* 5*d.* stock in the 3 per cents., standing at $87\frac{1}{8}$, and what income will the purchaser receive?

85. A, B, and C can do a piece of work in 20 days; A and B can do it in 40 days, and B and C in 30 days; in how many days can each do it alone?

86. A cistern can be filled by means of a tap in 45 mins., and emptied by a plug in 30 mins.; supposing the cistern to be full and both tap and plug to be opened, at what time will the cistern be emptied?

87. Find the compound interest on £100 for 3 years at 100 per cent. per annum; how many years would it take for the simple interest to amount to the same sum at the same rate?

88. Two men, A and B, are walking in the same direction; A is 18 *miles ahead of* B, but B travels 7 miles whilst A travels 4; how many

miles from the place of starting will B have walked before he overtakes A?

89. The sum of 11*s.* $1\frac{1}{4}$*d.* was divided among 4 people in the ratios of $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$; what was the share of each?

90. Simplify $(\frac{5}{11}+\frac{3}{7})\times(\frac{8}{13}+\frac{1}{9})\times(\frac{3}{5}+\frac{12}{19})\times(1\frac{3}{17}+\frac{1}{3})$.

91. By selling an article at 4*s.* a person loses 7 per cent.; what must he sell it at to gain 5 per cent.?

92. A rate of $5\frac{1}{4}$*d.* in the £ was levied in a parish where the ratable value was £50,967; what was the amount of the rate?

93. A tank has 3 taps; the first and second will empty it in 15 hours; the second and third in 10 hours; and the first and third in 12 hours; how long will they be emptying the tank when all are working together?

94. At what price must I buy into the 3 per cents. to receive interest at the rate of $3\frac{1}{4}$ per cent.?

95. If 1 lb. of tea is worth 50 oranges, and 70 oranges are worth 84 lemons, what does tea cost per lb. if a lemon costs a penny?

96. The amount of a certain principal at a certain rate per cent. is £750, and the interest is $\frac{1}{4}$ of the principal; what is the principal and what is the rate per cent.?

97. If 4 men in $7\frac{2}{3}$ days earn £11 10*s.*, how much will 7 men earn in $\frac{4}{19}$ of a day?

98. What would be the difference of income produced by £1,000 invested in the 4 per cents. at 98, and in the $3\frac{1}{2}$ per cents. at 94?

99. If 4 men *or* 6 women can do a piece of work in 20 days, how long will 3 men *and* 5 women take to do it in?

100. Find the value of £$4\frac{1}{6}+11\frac{1}{3}$*s.*$+7\frac{2}{12}$*d.*, and reduce the result to the decimal of 3*s.* $9\frac{3}{4}$*d.*

101. A and B start at a distance of 64 miles from each other. A walks $3\frac{1}{2}$ miles in the hour, B $2\frac{1}{2}$; if they start at the same time, how many miles will each walk before they meet?

102. A wine merchant bought 56 gallons of brandy at 22*s.* 6*d.* per gallon, but 7 gallons were lost; at what price per gallon must he sell the remainder to gain 15 per cent. on the whole?

103. A gentleman left by will to A £1,000, to B £950, to C £800, to D £750. His estate produced only £2,900; how much will each legatee receive?

104. Find the value of £·6+·3125*s.*+·2 guineas.

105. In what time will £355 15*s.* produce £56 18*s.* $4\frac{3}{4}$*d.* of interest at the rate of $4\frac{1}{2}$ per cent. per annum?

106. How many yards of paper 2 ft. 3 in. wide are required to paper a room 24 ft. long, 18 ft. 9 in. wide, and 14 ft. 2 in. high?

107. If a person accepts a present payment of £172 11*s.* 8*d.* instead of £174 14*s.* $9\frac{3}{4}$*d.* due 3 months hence, what rate of interest does he allow?

108. The length of a room is 15 ft., the breadth 10 ft., and the height 9 ft. 9 in.; find the expense of painting the walls and ceiling at 1*s.* 9*d.* per square yard.

109. If a brick measures 8 in. by 4 in. by $2\frac{1}{2}$ in., how many are there in a stack 24 ft. long, 6 ft. wide, and 5 ft. high?

110. Reduce $\cdot 53\dot{5}71428$ and $1\cdot 2\dot{1}42857$ to vulgar fractions in lowest terms.

111. When a pound sterling is worth on the exchange 25·05 francs, a man sells 45 pieces of 20 francs each at 16*s.* 2*d.* each; how much does he gain?

112. Find the compound interest on £300 for 3 years at $2\frac{2}{3}$ per cent.

113. Find the value of $2\frac{5}{6\frac{2}{3}} + (\frac{1}{7} \times \frac{4}{5}) - 2\frac{1}{2}$ of $(\frac{2}{3} \div \frac{7}{9})$.

114. A had a sum of money, of which he paid away $\frac{1}{3}$, then $\frac{1}{2}$ of the remainder, then $\frac{2}{5}$ of what was still left, and found that he had 1*s.* less than $\frac{1}{8}$ of $\frac{3}{4}$ of the whole; what sum had he at first?

115. A room is 27 ft. 6 in. long, and 19 ft. 9 in. broad; what length of carpet $\frac{3}{4}$ yd. broad would be required to cover the room, and what would be the cost at 4*s.* 3*d.* a yard?

116. What would be the difference of income produced by investing in the 3 per cents. at $85\frac{1}{2}$ the sum of £1,060, previously lent on a mortgage at $4\frac{1}{2}$ per cent.?

117. Reduce $3\cdot 71\dot{2}4\dot{3}$ to a vulgar fraction in its lowest terms.

118. Divide ·02223758 by ·00001927.

119. What is the largest sum of money contained exactly in both £25 10*s.* 6*d.* and £42 7*s.*?

120. A tradesman's prices are 20 per cent. above cost price. If he allows a customer 5 per cent. discount on his bill, what profit does he make?

121. If when the income tax is 7*d.* in the £ a man has £291 5*s.* left after paying the tax, how much will he have when the tax is reduced to 5*d.*?

122. Reduce $\cfrac{1}{2+\cfrac{1}{3+\frac{1}{4}}}$ to a decimal.

123. Find the discount on £972 due 10 months hence at $5\frac{1}{4}$ per cent. per annum.

124. Simplify $\dfrac{(\frac{4}{9} \times \frac{3}{5}) + (\frac{3}{5} \times \frac{3}{7}) + (\frac{3}{7} \times \frac{2}{7})}{(\frac{4}{9} \times \frac{3}{5}) + (\frac{4}{5} \times \frac{4}{9}) + (\frac{4}{5} \times \frac{2}{7})}$

125. What decimal of a week is $\frac{1}{20}$ of 3 days 5 hours?

126. By selling a horse for £82 5*s.* I lost $5\frac{1}{2}$ per cent.; what should I have gained per cent. by selling for £94?

127. How much land worth 17*s.* 6*d.* an acre must be added to a farm *of 51 acs.* 2 rds. 20 pls. worth £1 14*s.* 6*d.* an acre, that the average value *of the whole* may be £1 2*s.* 9*d.* an acre?

128. A can do a piece of work in 12 hours, B in 4 hours, and C in 3 hours. A, B, and C all work together for half an hour, when A leaves off; how long will it take B and C to finish the work?

CHAPTER XI.

ROOTS.

SECTION I.—SQUARE ROOT.

When a factor is multiplied by itself any number of times the *product* is said to be a *power* of that factor, and the *factor* is said to be a *root* of the power.

Thus:—

(1) $5 \times 5 = 25 =$ 2nd *power* of 5; and 5 is the 2nd *root* of 25.

(2) $5 \times 5 \times 5 = 125 =$ 3rd *power* of 5; and 5 is the 3rd *root* of 125.

(3) $5 \times 5 \times 5 \times 5 = 625 =$ 4th *power* of 5; and 5 is the 4th *root* of 625.

The 2nd power is called the *square*, and usually written thus 5^2.

The 2nd root is called the *Square Root*, and is usually written $\sqrt[2]{25}$, or $\sqrt{25}$.

The 3rd power is called the *cube*, and is usually written 5^3.

The 3rd root is called the *Cube Root*, and is usually written $\sqrt[3]{125}$.

The 4th power is written 5^4, and the 4th root $\sqrt[4]{625}$.

The small figure is called the *index*, because it points to the particular power or root required.

Many problems in arithmetic and mensuration necessitate the *extraction* of the square and cube roots. As the proper understanding of the processes is based on algebraic principles, we prefer in this place to give the methods only.

Ex. 1. Find the square root of 54756.

$$
\begin{array}{rl}
 & 5\dot{4}7\dot{5}\dot{6}(234 \\
 & 4 \\
43) & 147 \\
 & 129 \\
464) & 1856 \\
 & 1856 \\
 & \cdots\cdots
\end{array}
$$

(1) Mark off the given number into periods of two digits each, beginning at the units.

(2) Find the nearest square root [2] of the 1st period on the left hand

[5], and set the root to the right of the given number, in the same way as a quotient number in division.

(3) Place the square [4] under the first period [5] and subtract. To the remainder [1] bring down the next period [47], making a dividend of 147.

(4) Double the root [2] above mentioned [=4], and find how many times it is contained in the new dividend, omitting the last figure (4 into 13 goes 3 times). Set this result [3] to the right both of the part of the root already found [making 23] and of the divisor [making 43].

(5) Multiply the augmented divisor by the last quotient figure [=129] and subtract from the dividend.

(6) To the remainder [18] bring down the next period [56] for a new dividend. Double the part of the root already obtained [making 46] and use this as a trial divisor for the new dividend omitting the last figure [46 goes into 185 four times]. Set this result [4] to the right both of the root already obtained and of the divisor.

7. Multiply the augmented divisor [464] by the last quotient figure [=1856] and subtract from the dividend.

Ex. 2. Find the square root of 38427601.

$$
\begin{array}{rl}
 & 3\dot{8}4\dot{2}76\dot{0}1\,|\,6199 \\
 & 36 \\
\hline
121) & 242 \\
 & 121 \\
\hline
1229) & 12176 \\
 & 11061 \\
\hline
12389) & 111501 \\
 & 111501 \\
\hline
 & \cdots\cdots \\
\hline\hline
\end{array}
$$

Note that in this example the first divisor 12 goes into the first dividend (omitting the last figure) *twice*. But 124×2 gives a product [248] greater than the dividend [242]. Hence 2 is too large for the second figure in the root, and we substitute *one less*.

Note also that the remainder may be as large or even larger than the subtrahend.

Ex. 3. Find the square root of 9054081.

$$
\begin{array}{rl}
 & \dot{9}0\dot{5}4\dot{0}8\dot{1}\,|\,3009 \\
 & 9 \\
\hline
6009) & 054081 \\
 & 54081 \\
\hline
 & \cdots\cdots \\
\hline
\end{array}
$$

Note the position and effect of the ciphers.

Ex. 4. Find the square root of 591·4624.

$$
\begin{array}{r}
\dot{5}9\dot{1}\cdot4\dot{6}2\dot{4}(24\cdot32 \\
4 \\
\hline
44)191 \\
176 \\
\hline
483)\ 1546 \\
1449 \\
\hline
4862)\quad 9724 \\
9724 \\
\hline
\cdots\cdot \\
\hline\hline
\end{array}
$$

NOTE.—If the number of decimal places is odd make them even by adding a cipher, and point off into periods of two digits each, commencing as before with the units.

Ex. 5. Find the square root of $\frac{169}{289}$.

To find the square root of a fraction we extract the square root of the numerator and denominator separately—

$$\sqrt{169}=13 \text{ and } \sqrt{289}=17.$$

$$\therefore \quad \sqrt{\tfrac{169}{289}}=\tfrac{13}{17}.$$

NOTE.—If the numerator and denominator are not exact squares it is best to reduce the fraction to a decimal, and then find the square root.

Ex. 6. Find the square root of $3\frac{241}{737}$.

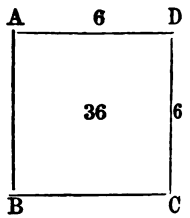
$$3\tfrac{241}{737}=\tfrac{2452}{737}=3\cdot327000131.$$

$$
\begin{array}{r}
3\cdot32\dot{7}000131(1\cdot82403 \\
1 \\
\hline
28)232 \\
224 \\
\hline
362)\ 870 \\
724 \\
\hline
3644)14601 \\
14576 \\
\hline
364803)\ 1253100 \\
1094409 \\
\hline
158691 \\
\hline\hline
\end{array}
$$

Comparatively speaking very few numbers are perfect squares, but by adding decimal places we can find the square root to any assigned degree of accuracy.

Ex. 7. The area of a square field is $62\frac{1}{2}$ acres ; find the length of the side.

Let A, B, C, D represent a square 6 in. in the side. Then the area $= 6 \times 6 = 36$, and the length of the side $= \sqrt{36} = 6$.



In the same way, to find the length of the side of the square field, reduce the acres to square yards, and extract the square root of the number.

1 acre $= 4840$ sq. yds.

$\therefore$ $62\frac{1}{2}$ acres $= 302500$ sq. yds.

And $\sqrt{302500} = 550$

Hence length of side of field $= 550$ yards.

To find a mean proportional to any two numbers, extract the square root of the product of the given numbers.

Ex. 8. Find a mean proportional to the two numbers 3 and 27.

$3 \times 27 = 91$ and $\sqrt{91} = 9$

Hence $3 : 9 :: 9 : 27$.

To find a third proportional to any two numbers, square the second number and divide by the first.

Ex. 9. Find a third proportional to the two numbers 8 and 12.

$12^2 = 144$, and $\frac{144}{8} = 18$, viz. the third proportional.

Ex. 10. A house is 45 feet high, and the nearest point on which the foot of a ladder can be placed is 16 feet from the wall of the house ; find the length of the ladder which will exactly reach to the top of the house.

In the figure the angle at B is a right angle

And the sq. on A C = sq. on A B + sq. on B C*

and substituting the numbers

the sq. on A C $= 45^2 + 16^2$

$= 2025 + 256$

And the side A C $= \sqrt{2281} = 47{\cdot}7$ feet.

NOTE.—The 4th root is found by extracting the square root of the square root.

EXERCISES.

1. Find the square root of the following :—

(1) 961	(4) 94249
(2) 1600	(5) 160801
(3) 4225	(6) 277729

* See Euclid, Book I. Prob. 47.

2. Find the square root of—

(1) 57600	(4) 492804
(2) 419904	(5) 51969681
(3) 133225	(6) 57426084

3. Find the square root of—

(1) 210681	(4) 2570591401
(2) 94249	(5) 3915380329
(3) 40960000	(6) 182493081

4. Find the square root of—

(1) $62\frac{33}{67}$	(4) 15661·521316
(2) $83\frac{123}{529}$	(5) ·0019140625
(3) $35\frac{142}{343}$	(6) $1383\frac{21}{25}$

5. Find the square root of ·0529, ·00367236, and ·008836.

6. What is the square root of 2? of 3? of 5?

7. Extract the square root of $1040\frac{1}{16}$, and of 50·013184.

8. Find the square root of 8, and of $17\frac{3}{8}$ to 4 places of decimals.

9. Find the square root of ·6775609, and of 1·00060009.

10. Extract the square root of 1·7 and 1·9, correct to four places of decimals.

11. Find the 4th root of 2401, and of 22801.

12. A floor is 24 feet long and 15 feet wide; what is the distance between the opposite corners?

13. How many rods of fence will enclose 10 acres in the form of a square?

14. A room is 5 yards from one corner to the opposite, and 4 yards long; find the width.

15. Find the square root of 6005320036, and of $\frac{3757}{65533}$.

16. Extract the square root of 191810·713444 to 3 places of decimals.

17. A square deer-park, including 1,600 acres of land and a lake of 960 acres is to be enclosed by a wall; what will be its length?

18. Find the length of the side of a square having an area equal to the rectangle whose sides are 513 yds. 1 ft. 11 ins. and 1,628 yds. 11 ins.

19. A room is 20 ft. long 16 ft. wide and 12 ft. high; what is the distance from one of the lower corners to the opposite upper corner?

20. Find the mean proportional between 4 and 16, and a third proportional to $1\frac{3}{4}$ and $3\frac{1}{2}$.

21. Find the mean proportional between 2012 and 12575, and also between 6345 and 11280.

22. Find a third proportional to 1331 and 968.

23. Two columns 160 ft. and 85 ft. high respectively are placed 30 yards apart; find the distance from the top of the one to the top of the other. *(Correct to 4 places.)*

24. Two boys start from the same point; one walks 96 furlongs due north, and the other 72 rods due east; how far are they apart?

Section II.—Cube Root.

The process for the extraction of the cube root is based on algebraic principles. As in square root, we give the rule only, and illustrate by examples.

Ex. 1. Find the cube root of 15625.

$$
\begin{array}{rl|l}
 & & \dot{1}5\dot{6}2\dot{5}(25 \\
 & & 8 \\
\hline
\text{Trial Divisor } 3\times 20^2 = & 1200 & 7625 \\
3\times 20\times 5 = & 300 & \\
5^2 = & 25 & \\
\hline
\text{Real Divisor} & 1525 & 7625
\end{array}
$$

(1) Mark off the given number into periods of 3 digits each, beginning at the units end.

(2) Find the nearest cube root [2] of the first period [15]. Set down the figure 2 as the first figure in the required root. Cube it [8] and subtract from the first period. Bring down the next period [625] to the right of the remainder [making 7625] as a dividend.

(3) Take the figure in the quotient, consider it as tens, square, and multiply by 3. Make this number the trial divisor. The quotient [5] will be the 2nd figure of the root.

(4) For the real divisor, take 3 times the square of the 1st part [considered as tens] + 3 times the product of the first [considered as tens], into the second + the square of the second. Multiply this sum by the second figure of the root, and subtract the result.

If more periods remain to be brought down, treat the 2 figures in the root already found exactly in the same way as the 1st figure was treated, and complete the remaining part of the process exactly as before.

Ex. 2. Find the cube root of 61162984.

$$
\begin{array}{rl|l}
 & & 6\dot{1}1\dot{6}2\dot{9}8\dot{4}(394 \\
 & & 27 \\
\hline
3\times 30^2 = & 2700 & 34162 \\
3\times 30\times 9 = & 810 & \\
9^2 = & 81 & \\
\hline
 & 3591\times 9 = & 32319 \\
\hline
3\times 390^2 = & 456300 & 1843984 \\
3\times 390\times 4 = & 4680 & \\
4^2 = & 16 & \\
\hline
 & 460996\times 4 = & 1843984
\end{array}
$$

Ex. 3. Extract the cube root of 27·379766744.

$$
\begin{array}{rl|l}
 & & 27\cdot\dot{3}7\dot{9}76\dot{6}74\dot{4}(3\cdot014 \\
 & & 27 \\
 & & \hline
 & & 379766 \\
3\times300^2= & 270000 & \\
3\times300\times1= & 900 & \\
1^2= & 1 & \\
 & \overline{270901} & \hline
 & & 270901 \\
 & & \hline
 & & 108865744 \\
3\times3010^2= & 27180300 & \\
3\times3010\times4= & 36120 & \\
4^2= & 16 & \\
 & \overline{27216436\times4=} & 108865744 \\
\end{array}
$$

NOTE.—The first trial divisor $=3\times30^2=2700$, a larger number than the period 379 brought down; hence the second figure of the root is 0.

Ex. 4. Find the cube root of $3\frac{1409}{2197}$.

$3\frac{1409}{2197}=\frac{8000}{2197}$ and cube root of $8000=20$

and cube root of $2197=13$

$\therefore \sqrt[3]{\frac{8000}{2197}}=\frac{20}{13}=1\frac{7}{13}$ *Ans.*

NOTE 1.—If the numerator and denominator are not perfect cubes, reduce the fraction to a decimal, and then find cube root.

NOTE 2.—The *sixth* root is found by extracting the square root, and then the cube of this square root.

The *eighth* root is found by extracting the square root 3 times.

The *ninth* root is found by the taking the cube root of the cube root.

Ex. 5. A cubical block of granite contains $3\frac{3}{8}$ cubic feet; find the length of one side.

$$3\frac{3}{8}=\frac{27}{8}$$

and $\sqrt[3]{\frac{27}{8}}=\frac{3}{2}=1$ ft. 6 in.

EXERCISES.

1. Find the cube root of—

(1) 13824
(2) 42875
(3) 274625
(4) 185193
(5) 405224
(6) 912673

2. Find the cube root of—

(1) 262144 (4) 128024064
(2) 48228544 (5) 7189057
(3) 2460375 (6) 1194389981

3. Find the cube root of the following numbers, correct to 3 places of decimals—

(1) 15·625 (4) 12·3456
(2) ·074256 (5) 31
(3) 56·47 (6) ·00037

4. Find the cube root of $11\frac{25}{64}$ and $\frac{1331}{4913}$.

5. Find the cube root of 515849608.

6. Extract the cube root of 27054036008.

7. A cubical box contains 19683 cubic inches; what is the length of its edge?

8. A block of wood in the form of a cube contains 41063·625 cubic inches; what is the length of its edge?

9. Find the sixth root of 117649, and the eighth root of 390625.

10. Find the cube root of ·1, ·2, and ·3; correct to 3 places.

11. Find the cube root of ·0047, ·0001, and ·00000001; correct to 3 places.

12. Find the cube root of $\cdot\dot{2}9\dot{6}$.

CHAPTER XII.

MISCELLANEOUS EXERCISES.

1. If a certain number be multiplied by 22, and 64 be subtracted from the product, and the remainder be divided by 4, the quotient number will be 50; what is the number?

2. Express £7 3*s.* 6½*d.* as the decimal of £1.

3. Divide 240 lbs. into quantities having the ratio of 1, 2, 3, 4.

4. Find the discount per cent. when 19*s.* 10½*d.* is taken from £39 15*s.* in consideration of cash.

5. Convert $\cdot\dot{3}07692\dot{}$ and $\cdot\dot{3}82\dot{1}42857\dot{}$ to their equivalent vulgar fractions.

6. Find the present worth of £1,496 due in 3 years allowing simple interest at 4½ per cent.

7. If by buying at 15*s.* 6*d.* a yard, and selling at a gain of 15 per cent. *on the outlay,* I realised a profit of £100, how many yards did I buy?

8. A bill of £95 18*s.* 7*d.* was paid by an equal number of sovereigns, half-sovereigns, crowns, shillings, sixpences, threepenny pieces, pence, halfpence, and farthings; how many were there of each?

9. Find the value of $\cdot 41\dot{6}$ of a lb.

10. Divide $4\cdot 1\dot{9}\dot{4}$ by $\cdot 2\dot{7}$.

11. When last before noon are the hands of a clock together?

12. At what rate per cent. simple interest will £300 amount to £373 10*s.* in 7 years?

13. Reduce seven inches and two-thirds to the decimal of a mile.

14. Divide £333 6*s.* 8*d.* among A, B, and C, so that B may have $1\frac{3}{8}$ of A's share, and C as much as A and B together.

15. If a train which runs at the rate of 45 miles an hour leaves London 1 hour and 20 minutes after another train whose rate is 30 miles an hour and overtakes it at Bristol; what is the distance between London and Bristol?

16. Reduce $3\cdot 74\dot{2}4\dot{9}$ to a vulgar, and $\frac{2}{7}$ to a decimal fraction.

17. A cistern holds $243\frac{3}{4}$ cubic feet of water; its length is 11 ft. 3 ins., and depth 3 ft. 4 in.; find its width.

18. Arrange $\frac{19}{21}$, $\frac{21}{25}$, $\frac{11}{13}$, $\frac{29}{27}$ in order of magnitude.

19. Find the number of yards in the side of a square park containing 109 acs. 3 rds. 8 pers. 9 sq. yds.

20. If by selling at 17*s.* 6*d.* per yard I gain 12 per cent. on the outlay, what do I gain or lose per cent. by selling at 16*s.* 3*d.* per yard?

21. Find the value of $\cdot 34\dot{0}\dot{9} \times 81\dot{4} \div 3748\dot{1}$.

22. A railway cutting for a quarter of a mile averages 28 ft. wide and 17 ft. deep; find the number of cubic yards of earth taken out.

23. Find the cost of lining 6 boxes, each 5 in. long 4 in. wide, and 2 in. deep with gold leaf at $1\frac{1}{2}d.$ a square inch.

24. What is the present value of £1,512 7*s.* 6*d.* due in 4 years at $4\frac{1}{2}$ per cent. per annum simple interest?

25. Water expands when freezing so that a cubic foot of water becomes 1·089 of ice; find how many cubic feet of water there are in an iceberg estimated to be 900 ft. long, 88 ft. broad, and 220 ft. high?

26. A can do a piece of work in 12 hours; A and B can do the same in 9 hours; how long will B take to do it by himself?

27. A cistern is 18 ft. long and 16 ft. broad; how deep must it be to hold as much as another cistern 20 ft. long, 15 ft. broad, and 27 ft. deep?

28. What is the smallest sum of money with which I can purchase either pigs at 16*s.* each, lambs at £2 each, calves at £5 each, or heifers at £8 5*s.*?

29. Simplify $\dfrac{9\frac{5}{8} \text{ of } \frac{2}{11} + 6\frac{3}{5} \text{ of } \frac{2}{10}}{6\frac{2}{10} \text{ of } \frac{5}{8} \quad 4\frac{1}{3} \text{ of } 2\frac{10}{11}}$

30. Received to-day (Sept. 29) £456 10*s.* for a debt of £490 14*s.* 9*d.* discount at 5 per cent. per annum; when should the debt have been payable without discount?

31. A steamer sails a mile down stream in 5 minutes, and a mile up stream in 7 minutes; how far down stream and back can it go in one hour?

32. How many bricks will it take to build a wall 10 yds. long, 6 ft. high, and 18 in. thick, each brick being 8 in. long, 4 in. wide, and $2\frac{1}{2}$ in. thick?

33. Find the depth of a reservoir containing 1,089,000 gallons of water, and covering an acre of ground. (1 acre = 4,840 square yards; 1 cubic foot of water weighs 1,000 ounces; and 1 gallon of water weighs 10 lbs.)

34. A carries on a business for 3 months, when B joins him. At the end of 6 months more they have made a profit of £115. B's original capital was $\frac{4}{5}$ of A's, and A's capital amounted to $2\frac{1}{2}\frac{1}{3}$ of the profits; how ought the profits to be divided?

35. The difference between the simple and compound interest of a certain sum for 3 years at 5 per cent. per annum is £1; find the sum.

36. A hare starts 50 leaps before a dog, and leaps 4 times whilst the dog leaps 3 times; but 2 of the dog's leaps equal 4 of the hares; how many leaps must the dog take to catch the hare?

37. A regiment lost 8 per cent. of its men in a battle, and 25 per cent. of those that remained died from sickness, and it then mustered 621 men; how many men were in the regiment at first?

38. What is the cost of papering a room 16 ft. 7 in. long, 13 ft. 5 in. broad, and 10 ft. 6 in. high, with paper 1 ft. 8 in. wide at 10*d.* per yard?

39. Find the present worth of £540 due 7 months hence at $3\frac{1}{2}$ per cent. simple interest.

40. A and B together do a piece of work in 11 days; A and C can do the same piece of work in 5 days; and B and C can do it in 8 days; how long would it take each working alone!

41. I sold goods at 25 per cent. gain, and, investing the proceeds in goods, sold at 20 per cent. loss; did I gain or lose, and how much?

42. How many cubical blocks 4 in. on the edge are equivalent to a block of wood 8 ft. long 4 ft. wide and 2 ft. thick?

43. How many iron balls 2 in. in diameter will weigh as much as an iron ball 8 in. in diameter? [The volume of a sphere = the cube of the radius × 4·18878, or the cube of diameter × ·52359].

44. There are two rectangular fields equal in area. The one is a third of a mile long and 990 ft. broad; the other is a square; find the length of a side of the square.

45. If by selling at 17*s.* a yard I lose 20 per cent. on the outlay, what do I gain or lose per cent. by selling at £1 a yard?

46. A and B together possess £120, A and C £140, and A, B, and C together £170; what does each possess?

47. Find the square root of 102·030201, and the cube root of 405·224.

48. If eggs be bought at 5 for 4*d.*, at what price must they be sold to gain 20 per cent.?

49. A man buys land at £42, and having built upon it, sells the land and houses so as to gain 20 per cent. on his whole outlay and receives £9,004 16*s.*; how much did he spend in building?

50. Simplify $\frac{\frac{2}{9}-\frac{2}{7}+\frac{1}{8}-\frac{1}{6}+\frac{1}{5}}{\frac{2}{3}-\frac{1}{4}+\frac{3}{5}-\frac{1}{7}}$

51. The depth of a cistern whose base contains 1,344 square inches is 3 ft. 9 in.; find the depth of the same quantity of water in another cistern whose base contains 1,088 square inches.

52. A takes 10 days to do what B can do in 7 days; but B is paid per day $\frac{1}{3}$ as much again as A. If A takes 25*s.* for a certain work, what will B take for the same?

53. Multiply $1\cdot\dot{0}47619\dot{}$ by $\cdot\dot{4}\dot{2}$, and find the square root of the result.

54. Find the value of $\frac{1\cdot\frac{3}{7}\times 1\frac{1}{4}}{4\frac{1}{2}\times 8\frac{1}{3}}$ of £22 3*s.* 2$\frac{1}{4}$*d.*

55. If the cost of building a wall 135 yds. 2 ft. long and 1 ft. 9 in. thick be £17 16*s.* 1$\frac{1}{2}$*d.*, what will be the cost of building a wall of the same height 86 yds. 1 ft. long and 1 ft. 10 in. thick?

56. Find the value of 100,000 rupees in francs, 1 rupee being reckoned as worth 1*s.* 10$\frac{1}{2}$*d.*, and 25 francs as £1.

57. Divide £900 between A, B, and C, giving to A twice as much as to B, and to C half as much as to A and B together.

58. A sold cloth to B, and gained 10 per cent.; B sold on to C, and gained 10 per cent.; C sold on to D for £726, and gained 10 per cent.; what did A give for it?

59. Simplify $\frac{1\frac{5}{17}-\frac{5}{7}}{1\frac{2}{5}+\frac{2}{19}}\div\frac{11\frac{9}{11}-\frac{2}{13}}{1\frac{1}{3}+\frac{9}{23}}$

60. If I lend a sum of money to a friend at 4$\frac{3}{4}$ per cent. simple interest, and he return me £877 12*s.* 6*d.* at the end of 4 years, what is the amount of the loan?

61. Find the G. C. M. of £12,148 10*s.* 7*d.* and £17,410 6*s.* 3*d.*

62. Find the whole surface of a room 22 ft. 5 in. long, 18 ft. 4 in. broad, and 10 ft. 9 in. high.

63. A tradesman professes to retail goods at 10 per cent. profit, but adulterates them by adding $\frac{1}{4}$ of an inferior article which cost $\frac{4}{5}$ of the price of the better; how much per cent. profit does he make?

64. A room is half as high and twice as long as it is wide. If the floor contains 3,362 square feet, what is the cost of papering the walls at 9*d.* per square yard?

65. If by selling at 17*s.* a yard I lose 15 per cent. on the outlay, at what rate must I sell to gain 15 per cent.?

66. Simplify $\frac{14\frac{1}{4}-6\frac{1}{2}}{3\frac{1}{3}+7\frac{1}{8}} \div \frac{4\frac{1}{12}+6\frac{1}{8}}{9\frac{1}{8}-3\frac{1}{4}} + \left(30\frac{7}{18}-22\frac{3}{8}\right)$

67. Find the side of a square field measuring $8\frac{1}{10}$ acres.

68. If £1,000 be invested in the 3 per cents. at $92\frac{1}{2}$, what annual income will be derived from the investment, allowing $\frac{1}{8}$ per cent. for brokerage, and deducting 3*d.* in the £ for income tax?

69. If 12 masons can build a wall 400 ft. long, 12 ft. high, and 18 in. thick in $13\frac{3}{4}$ days of $10\frac{1}{2}$ hours each, how long will it take 5 masons to build a wall 250 ft. long, 18 ft. high, and $22\frac{1}{2}$ in. thick, working $12\frac{1}{2}$ hours a day?

70. 4,246 yards are bought for £337 17*s.* 6*d.*; at what price must they be sold per yard so as to clear a profit of $23\frac{1}{2}$ per cent.?

71. Extract the 4th root of 1·04060401, and the cube root of 731·457055300301.

72. Bought $176\frac{1}{4}$ cwt. for £246 8*s.* Allow 12 per cent. for waste; at how much a lb. must the remainder be sold to give $17\frac{1}{2}$ per cent. profit on the outlay?

73. An article of commerce passes successively through the hands of 3 dealers, each of whom, in selling, adds as his profit 10 per cent. of the price at which he bought it; if, under these circumstances, goods are sold to the 3rd dealer for £33 5*s.* 6*d.*, what did the first dealer pay for them?

74. If £1 sterling = 10 florins = 100 cents. = 1,000 milrees, express £2 11*s.* 11*d.* in each coinage.

75. What must be the price of consols in order that, after deducting income tax of 5*d.* in the £, there may remain a clear return of $3\frac{1}{4}$ per cent. on the cost?

76. Two men, A and B, working together, can do a piece of work in 10 days; but if after 4 days A stops working, B can finish the work in 14 days more, compare their rates of working.

77. There are two equal glasses, both full of mixtures of spirit and water, the one contains 1 of spirit to 10 of water, and the other 1 of spirit to 8 of water; when the two are poured together into a tumbler, what is the strength of the mixture?

78. Extract the cube root of 37·41 to 3 decimals.

79. A commission agent offers to take either $2\frac{1}{4}$ per cent. on the money passing through his hands, or $\frac{1}{3}$ of the profits of his sales. The sales amount to £300,000, of which 35 per cent. produce no profit, but the remainder realise a profit of $10\frac{1}{2}$ per cent.; what difference will it make to the agent which of his offers his customers accept?

80. A man owes £300 due in 5 months, £800 due in 3 months, and £200 due in 8 months; if he pays $\frac{1}{2}$ of the whole in 2 months, when ought the other half to be paid?

81. The French 5 per cents. being at $105\frac{3}{8}$, and the English 3 per cents. at $96\frac{1}{8}$; find the loss of income caused by transferring £5,000 French stock into English stock, and the value of the stock in English money.

82. Two watermen rowing, with the tide flowing at the rate of $2\frac{1}{4}$ miles an hour, go 20 miles in $3\frac{1}{3}$ hours; in what time would they row this distance against the ebb of the same tide?

83. A tradesman buys 500 yards of cloth at 9*s.* $7\frac{5}{8}$*d.* per yard, but has £44 0*s.* $2\frac{1}{2}$*d.* thrown off for cash; at what price per yard can he afford to sell so as to gain $33\frac{1}{3}$ per cent.?

84. Required the length of the edge of a cube of metal worth £10,481 1*s.* 4*d.* at 10*s.* 5*d.* per cubic inch.

85. A, who travels $3\frac{1}{2}$ miles an hour, starts $2\frac{1}{2}$ hours before B, who goes the same road $4\frac{1}{2}$ miles an hour; when will B overtake A?

86. A steamer is $14\frac{3}{4}$ days on a voyage. For the first 3 days she makes 13 miles an hour; in the next $6\frac{1}{2}$ days she improves on that speed 17 per cent.; and for the remaining time she runs 390 miles in the 24 hours; what distance has she gone, and what was her average speed per hour?

87. £12,540 has to be divided between A, B, and C, so that A shall receive $\frac{3}{7}$ of A's and B's shares, and B shall receive $\frac{2}{5}$ of A's and C's; find the share of each.

88. If 70 yards of carpet 2 ft. 6 in. wide be bought at 3*s.* 6*d.* a yard, to cover a room 24 ft. 9 in. long and 16 ft. 8 in. wide, and the remnant sold at 2*s.* 6*d.* a yard, find how much the carpet for the room costs.

89. The section of a telegraph cable is $\frac{1}{2}$ an inch in diameter, the central wire is $\frac{1}{8}$ of an inch in diameter, and there are 4 protecting wire $\frac{1}{16}$ inch section embedded in the hempen cable; what percentage of the cable consists of metal, and what of hemp?*

90. I have sold 50 bushels of wheat for A, and 60 bushels for B, receiving £30 for both lots; if A's wheat is worth 20 per cent. more than B's, how much ought I to pay to each?

91. How many cubes whose edges are $\frac{3}{4}$ in. long can be contained in a box, of which the base is 18 square inches and height $7\frac{1}{2}$ inches?

92. What sum must be laid out in the 3 per cents. at 92 in order to produce an income of £300 a year after income tax of 10*d.* in the £ has been deducted?

93. A cistern is filled by 2 pipes, one of which will fill it in 2 hours and the other in 3 hours; it is emptied by 3 pipes, the first of which

* In a circle (1) circumference = diameter × 3·14159; (2) area = $\frac{\text{diameter}}{2} \times \frac{\text{circumference}}{2}$

or area = square of diameter × ·7854
or area = square of radius × 3·14159
or area = square of circumference × ·07958.

will empty it in 5 hours, the second in 6 hours, and the third in $7\frac{1}{2}$ hours; when the cistern is empty, and all the pipes opened, in what time will it be filled?

94. Find the sixth root of 1061520150601.

95. Find the weight of an inch rainfall over $\frac{1}{2}$ an acre of ground. [A cube foot of water weighs 1,000 ozs.]

96. 102,900 bricks measuring 9 in. by 4 by $2\frac{1}{2}$ are stacked in the form of a cube; find the length of the side.

97. Simplify $\frac{\cdot 75 \times \cdot 58\dot{3}}{1\cdot 3571428} \times \frac{2\cdot 8}{\cdot 35} \times 2\cdot 7\dot{1}4285$.

98. It is found that if a number of beans be divided into lots of 63, of 84, or of 14, there is always the same remainder, 11; the number is known to be between 700 and 800; how many beans are there?

99. What sum will be left out of the amount realised by £1,000 of stock standing at $91\frac{1}{10}$, if enough be set aside to clear £43 12*s.* in $1\frac{3}{8}$ years, at $3\frac{1}{2}$ per cent. per annum simple interest?

100. A and B formed a partnership for one year; A put in £2,000, and B £800; how much more must B put in at the close of 6 months to receive one-half of the profits?

101. A gentleman, after paying 9*d.* in the £ income tax on his whole income, and other taxes at 10*d.* in the £ on the remainder, has £5,180 3*s.* 6*d.* left; find his gross income.

102. Find the whole surface of a rectangular block 3·27 ft. long, 2·58 ft. broad, and ·13 ft. thick.

103. A sum of £250 17*s.* 6*d.* is transmitted through Paris to New York. £1 = 24 francs 79 cents French; and 6 francs 20 cents French = 2 dollars 25 cents American; what is the value of the sum in American currency?

104. A man made his will that the *first four* legatees are to be paid in full before anything is given to the others. He bequeaths to A £3,750, to B £3,525, to C £3,280, to D £2,000, to E £1,895, to F £1,750, to G £1,668, to H £1,745, to K £1,475, and to L £1,067. The whole estate turns out to be worth £19,575; how should the money be distributed?

105. How many gallons of water will a circular cistern contain which is 6 ft. deep and 4 ft. in diameter. [1 gallon of water = 277·274 cubic inches.]

106. In a mile race A gives B 50 yards, B passes the winning post 5 minutes after the start, and A passes it 5 seconds later; which would win in an even race, and by what distance?

107. The weights of equal volumes of aluminium and silver are as $2\frac{3}{4}$ *to* $10\frac{1}{2}$, and the values of equal weights are as $3\frac{1}{2}$ to $6\frac{1}{2}$; compare the *value of* equal volumes.

108. Two trees stand on opposite sides of a stream 40 ft. wide, the height of one tree is to the width of the stream as 8 to 4, and the width of the stream is to the height of the other tree as 4 to 5; what is the distance between their tops?

109. A's share is $\cdot 0\dot{9}$ of B's, B's is 3·08 of C's, C's is 1·1 of D's; C's share is £2,750; find A's, B's, and D's shares.

110. Simplify $\frac{\frac{5}{6}+\frac{4}{11}}{\frac{3}{5}+\frac{5}{17}} \div \frac{\frac{3}{4}+\frac{7}{19}}{\frac{6}{7}+\frac{3}{13}}$

111. If tin foil is sold in sheets 10 in. wide, what length of it will be required to line a box 16 in. wide, 15 in. high, and 20 in. long?

112. The centigrade thermometer, in which water freezes at zero and boils at 100°, stands at 25°; what will Fahrenheit's thermometer mark, in which water freezes at 32° and boils at 212°?

113. Consols being at $94\frac{3}{4}$, new 3 per cents. at $93\frac{1}{8}$, and India 5 per cents. at $113\frac{1}{4}$, determine the interest on £100 invested in each of these securities.

114. Two persons have invested £11 17*s.* $2\frac{3}{4}$*d.*, and £17 16*s.* $8\frac{1}{2}$*d.*, and the return is £46 2*s.* $0\frac{3}{4}$*d.*; find, within a farthing, what the share of each must be.

115. Find the length of the edge of a cube which contains 56 cubic feet 568 cubic inches.

116.* A cask weighing 1 cwt. 6 lbs. 2 ozs. floats in a cubical cistern of water whose side is 2 ft. 6 in. On the removal of the cask, find how much the water will sink in the cistern, supposing a cubic foot of water weighs 63 lbs.

117. I have £26,080 to invest, and the 3 per cents. are at 80. I wait 21 days for a fall, but the funds rise steadily $\frac{1}{8}$ per cent. per day, and on the 11th day I invest, paying the broker $\frac{1}{8}$ per cent.; what loss of capital do I sustain by my delay, and what will the be amount of my annual income from the investment!

118. A gave $\frac{7}{10}$ of what she had in her purse to B; B gave to C $\frac{15}{28}$ of what she received from A; C gave to D $\frac{2}{3}$ of what she received from B; if D received 7*s.* $0\frac{1}{2}$*d.*, how much had A in her purse?

119. There are two adjacent reservoirs. The one which is 84 ft. long, 27 ft. wide, and 15 ft. deep, is filled by a system of steam pumps in 9 hours. The same pumps take 48 hours to fill the other, which is 90 ft. long by 42 ft. wide; what is its depth?

120. My bookseller's bill amounts to £12 6*s.* 8*d.* after he has allowed a discount of 3*s.* 4*d.* in the £ on a certain portion of it. On £4 of the original bill no discount was allowed; what advantage have I derived from the allowance of discount?

* A body *floating in water* displaces a weight of water equal to its own weight.

121. Find the value of $\frac{1}{4}$ of 17*s.* 8*d.* + 2·875 of 1*s.* − $\frac{3}{4}$ of $\frac{5}{8}$ of 5*s.* 4*d.* + ·26$\dot{3}$ of 25*s.*, and reduce the result to the decimal of £5.

122. Find the length of the side of a cubical tank which contains 15 cwt. 7 lbs. 8 ozs. of water. [One cubic foot of water weighs 1,000 ozs.]

123. A clock which gains one minute per day is at 12 o'clock (noon) on Jan. 1st 20 minutes behind another clock which loses 29 seconds daily; when will the clocks show the same time?

124. Divide £2,025 among A, B, C, D, and E, so that A's share is to B's share as 1 to 2; C's share to B's share as 5 to 4; D's share to C's share as 6 to 5; and E's share to D's share as 4 to 3.

125. Find the value of $\sqrt{\cdot\dot{2}\dot{7} \times 1\cdot\dot{0}\dot{9}}$.

126. Took money out of a bank, giving 2$\frac{1}{2}$ per cent., and bought five £20 shares bearing interest at 8$\frac{1}{2}$ per cent., the share to be paid in six equal instalments. I at once paid up five instalments, and at the end of a year and a half, the property having increased 17$\frac{1}{2}$ per cent. in value, I sold my shares; what did I gain by the whole transaction?

APPENDIX.

MANY problems in Commercial Arithmetic may be more easily worked by turning the £ *s. d.* into the *decimal of a* £ by *Mental Arithmetic*, and then, when the answer is obtained in a decimal form, the decimal fraction may be changed into shillings and pence at sight.

A. *To reduce £ s. d. to the decimal of* £1.

I. Write down the pounds as the integral part of the decimal.

II. Write down as many *tenths* as there are *florins*.

III. (1) Reckon, mentally, 50 *thousandths, or* ·050 if there is an *odd shilling* left.

(2) ,, ,, 25 *thousandths, or* ·025 if there is *sixpence* left.

(3) ,, ,, *as many thousandths* as there are farthings remaining.

Write down the sum of (1), (2), and (3) next to II. for the decimal to 3 places.

IV. When there are any farthings in III. (3), the remaining figures of the decimal may be found by dividing the thousandths in III. (3) by 24.

Ex. 1. £7 17*s.* 9*d.* = £7 + 8 florins + 1 shilling + 6 pence + 12 farthings

$$= £\left(7 + \cdot 8 \quad + \cdot 050 \quad + \cdot 025 \quad + \cdot 012 + \frac{\cdot 012}{24}\right)$$

$$= £7 \cdot 8875.$$

Ex. 2. £68 16*s.* 6*d.* = £68 + 8 florins + 6 pence

$$= £(68 + \cdot 8 \quad + \cdot 025)$$

$$= £68 \cdot 825.$$

Ex. 3. £3 18*s.* 5½*d.* = £3 + 9 florins + 21 farthings

$$= \left(£3 + \cdot 9 \quad + \cdot 021 + \frac{\cdot 021}{24}\right)$$

$$= £3 \cdot 921875.$$

Ex. 4. £561 12*s.* 8*d.* = £561·6333.

NOTE.—The thousandths in III. (2) and (3) may be found by taking the farthings as thousandths; but if 24 farthings, or more than 24, one-thousandth must be added.

B. *To reduce a decimal fraction of £1 to shillings and pence.*

To obtain the shillings :—

(1) For every *tenth* reckon a *florin.*

(2) If there be 50 or more than 50 thousandths reckon one sh

(3) Write down the sum of (1) and (2).

To obtain the pence :—

(4) If the remaining thousandths amount to, or exceed, 25, r sixpence for the 25; and if any thousandths still remai them farthings and reduce to pence.

or (4) When the shillings have been found, take the remaining sandths as farthings, deducting one if there be 25 or thousandths.

Ex. 1. ·792 = ·7 + ·050 + ·025 + ·017, *or* ·7 + ·050 + ·042
= 14*s.* + 1*s.* + 6*d.* + 4½*d.*, *or* 14*s.* + 1*s.* + 10½*d.*
= 15*s.* 10½*d.*

Ex. 2. ·8875 = ·8 + ·050 + ·025 + ·012, *or* ·8 + ·050 + ·037
= 16*s.* + 1*s.* + 6*d.* + 3*d.*, *or* 16*s.* + 1*s.* + 9*d.*
= 17*s.* 9*d.*

Ex. 3. ·825 = ·8 + ·025
= 16*s.* 6*d.*

Ex. 4. ·2354 = ·2 + ·025 + ·010, *or* ·2 + ·035
= 4*s.* 8½*d.*

All decimal places after 3 may be neglected.

C. *Reasons for the foregoing rules.*

The reasons for these rules may be seen from the following:—

Our sovereign is divided into 10 parts called florins.

Therefore 2*s.* or 1 florin may be written £·1.
12*s.* or 6 „ „ „ £·6.

And so on.

The sovereign is also divided into 960 equal parts called farthings. pose it to be divided into 1000 equal parts—

Then 960 farthings are equal to the 1000 parts

Dividing by 20, 48 farthings or 1 shilling are equal to 50 of these pa
„ 40, 24 „ or 6 pence „ „ 25 „ „
„ 960, 1 farthing is equal to $\frac{1000}{960}$ of these parts
$= 1\frac{40}{960} = 1\frac{1}{24}$ parts.

From the preceding it appears that—

1 florin may be written as £·1.
1 shilling „ „ £·050.
1 sixpence „ „ £·025.
1 farthing „ „ $£\cdot001 + \frac{\cdot001}{24}$

ANSWERS

TO THE EXERCISES.

CHAPTER I.

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Ex. 2. 1236
„ 3. 1057
„ 4. 9939

Ex. 5. 744
„ 6. 2447

Section II. Page 1.

Ex. 2. 223, 283
„ 3. 1567; 1530; 1510
„ 4. 34615

Ex. 5. 876
„ 6. 9853

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Ex. 2. 52936
„ 3. 3219
„ 4. 38440

Ex. 5. 188615
„ 6. 4805; 187

Section IV. Page 2.

Ex. 2. 222
„ 3. 103
„ 4. 40, 2 more

Ex. 5. 3 miles
„ 6. Officer £100;
Sailor £50

Section V. Page 3.

Ex. 2. £662 17*s.* 6½*d.*
„ 3. £913 12*s.* 7*d.*

Ex. 4. 8 ac. 0 ro. 30 pls.
„ 5. 56 lbs.

Section VI. Page 4.

Ex. 2. £27 6*s.* 9¾*d.*
„ 3. £1412 14*s.* 2*d.*
„ 4. £3 15*s.* 7*d.*; £3 8*s.* 10*d.*

Ex. 5. 3*s.*, 1*s.*
„ 6. 2 qts.; 1 qt.

Section VII. Page 4.

Ex. 2. £13458 14*s.* 7*d.*
„ 3. £1055 4*s.* 3*d.*
„ 4. £847 17*s.* 10*d.*

Ex. 5. £164 9*s.* 0*d.*
„ 6. 150 ac. 1 ro.

Section VIII. Page 5.

Ex. 2. £2 13*s.* 9*d.*
„ 3. £3 5*s.* 2½*d.*
„ 4. 5 lbs. 5 oz. 10 dwt. 3⅜ grs.

Ex. 5. 17 lbs.
„ 6. 5 lbs.

Section IX. Page 5.

Ex. 2. 237
„ 3. 48 sheep
„ 4. 13 lbs.

Ex. 5. 20
„ 6. 72 shirts.

Section X. Page 6.

Ex. 2. 5
„ 3. 480
„ 4. 252

Ex. 5. 15
„ 6. 7

Section XI. Page 7.

Ex. 2. 80; 40; 20
„ 3. 6*s.*; 3*s.*; 2*s.* 3*d.*; 2*s.* 3*d.*
„ 4. Each son £2500; Each daughter £1666⅔

Ex. 5. 4; 8; 16; 32
„ 6. 1 ac. 2 ro.; 2 ac. 4 r
2 ac. 1 ro.; 2 ac. 1 ro.

Section XII. Page 8.

Ex. 2. £18 14*s.*
„ 3. £7 11*s.* 3*d.*
„ 4. £3 4*s.* 8*d.*

Ex. 5. 29⅓ galls.
„ 6. watch 25 guineas; chain 5 guineas

Section XIII. Page 9.

Ex. 2. Sugar 6*s.* 4*d.*
„ 3. 2*s.* 3 17/20*d.*
„ 4. £118 2*s.* 6*d.*

Ex. 5. £34 8*s.* 10*d.*
„ 6. Receives £14; ga
£3 10*s.*

SECTION XIV. Page 9.

Ex. 2. $1\frac{1}{2}d.$
„ 3. $4s.$
„ 4. £14 11*s.* 8*d.* gain

Ex. 5. 16*s.* 6*d.* per table
„ 6. 18*s.* 8*d.* per cwt. ; 2*d.* per lb.

SECTION XV. Page 10.

Ex. 2. £87 16*s.* $6\frac{3}{4}d.$
„ 3. £4 ; £$6\frac{2}{3}$; £$9\frac{1}{3}$
„ 4. £487 7*s.*

Ex. 5. £48 19*s.* 4*d.*
„ 6. 9*s.* $2\frac{8}{13}d.$

SECTION XVI. Page 10.

Ex. 2. £1 9*s.*
„ 3. £1 18*s.* 3*d.*
„ 4. 17*s.* 6*d.*

Ex. 5. £25
„ 6. £112 10*s.*

SECTION XVII. Page 10.

Ex. 3. $2\frac{1}{2}$ ft.
„ 4. 16 loads

Ex. 5. 960
„ 6. Painting £1 15*s.* 4*d.*

SECTION XVIII. Page 12.

Ex. 2. 120 ft.
„ 3. 7040 cu. ft.
„ 4. 10 ft. 8 in.

Ex. 5. 2197 cu. in.
„ 6. 2975 cu. ft.

SECTION XIX. Page 13.

Ex. 4. £9 11*s.* $7\frac{1}{2}d.$
„ 5. 17*s.* 3*d.*
„ 6. £227 15*s.* $7\frac{1}{2}d.$
„ 7. 7*s.* $8\frac{1}{4}d.$
„ 8. £2 2*s.*

Ex. 9. £5
„ 10. £1 3*s.* $5\frac{1}{4}d.$
„ 11. £21 12*s.* 3*d.*
„ 12. £2 19*s.* 2*d.*

MISCELLANEOUS EXERCISES. Pages 14, 15, 16, 17, 18, 19, 20.

Ex. 1. 4005
„ 2. 33825 grs.
„ 3. 601·050
„ 4. 13*s.* 6*d.*
„ 5. 700 guis.
„ 6. 516
„ 7. £2 3*s.* $11\frac{1}{2}d.$
„ 8. 63

Ex. 9. 15525 ; 14515
„ 10. 15*s.*
„ 11. £1 8*s.* 4*d.*
„ 12. $2\frac{1}{4}d.$
„ 13. 1416 13*s.* 4*d.*
„ 14. 660
„ 15. 2*s.* $11\frac{1}{4}$*d.*
„ 16. 4*d.*

Ex. 17. 372
„ 18. 86 times
„ 19. 256
„ 20. 9*s.* 6*d.*
„ 21. 8*s.* 1*d.*
„ 22. 864 miles
„ 23. 64 yds.
„ 24. 63 ; 45 ; 27
„ 25. £6
„ 26. 294
„ 27. 62921
„ 28. £7947 2*s.* 10*d.*
„ 29. 19
„ 30. £1 6*d.*
„ 31. £14 19*s.*
„ 32. £1 13*s.* 7½*d.*
„ 33. £13 1*s.*
„ 34. £120
„ 35. £69 7*s.* 6*d.*
„ 36. £1 6*d.*
„ 37. $2830\frac{250}{341}$
„ 38. £3 8*s.* 9*d.*
„ 39. 2422 ac. 0 ro. 12 po.
„ 40. 1025
„ 41. 24 weeks
„ 42. £19 2*s.* 8*d.*
„ 43. 2*s.* 4*d.*
„ 44. 9 miles 16 yds. 2 ft. 10 in.
„ 45. 163 lbs.
„ 46. £242 14*s.* 2*d.*
„ 47. 11 min. $9\frac{1}{11}$ sec.
„ 48. £254 13*s.* 1½*d.*
„ 49. £54 3*s.* 4*d.*
„ 50. 7 lds. 1 qr. 4 bus. 2 pks.
„ 51 £12 7*s.* 9½*d.*
„ 52. 14 cwts. 2 qrs. 11 lbs.
„ 53. 5 tons 3 qrs. 24 lbs. 3 ozs.
„ 54. £3 1*s.* 6¾*d.*
„ 55. 21
„ 56. 448 ozs.
„ 57. 18
„ 58. 2120 acrs.
„ 59. 35
„ 60. £50 8*s.*
„ 61. 151 ts. 1 cwt. 2 qrs. 22 lbs.
„ 62. £2 1*s.* 9*d.*
„ 63. £264 13*s.* 9*d.*

Ex. 64. £44 15*s.* 3*d.*
„ 65. £2 8*s.* 4*d.*
„ 66. 1*s.* 4½*d.*
„ 67. £15
„ 68. 210998
„ 69. 540
„ 70. 2880 steps
„ 71. 5 ac. 17 pls. 14¼ yds.
„ 72. £5 10*s.* 3*d*
„ 73. £3 15*s.* 3*d.*
„ 74. 5*s.* 4*d.*
„ 75. 47 tons 12 cwt. 26 11 ozs.
„ 76. 19000
„ 77. 3549
„ 78. 14 ac. 20 po.
„ 79. £3 10*s.* 9*d.* ; last 1 £5 14*s.* 4*d.*
„ 80. 32
„ 81. 16 cu. yds. 8 cu. ft. 1 cu. in.
„ 82. 27 spoons
„ 83. £15
„ 84. 56 bags
„ 85. 12 ozs. tea, 2½ lbs. su
„ 86. £54 12*s.*
„ 87. 24
„ 88. 12 days
„ 89. 3 feet
„ 90. 5 dwt. 3 grs.
„ 91. 13 days
„ 92. 8 pence
„ 93. 60 lbs.
„ 94. 2*s.* 6*d.*
„ 95. £9 15*s.*; nothing
„ 96. £4 13*s.* 4*d.*
„ 97. 1 hr. 54 min.
„ 98. £1250
„ 99. £5 4*s.* 2*d.*
„ 100. Gain = 17*s.* 2½*d.*
„ 101. Loss = 7*s.*
„ 102. 15750
„ 103. 6*d.*
„ 104. £13069 7*d.*
„ 105. £68 15*s.*
„ 106. 40 miles 7 fur. 10 5 yds.
„ 107. £20 12*s.* 9*d.*

Ex. 108. Large wheel 1056 times; small wheel 3168 times
„ 109. £10 8*s.* 4*d.*
„ 110. 209
„ 111. 280
„ 112. A £6 2*s.*; B £12 4*s.*; C £24 8*s.*
„ 113. 120
„ 114. 40½ yds.
„ 115. 48 persons
„ 116. 361 sq. yds.
Ex. 117. 27 cu. ft.
„ 118. 1 ro. 1 pl. 25¾ sq. yds.; 10 pl. 14 sq. yds.
„ 119. 170
„ 120. 256
„ 121. 80,000
„ 122. 9 miles an hour
„ 123. 5*s.* per bush.
„ 124. £4095
„ 125. 2¼ per lb.
„ 126. 3 miles 420 yds.
„ 127. 100 miles

CHAPTER II.

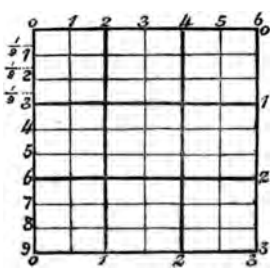
INTRODUCTORY FRACTIONS.

SECTION I. Page 22.

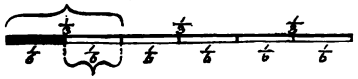
Ex. 1. £1 8*s.*
„ 2. £3 5*s.* 3*d.*
„ 3. 5*s.* 11*d.*
„ 4. £6 18*s.*
„ 5. 26 years
„ 6. £3150; £5250
„ 7. 5*d.*
„ 8. £708 15*s.*
„ 9. 1644⅘ lbs.
„ 10. £1307 4*s.*
Ex. 11. £50
„ 12. £1 3*d.*
„ 13. £4 12*s.* 7½*d.*
„ 14. £1 1*s.* 9⅜*d.*
„ 15. 1837½ lbs.
„ 16. £70 5*s.* 6*d.*
„ 17. £2 5*s.*
„ 18. 17*s.* 10*d.*
„ 19. £109 5*s.* 9*d.*
„ 20. £21306 6*s.* 11¼*d.*

SECTION II. Page 24.

Ex. 1. $\frac{1}{3}=\frac{2}{6}=\frac{3}{9}$



Ex. 2. $\frac{1}{3}-\frac{1}{6}=\frac{1}{6}$



Ex. 3. $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

Ex. 4. (1) $4\frac{1}{2}, 2\frac{1}{7}, 11, 11\frac{1}{2}, 17\frac{1}{6}, 18\frac{2}{17}$;
(2) $7\frac{1}{8}, 40\frac{13}{19}, 100\frac{41}{376}, 400$

„ 5. (1) $\frac{2}{3}, \frac{15}{7}, \frac{8}{17}, \frac{3}{2}, \frac{16}{9}, \frac{315}{17}$;
(2) $\frac{127}{18}, \frac{311}{16}, \frac{1158}{6}, \frac{224}{\frac{205}{6}}$

Ex. 6. $\frac{16}{20}$ and $\frac{15}{20}$
„ 7. $\frac{3}{7}$
„ 8. $\frac{6}{7}$ by $\frac{1}{12}$
„ 9. $2\frac{21}{27}$, $2\frac{22}{27}$; second gtr. by $\frac{1}{27}$
„ 10. (1) $\frac{9}{10}$, $\frac{9}{10}$, 10, $1\frac{1}{3}$, 5
(2) $\frac{13}{18}$, $\frac{14}{18}$, $\frac{25}{36}$, $\frac{3}{4}$
„ 11. $\frac{1}{10}$

Ex. 12. £3000
„ 13. $\frac{1}{8}$, $\frac{1}{8}$
„ 14. $\frac{6}{9}$, $\frac{6}{9}$
„ 15. $\frac{5}{6}$, $\frac{1}{2}$
„ 16. 9, $8\frac{2}{7}$, $\frac{32}{5}$, $\frac{42}{3}$, $\frac{22}{11}$
„ 17. 1*d.*
„ 18. 9 gallons
„ 19. 120 gallons

CHAPTER III.

PRACTICE AND BILLS OF PARCELS.*

SECTION I.—SIMPLE PRACTICE. Pages 30, 31, 32.

Ex. 1. $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{12}$, $\frac{1}{16}$
„ 2. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$
„ 3. *Of* 1*s.*—
$6d. = \frac{1}{2}$, $4d. = \frac{1}{3}$, $3d. = \frac{1}{4}$,
$2d. = \frac{1}{6}$, $1\frac{1}{2}d. = \frac{1}{8}$, $1d. = \frac{1}{12}$
Of 2*s.* 6*d.*—
$1s.3d. = \frac{1}{2}$, $10d. = \frac{1}{3}$, $7\frac{1}{2}d. = \frac{1}{4}$
$6d. = \frac{1}{5}$, $5d. = \frac{1}{6}$, $3\frac{3}{4}d. = \frac{1}{8}$
$3d. = \frac{1}{10}$, $2\frac{1}{2}d. = \frac{1}{12}$,
$2d. = \frac{1}{15}$, $1\frac{1}{2}d. = \frac{1}{20}$
$1\frac{1}{4}d. = \frac{1}{24}$, $1d. = \frac{1}{30}$
„ 4. 1*s.* $9d. = \frac{1}{4}$ of 7*s.*
2*s.* $3d. = \frac{1}{8}$ of 6*s.* 9*d.*
1*s.* $11d. = \frac{1}{2}$ of 7*s.* 8*d.*
1*s.* $7d. = \frac{1}{3}$ of 12*s.* 8*d.*
$9d. = \frac{1}{11}$ of 8*s.* 3*d.*
$\frac{3}{4}d. = \frac{1}{18}$ of 9*d.*

„ 5.

10	0	10	0	10	0
5	0	6	8	5	0
2	6	2	0	1	0
0	5	0	3	0	6
	—		—	0	3
0	$0\frac{1}{2}$		—	0	$0\frac{1}{4}$
17	$11\frac{1}{2}$	18	11	16	$9\frac{1}{4}$

$10s. - 2s. - 1s. - 6d. - \frac{1}{2}$
$= 13s. 6\frac{1}{2}d.$
$10s. - 5s. - 4s. - 10d.$
$= 19s. 10d.$
$6d. - 3d. - \frac{1}{2}d. = 9\frac{1}{2}d.$

Ex. 6. £103 10*s.*; £142 10*s.*
„ 7. £68 8*s.*; £82 4*s.*
„ 8. £927 10*s.*; £887 15*s.*
„ 9. £798 15*s.*; £782 15*s.* 6
„ 10. £123 15*s.*; £4636 10*s.*
„ 11. £18 19*s.* 9*d.*; £5924 2*s.*
„ 12. £12 6*d.*; £1672 8*s.*
„ 13. £118 8*s.*; £300 16*s.*
„ 14. £13 7*d.*; £27126 14*s.* 6
„ 15. £2580 12*s.*; £32763 1(
„ 16. £153 17*s.* $1\frac{1}{2}d.$
£1909 13*s.* $8\frac{1}{4}d.$
„ 17. £322 10*s.*; £11 6*s.* 3*d.*
„ 18. £160 1*s.* 9*d.*; £26325 1(
„ 19. £232 8*s.* $1\frac{1}{2}d.$; £9906 1
„ 20. £2200 12*s.* 6*d.*
£1010 15*s.* 10*d.*
„ 21. £124 5*s.* $7\frac{1}{2}d.$
£2767 6*s.* 7*d.*
„ 22. £15 4*s.* 7.; £6353 5*s.*
„ 23. £51 17*s.* $3\frac{1}{2}d.$
£50 12*s.* 11*d.*
„ 24. £5295 16*s.* 8*d.*
£27995 16*s.* 8*d.*
„ 25. £4018 15*s.*; £5615 12*s.*(
„ 26. £4461 9*s.* 2*d.*
£4923 19*s.* 2*d.*
„ 27. £6493 5*s.* $1\frac{1}{2}d.$
£11013 1*s.* $5\frac{1}{4}d.$
„ 28. £6943 15*s.*
£7192 19*s.* $4\frac{1}{2}d.$

* All answers given to fractions of one penny.

Ex. 29. £297 14*s.* 1½*d.*
£317 5*s.* 7¼*d.*
„ 30. £9478 3*d.*
£11584 4*s.* 9*d.*
„ 31. £8 1*s.*
„ 32. £17 17*s.* 5½*d.*
„ 33. 1936 15*s.* 10*d.*
„ 34. £86 14*s.* 6½*d.*
„ 35. £3150 17*s.* 3¾*d.*
„ 36. £253 6*s.* 0$\frac{21}{40}$*d.*
„ 37. £904209 4*s.* 1½*d.*
„ 38. £1380 15*s.*
„ 39. £19019 19*s.* 7½*d.*
„ 40. £196 17*s.* 6*d.*
„ 41. £4834 11*s.* 5¼*d.*
„ 42. £12 12*s.* 1*d.*
„ 43. £63907 5*s.* 2¼*d.*
„ 44. £15 10*s.* 1*d.*
„ 45. £26027 12*s.* 1*d.*
„ 46. £93 3*s.* 10*d.*
„ 47. £3197 5*s.*

Ex. 48. £12886 8*s.* 10¼*d.*
„ 49. £76 9*s.* 6*d.*
„ 50. £6963 15*s.* 11¼*d.*
„ 51. £15711 3*s.* 9*d.*
„ 52. £13927 11*s.* 10½*d.*
„ 53. £2393 9*s.* 10½*d.*
„ 54. £47981 17*s.* 3*d.*
„ 55. £12 5*s.*
„ 56. £5217 10*s.* 0¾*d.*
„ 57. £922 10*s.*
„ 58. £131 11*s.* 6⅛*d.*
„ 59. £5436 2*s.* 3*d.*
„ 60. £188 8*s.*
„ 61. £19872 6*s.* 1½*d.*
„ 62. £71 3*s.* 4*d.*
„ 63. £43 12*s.*
„ 64. £1122 6*s.* 10⅛*d.*
„ 65. £2 2*s.*
„ 66. £4020 16*s.* 7$\frac{1}{14}$*d.*
„ 67. £625 19*s.* 0⅜*d.*

Section II.—Compound Practice. Pages 33, 34, 35.

Ex. 1. £8 3*s.* 0½*d.*
„ 2. £7 16*s.* 1⅓*d.*
„ 3. £121 4*s.* 2*d.*
„ 4. £360 18*s.* 9*d.*
„ 5. £69 7*s.* 6*d.*
„ 6. 19*s.* 4½*d.*
„ 7. £17955 9*s.* 0$\frac{1}{48}$*d.*
„ 8. £27 19*s.* 1½*d.*
„ 9. £199 4*s.* 3¼*d.*
„ 10. £60 9*s.* 6⅜*d.*
„ 11. £3 18*s.* 9*d.*
„ 12. £1 9*s.* 6⅜*d.*
„ 13. £11 4*s.* 2¾*d.*
„ 14. £3 13*s.* 1½*d.*
„ 15. £1 11*s.* 7½*d.*
„ 16. £36 5*s.* 2⅛*d.*
„ 17. £44 9*s.* 2*d.*
„ 18. £381 4*s.* 7*d.*
„ 19. £496 1*s.* 9$\frac{15}{64}$*d.*
„ 20. 6*s.* 6¾*d.*
„ 21. £83 1*s.* 11¼*d.*
„ 22. £7 11*s.* 10⅛*d.*
„ 23. 9*s.* 10½*d.*
„ 24. £12 6*s.* 6*d.*

Ex. 25. £14 9*s.* 3*d.*
„ 26. £7889 17*s.* 3½*d.*
„ 27. £839 14*s.* 4½*d.*
„ 28. £1 18*s.* 6*d.*
„ 29. £490 10*s.* 10*d.*
„ 30. £714 18*s.* 9*d.*
„ 31. £19 12*s.* 1$\frac{5}{16}$
„ 32. £10 1*s.* 6¾*d.*
„ 33. £35 10*s.* 0¾*d.*
„ 34. £576875
„ 35. £51 8*s.* 0$\frac{3}{16}$*d.*
„ 36. 3*s.* 7½*d.*
„ 37. £46 1*s.* 6¼*d.*
„ 38. £26 11*s.* 4½*d.*
„ 39. £409 19*s.* 1$\frac{37}{80}$*d.*
„ 40. £31 17*s.* 6¾*d.*
„ 41. £2 8$\frac{9}{10}$*d.*
„ 42. £63 10*d.*
„ 43. £4 11*s.* 9½*d.*
„ 44. £46 12*s.* 8½*d.*
„ 45. £178 4*s.* 4⅜*d.*
„ 46. £1 5*s.* 4*d.*
„ 47. £36 10*s.* 2¼*d.*
„ 48. £390 14*s.* 3*d.*

Ex. 49. £57 10*s.* $10\frac{3}{80}$*d.*
„ 50. £3 8*s.* $2\frac{1}{4}$*d.*
„ 51. £211 7*s.* $2\frac{1}{4}$*d.*
„ 52. £5 7*s.* 1*d.*
„ 53. £723 5*s.* $4\frac{1}{2}$*d.*
„ 54. £513 6*s.* $6\frac{1}{4}$*d.*

Ex. 55. £4 15*s.* 7*d.*
„ 56. £561 1*s.* 3*d.*
„ 57. £1909 2*s.* $11\frac{1}{2}$*d.*
„ 58. £2495 5*s.* $3\frac{1}{32}$*d.*
„ 59. £50 6*s.* 3*d.*
„ 60. £654 7*s.* 6*d.*

Section III.—Bills of Parcels. Pages 36, 37, 38.

		£	*s.*	*d.*
Ex. 1.	£3 4*s.*, £1 2*s.* 4*d.*, £4 11*s.* $2\frac{1}{2}$*d.*	8	17	$6\frac{1}{2}$
„ 2.	14*s.* 3*d.*, 17*s.* 3*d.*, 11*s.* $8\frac{1}{4}$*d.*, 13*s.* 6*d.* . . .	2	16	$8\frac{1}{4}$
„ 3.	2*s.* $11\frac{3}{4}$*d.*, 7*s.* $10\frac{1}{2}$*d.*, 2*s.* 9*d.*, 5*s.* $0\frac{3}{4}$*d.* . . .	0	18	8
„ 4.	£27 3*s.* 9*d.*, £3 6*s.* $11\frac{1}{4}$*d.*, £7 16*s.*	38	6	$8\frac{1}{4}$
„ 5.	7*s.* $2\frac{1}{4}$*d.*, 7*s.* $9\frac{3}{4}$*d.*, 10*s.* $4\frac{1}{4}$*d.*, 7*s.* $3\frac{3}{4}$*d.* . . .	1	12	8
„ 6.	8*s.* 9*d.*, 3*s.*, 7*d.*, 1*s.* 6*d.*, 7*s.* 6*d.*, 9*s.* 9*d.* . . .	1	11	1
„ 7.	5*s.* 6*d.*, 6*s.* $3\frac{3}{4}$*d.* = 11*s.* $9\frac{3}{4}$*d.* Change from £5 = .	4	8	$2\frac{1}{4}$
„ 8.	9*s.* 9*d.*, £1 1*s.* $10\frac{1}{2}$*d.*, 4*s.* $8\frac{1}{4}$d., 5*s.* $1\frac{3}{4}$*d.*, 11*s.* 1*d.*, 9*s.* $8\frac{1}{4}$*d.*	3	2	$2\frac{3}{4}$
„ 9.	£26 5*s.* 2*d.*, £40 9*s.* 9*d.*, £61 5*s.*, £39 7*s.* 6*d.* . .	167	7	5
„ 10.	14*s.*, 5*s.*, 7*s.* $10\frac{1}{2}$*d.*, 3*s.* $0\frac{3}{4}$*d.*, 1*s.* 4*d.*, from £2 = .	0	8	$8\frac{3}{4}$
„ 11.	4*s.* 2*d.*, 7*s.* 10*d.*, 6*s.*	0	18	0
„ 12.	17*s.*, 2*s.* $4\frac{1}{2}$*d.*, 3*s.* 9$\frac{1}{2}$*d.*, 9*s.* 2*d.*, 4*s.* 3*d.*, 9*s.* 6*d.*, 4*s.* $4\frac{1}{2}$*d.*	2	10	$5\frac{1}{2}$
„ 13.	£19 17*s.* 3*d.*, £9 10*s.*, 19*s.* $4\frac{1}{2}$*d.*, 13*s.* $1\frac{1}{2}$*d.* . .	30	19	9
„ 14.	£8 6*s.* 6*d.*, 6*s.*, 8*s.* $1\frac{1}{2}$d.	9	0	$7\frac{1}{2}$
„ 15.	£6 6*s.* 6*d.*, £1 11*s.* $7\frac{1}{2}$*d.*, £3 2*s.*, 18*s.* 4*d.* Error too little	0	0	$9\frac{1}{4}$
„ 16.	3*s.* $11\frac{1}{4}$*d.*, 9*s.* $8\frac{1}{4}$*d.*, 12*s.* $6\frac{1}{2}$*d.*, 5*s.* 3*d.*, $10\frac{1}{2}$*d.*, 14*s.* $10\frac{1}{4}$*d.* 3*s.* $2\frac{1}{4}$*d.*, 6*s.* $10\frac{1}{2}$*d.*, 1*s.* $5\frac{1}{2}$*d.*, 10*s.* $0\frac{3}{4}$*d.* $7\frac{1}{2}$*d.* + B. Fd. £1 17*s.* 5*d.*	5	6	$9\frac{1}{4}$
„ 17.	£61 8*s.* 4*d.*, £79 13*s.*, £8 7*s.* $0\frac{3}{4}$*d.*, £37 5*s.* $8\frac{1}{2}$*d.* .	186	14	$1\frac{1}{4}$
„ 18.	The weights are 13 lbs. 12 ozs., $11\frac{1}{2}$ lbs., 11 lbs. 13 ozs., 8 lbs. 12 ozs.			
„ 19.	The weights are 8 cwt. 3 qrs. 20 lbs., 10 cwt. 5 lbs., 2 qrs. $10\frac{1}{2}$ lbs.			
„ 20.	£46 17*s.* 6*d.*, £17 6*s.* 6*d.*, £15 5*s.* 6*d.*, £6 7*s.* 6*d.* .	86	7	0

Miscellaneous Exercises following Chapter III. Pages 38, 39, 40, 41.

Ex. 1. £3790 4*s.* 2*d.*
„ 2. £147 9*s.* $11\frac{3}{4}$*d.*
„ 3. 2481 persons
„ 4. £18 6*s.* 8*d.*
„ 5. £4791 6*s.* 9*d.*
„ 6. £6928 1*s.* $5\frac{1}{4}$*d.*
„ 7. £34 10*s.* $7\frac{1}{2}$*d.*
„ 8. £2 0*s.* $8\frac{3}{4}$*d.*

Ex. 9. £29 16*s.* $7\frac{1}{2}$*d.*
„ 10. 448 miles
„ 11. £9 8*s.* 3*d.*
„ 12. £839 14*s.* $4\frac{1}{2}$*d.*
„ 13. £5381 19*s.* 7*d.*
„ 14. £556 9*s.* 4*d.*
„ 15. £13 0*s.* $6\frac{1}{2}$*d.*
„ 16. £8095 4*s.* $2\frac{1}{4}$*d.*

Ex. 17. £17 10*s.*
„ 18. £3846 1*s.* 6*d.*
„ 19. £203 17*s.* 7*d.*
„ 20. 38
„ 21. £2767 16*s.* $8\frac{1}{2}d.$
„ 22. £4 18*s.* $3\frac{7}{8}d.$
„ 23. £95 3*s.* $8\frac{3}{4}d.$
„ 24. £1 10*s.* $6\frac{1}{2}d.$
„ 25. 10*s.* $9\frac{9}{13}d.$ saved per week
„ 26. £20 12*s.* $8\frac{5}{14}d.$
„ 27. £2 7*s.* $0\frac{13}{16}d.$
„ 28. 33 yds. 1 qr.
„ 29. 2 sq. mls. 400 ac.
„ 30. £1674 4*s.*
„ 31. A £525, B £210, C £105
„ 32. A 1*s.* $3\frac{3}{4}d.$, B 4*s.* $5\frac{3}{4}d.$
„ 33. 19*s.* 10*d.*
„ 34. £5955 18*s.* $0\frac{3}{4}d.$
„ 35. A £181 6*s.* 8*d.*,
B £90 13*s.* 4*d.*,
C £45 6*s.* 8*d.*,
D £22 13*s.* 4*d.*

Ex. 36. 5103 sq. ft.
„ 37. 26 ; 234
„ 38. 7*s.* 2*d.*
„ 39. 6 lbs.
„ 40. 2*s.* 3*d.*
„ 41. 13 tons
„ 42. 1*s.* $5\frac{3}{17}\frac{3}{8}d.$
„ 43. £1810 6*s.* $7\frac{1}{2}d.$
„ 44. £414 11*s.* $11\frac{31}{40}d.$
£571 11*s.* $1\frac{1}{8}d.$
£986 3*s.* $0\frac{9}{10}d.$
„ 45. £473 11*s.* $9\frac{1}{4}d.$
„ 46. £3105
„ 47. 24 in.
„ 48. $40\frac{1}{2}$ scores
„ 49. £8 13*s.* $7\frac{1}{8}d.$
„ 50. £79 0*s.* $7\frac{1}{2}d.$
„ 51. £1 5*s.* 8*d.*
„ 52. 1*s.* $10\frac{1}{4}d.$
„ 53. £256
„ 54. £29 5*s.* $2\frac{3}{4}d.$

CHAPTER IV.

RATIO AND PROPORTION. Pages 44, 45.

Ex. 1. $\frac{1}{4}, \frac{20}{7}, \frac{7}{3}, \frac{3}{7}$
„ 2. 75, 1
„ 3. $x=18$, $x=27$, x $10\frac{1}{2}$
„ 4. 5 : 3 : : 120 : 7, 2 ; 160 : 14 : : 80 : 7 ; 1020 : 204 : : 85 : 17
„ 5. 1080
„ 6. 225
„ 7. 98

Ex. 8. 18
„ 9. 228
„ 10. 242
„ 11. 4 : 3
„ 12. The first allows 1*d.* in 6*d.*, the second 1*d.* in 10*d.* ∴ 6 : 10
„ 13. 24000
„ 14. £16 13*s.* 4*d.*

CHAPTER V.

SIMPLE PROPORTION OR RULE OF THREE. Pages 50, 51, 52.

Ex. 1. 1 day
„ 2. 78 men
„ 3. £12 15*s.*
„ 4. 17*s.* $8\frac{4}{5}d.$

Ex. 5. 85 bush.
„ 6. £11 0*s.* 6*d.*
„ 7. 16*s.*
„ 8. 18 months

Ex. 9. 1080 miles
„ 10. 140 ac.
„ 11. 36 days
„ 12. £13 17*s.* $2\frac{3}{4}d.$
„ 13. £1 9*s.* $6\frac{3}{4}d.$
„ 14. 120 yds.
„ 15. £77 1*s.* $4\frac{4}{5}d.$
„ 16. 37 cwt.
„ 17. 625 apples
„ 18. £4 18*s.* $5\frac{1}{4}d.$
„ 19. 8 months
„ 20. £553 5*s.* 2*d.*
„ 21. 244 lbs.
„ 22. £65 10*s.* $2\frac{1}{2}d.$
„ 23. £100 5*s.* $4\frac{1}{2}d.$
„ 24. 108 miles
„ 25. $28\frac{1}{2}$ yds.
„ 26. 28 days
„ 27. £148 4*s.*
„ 28. 540 yds. $\frac{24}{25}$ ft.
„ 29. $2\frac{1}{2}$ lbs.

Ex. 30. £10 10*s.*
„ 31. £527 3*s.* $11\frac{1}{7}d.$
„ 32. $175\frac{1}{2}$ yds.
„ 33. £22 10*s.*
„ 34. 126
„ 35. £5 9*s.* 6*d.*
„ 36. £243 11*s.* 6*d.*
„ 37. £16 10*s.* $8\frac{8}{9}d.$
„ 38. 9*s.* 5*d.* per bush.
„ 39. 1*s.* $1\frac{1}{4}d.$
„ 40. £158 8*s.* 9*d.*
„ 41. £588 11*s.* $10\frac{4}{7}d.$
„ 42. 576 boys, 384 girls
„ 43. £248 10*s.* 6*d.*
„ 44. £256
„ 45. $27\frac{1}{2}$ per cent.
„ 46. £$113\frac{1}{3}$ per cent.
„ 47. 350 oranges
„ 48. $33\frac{1}{3}$ per cent.
„ 49. 9*s.* $4\frac{1}{2}d.$ per yd.
„ 50. 40 tons

CHAPTER VI.

COMPOUND PROPORTION. Pages 57, 58, 59.

Ex. 1. 7*s.* 6*d.*
„ 2. $15\frac{3}{4}$ acres
„ 3. $6\frac{2}{3}$ hours
„ 4. 40 horses
„ 5. 20 days
„ 6. 6 horses
„ 7. 7200 soldiers
„ 8. 8 days
„ 9. £114
„ 10. 12 men
„ 11. 66 days
„ 12. £31 9*s.* 4*d.*
„ 13. 300 men
„ 14. $503\frac{1}{3}$ yards
„ 15. £640
„ 16. 81 men
„ 17. $261\frac{1}{3}$ miles
„ 18. 8*d.*
„ 19. 9 weeks
„ 20. £36 17*s.* 9*d.*

Ex. 21. 5 months
„ 22. 12 men
„ 23. $51\frac{1}{3}$ ounces
„ 24. 60 shillings
„ 25. 14 wks. 2 days
„ 26. $22\frac{1}{2}$ days
„ 27. £3 2*s.* 6*d.*
„ 28. 450 miles
„ 29. $13\frac{1}{2}$ days
„ 30. 10 hours
„ 31. 6*d.*
„ 32. 200 bricks
„ 33. £37 10*s.*
„ 34. $22\frac{1}{2}$ days
„ 35. 1320 yds.
„ 36. $291\frac{2}{3}$ days
„ 37. $13\frac{1}{3}$ acres
„ 38. 3 days
„ 39. 60 men
„ 40. $12\frac{1}{2}$ ozs.

MISCELLANEOUS EXERCISES FOLLOWING CHAPTER VI. Pages 59, 60, 61, 62.

Ex. 1. £20, £15
„ 2. 33 men
„ 3. 20 horses
„ 4. 3000 blocks
„ 5. 18*s.*
„ 6. 12 men
„ 7. 12 hours
„ 8. 10*s.*
„ 9. £1, £1 5*s.*, £1 10*s.*
„ 10. 500 yards
„ 11. 18 men
„ 12. 30*s.*, 24*s.*, 18*s.*, 12*s.*, 6*s.*
„ 13. 11*s.* 6*d.*
„ 14. 165
„ 15. 45*s.* $8\frac{4}{13}$*d.* per qr.
„ 16. 5 hours, 225 miles
„ 17. Flagstaff $26\frac{1}{4}$ ft., tower $78\frac{3}{4}$ ft.
„ 18. £231 8*s.*
„ 19. 75000
„ 20. $10\frac{10}{11}$ minutes past 5
„ 21. 45 days
„ 22. $18\frac{1}{3}$ cwt. nitre, $3\frac{8}{9}$ charcoal, $2\frac{7}{9}$ sulphur
„ 23. 1 oz.
„ 24. $11\frac{3}{4}$ cwts.

Ex. 25. A £135 7*s.* $8d\frac{4}{13}$*d.*, B £184 12*s.* $3\frac{9}{13}$*d.*
„ 26. £1 5*s.* $3\frac{3}{4}$*d.*
„ 27. 9 ft.
„ 28. ·93 ac.
„ 29. A £100, B £80, C £40
„ 30. 1 lb. 5 ozs.
„ 31. 3 revolutions per second
„ 32. £5 16*s.* 8*d.*
„ 33. A £25, B 31, C £17, D £15
„ 34. 300 men
„ 35. A : D : : 1 : 2
„ 36. £591 18*s.* 8*d.*, £408 1*s.* 4*d.*
„ 37. £31 10*s.*
„ 38. £18
„ 39. £65 18*s.* $9\frac{15}{32}$*d.*
„ 40. £435
„ 41. $5\frac{1}{2}$ tons
„ 42. A £300, B £250
„ 43. 1000 men
„ 44. 7 died
„ 45. $18\frac{11}{18}$ cwt.
„ 46. 3 cwts.
„ 47. 255760 blocks, £3996 5*s.*
„ 48. 1·7

CHAPTER VII.

VULGAR FRACTIONS.

SECTION I.—G. C. M. Pages 65, 66.

Ex. 1. 10, 1, 15, 1001, 6, 11
„ 2. Measures of 18 are 2, 3, 6, 9, 18
„ „ 16 „ 2, 4, 8, 16
„ „ 34 „ 2, 17, 34
„ „ 45 „ 3, 5, 9, 15, 45
„ „ 66 „ 2, 3, 6, 11, 22, 33, 66
„ „ 156 „ 2, 3, 4, 6, 12, 13, 26, 39, 52, 78, 156
„ „ 216 „ 2, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72, 108, 216
„ 3. 2, 36, 721, 1234, 8, 48
„ 4. 37
„ 5. 2023

Ex. 6. 1
„ 7. 101
„ 8. 24

Ex. 9. 121
„ 10. 1771
„ 11. 9
„ 12. 217
„ 13. $5\frac{3}{4}d.$
Ex. 14. 385525
„ 15. 21, 42, 63, 84
„ 16. 13 plots 4 acres
„ 17. 15096, 33744
„ 18. 5 days 17 hours

Section II.—L. C. M. Pages 70, 71.

Ex. 1. 200, 90, 220, 112, 1575, 2520
„ 2. 3,651,056,640
„ 3. 17136, 9744, 2016, 4199, 102,960, 1728
„ 4. £21
„ 5. 363
„ 6. 27720
„ 7. 300 mls.
Ex. 8. £248 3*s.* $10\frac{1}{2}d.$
„ 9. 120 minutes
„ 10. £1 2*s.*
„ 11. 8
„ 12. 715
„ 13. 144
„ 14. 27, 396 or 99, 108 or 36, 297

Section III.—Reduction of Fractions. Pages 71, 72.

Ex. 1. $\frac{1}{3}, \frac{1}{13}, \frac{1}{2}, \frac{281}{6917}$
„ 2. $\frac{1}{7}, \frac{6}{7}, \frac{22}{24}$
„ 3. $116\frac{12}{17}, 13\frac{97}{105}, 3\frac{108}{301}$
„ 4. $\frac{3722}{513}, \frac{647}{5}, \frac{633}{317}$
„ 5. $\frac{52}{9}, \frac{338}{17}, \frac{1706}{31}$
„ 6. G. C. M. = 5, $\frac{1291}{1467}$
Ex. 7. G. C. M. = 378, $1\frac{22}{311}$
„ 8. $\frac{38}{63}, \frac{41}{107}$
„ 9. No, $\frac{551}{6006}$
„ 10. Yes, $\frac{527}{551}$
„ 11. $1\frac{6}{17}$
„ 12. $\frac{5}{8}, 5\frac{4}{151}, 2\frac{211}{23}$

Section IV.—Addition and Subtraction. Pages 74, 75.

Ex. 1. $26\frac{16}{63}, 20\frac{157}{210}, 1\frac{83}{140}, 77\frac{1936}{15903}$
„ 2. $12\frac{13}{63}, 5\frac{1}{2}, \frac{1}{2}, 9\frac{9}{140}, \frac{107}{168}$
„ 3. $\frac{13}{168}$
„ 4. $\frac{12}{25}$
„ 5. $\frac{17}{19}$
„ 6. $\frac{3}{8}$
„ 7. $\frac{1}{10}$
„ 8. $3\frac{277}{500}$
„ 9. $\frac{13}{18}$ gtr. by $\frac{1}{156}$
„ 10. $\frac{19}{8}, \frac{18}{8}$
„ 11. $\frac{798}{1613}$ of a day
„ 12. $\frac{850}{999}$
„ 13. $1\frac{1}{3}, 1\frac{23}{72}, 1\frac{111}{36}$
„ 14. $\frac{19}{30}$
„ 15. $3\frac{1}{9}s.$
„ 16. $\frac{9}{40}$ cwt.
„ 17. 18
„ 18. £4 8*s.* $8\frac{5}{12}d.$
Ex. 19. £8219 2*s.* $9\frac{17}{210}d.$
„ 20. £184 10*s.* $1\frac{1}{28}d.$
„ 21. 3 cwt. $4\frac{271}{272}$ lbs.
„ 22. £240
„ 23. $\frac{103}{210}$
„ 24. £2 14*s.* $3\frac{7}{11}d.$
„ 25. £9600
„ 26. $9\frac{23}{36}$
„ 27. £52 1*s.* $1\frac{32}{105}d.$
„ 28. $\frac{6}{35}$
„ 29. 14*s.* $3\frac{9}{11}d.$
„ 30. 8*s.* 4*d.*
„ 31. 225
„ 32. 36 days
„ 33. $\frac{16}{25}$ milk, $\frac{9}{25}$ water
„ 34. £7 10*s.*, £5, £3 15*s.*, £3
„ 35. £20
„ 36. 35 marbles

Section V.—Multiplication and Division of Fractions. Pages 79. 80.

Ex. 1. $\frac{1}{16}$
„ 2. $\frac{16}{27}$
„ 3. $\frac{112}{891}$, $16\frac{173}{225}$
„ 4. $\frac{1}{4}$.
„ 5. $3\frac{9}{10}$
„ 6. $22\frac{2}{7}$
„ 7. $47\frac{1}{16}$
„ 8. $\frac{308}{1875}$
„ 9. $5\frac{7}{11}$, $\frac{9}{16}$
„ 10. $\frac{551}{891}$, $26\frac{5}{32}$
„ 11. $\frac{2}{3}$
„ 12. 1
„ 13. $\frac{23}{33}$
Ex. 14. $\frac{81}{352}$
„ 15. $\frac{11}{13}$
„ 16. $49\frac{11}{15}$
„ 17. $39\frac{1}{2}$, 1
„ 18. 42
„ 19. 60
„ 20. $\frac{1}{576}$
„ 21. 4
„ 22. 18
„ 23. £6 16*s.* 6*d.*
„ 24. $39\frac{19}{459}$
„ 25. $\frac{4}{7}$
„ 26. £21 8*s.* $1\frac{1}{2}$*d.*

Section VI.—Addition, Subtraction, Multiplication, and Division of Fractions. Pages 83, 84, 85.

Ex. 1. $3\frac{1}{3}$
„ 2. $2\frac{11}{32}$
„ 3. The first by $\frac{1875}{2088}$
„ 4. 2 ; $13\frac{5}{63}$
„ 5. $7\frac{4061}{5182}$
„ 6. $\frac{11}{216}$
„ 7. $8\frac{1277}{1376}$
„ 8. $5\frac{13}{27}$
„ 9. $1\frac{17}{140}$
„ 10. $1\frac{2}{175}$
„ 11. $137\frac{59}{63}$; $3\frac{16}{21}$
„ 12. $12\frac{57}{176}$
„ 13. $\frac{3\frac{1}{2}}{5005}$
„ 14. $87\frac{74}{189}$
„ 15. $7\frac{7}{20}$
Ex. 16. $5\frac{229}{720}$
„ 17. $\frac{1}{2}$
„ 18. 1
„ 19. $\frac{115392}{207171}$; $\frac{215}{501}$
„ 20. $2\frac{788}{1849}$
„ 21. $\frac{15}{212}$
„ 22. $\frac{2}{15}$
„ 23. $17\frac{113}{218}$
„ 24. 6
„ 25. $\frac{280}{883}$
„ 26. $11\frac{120}{187}$
„ 27. $1\frac{10}{33}$
„ 28. $\frac{8}{9}$
„ 29. $2\frac{9}{34}$; $19\frac{289}{715}$
„ 30. 1

Section VII.—Reduction of Fractions. Pages 87, 88.

Ex. 1. $1\frac{1}{12}$
„ 2. $\frac{21}{100}$
„ 3. 8*d.*
„ 4. $\frac{1}{3}$
„ 5. 19*s.* 4*d.*
„ 6. £1 9*s.* $7\frac{1}{3}$*d.*
„ 7. $\frac{2}{7}$
„ 8. $\frac{1}{2}$
„ 9. The first by 10*d.*
„ 10. 3 tons 2 cwts. 12 lbs.
„ 11. $151\frac{1}{3}$
Ex. 12. 17 pence
„ 13. $\frac{6}{13}$
„ 14. The first by $6\frac{6}{35}$ minutes
„ 15. $\frac{3}{100}$ ton
„ 16. 4*s.* 6*d.*
„ 17. $\frac{1}{42}$ wk.
„ 18. $2\frac{2}{5}$; $\frac{27}{40}$
„ 19. 5*s.* 3*d.*
„ 20. $\frac{4}{7}$ cwt.
„ 21. $1\frac{3}{21}$ rood
„ 22. 1 rd. $15\frac{33}{121}$ po.

Ex. 23. $\frac{1}{7}$
„ 24. $\frac{29}{2880}$
„ 25. $\frac{4527}{6048}$; $\frac{19}{47}$
„ 26. $\frac{7}{15}$ hour

Ex. 27. $2\frac{224}{225}$
„ 28. $\frac{1025}{1792}$
„ 29. $\frac{716}{4576}$
„ 30. $\frac{303}{102400}$ sq. ml.

Miscellaneous Exercises following Chapter VII.
Pages 96, 97, 98, 99, 100.

Ex. 1. £5 12*s.* 6*d.*; £9 7*s.* 6*d.*; £13 2*s.* 6*d.*
„ 2. $111\frac{1}{3}$ lbs.
„ 3. $2\frac{1}{7}$ hrs.
„ 4. 7 months
„ 5. £945
„ 6. 15*s.* $3\frac{3}{4}$*d.*
„ 7. $6\frac{284}{205}$
„ 8. $28\frac{1}{2}$ hrs.
„ 9. 26,000 must be sent away
„ 10. 134505
„ 11. $20\frac{4}{7}$ ft.
„ 12. 11*s.* 6*d.*
„ 13. $34\frac{3}{8}$ cu. inches
„ 14. $15\frac{3}{4}$ ft.
„ 15. $\frac{522}{2640}$ fur.
„ 16. 15 days
„ 17. £66
„ 18. $13\frac{1}{2}$ weeks
„ 19. £9 12*s.* $7\frac{1}{2}$*d.*
„ 20. 11*s.* 4*d.*
„ 21. 350 men
„ 22. £13 $0\frac{30}{61}$*d.*
„ 23. $2\frac{1}{2}$ hrs.
„ 24. £4 1*s.* $4\frac{1}{2}$*d.*
„ 25. $\frac{23}{48}$ and $\frac{8}{9}$
„ 26. £6 11*s.* $7\frac{1}{6}$*d.*
„ 27. $22\frac{1}{2}$
„ 28. $\frac{2}{21}$
„ 29. $27\frac{3}{5}$ pecks
„ 30. 1 cwt. 1 qr. 1 lb. 1 oz.
„ 31. $\frac{81}{253}$
„ 32. £21351 6*s.* $11\frac{1}{4}$*d.*
„ 33. $\frac{3}{5}$
„ 34. 92 miles
„ 35. 1*s.* 3*d.*
„ 36. 4 min. $53\frac{1}{3}$ sec.
„ 37. $12\frac{2}{3}$ days
„ 38. 2000 tiles
„ 39. $\frac{3}{4}$*d.*

Ex. 40. £211 9*s.* 8*d.*
„ 41. 6600 men
„ 42. $31\frac{1}{3}$ days
„ 43. $\frac{260}{2149}$
„ 44. $\frac{10}{27}$
„ 45. $\frac{5}{7}$
„ 46. $\frac{23}{112350}$
„ 47. £586 2*s.* $\frac{24}{59}$*d.*
„ 48. 160 men
„ 49. £720
„ 50. $49\frac{1}{3}$ yds. £9 4*s.* $4\frac{1}{2}$*d.*
„ 51. $4\frac{3}{27}$
„ 52. 2*s.* 1*d.*
„ 53. $\frac{7}{10}$
„ 54. $112\frac{1}{2}$ cubic ft.
„ 55. $1\frac{3}{7}$
„ 56. $\frac{271}{348}$
„ 57. $1\frac{1}{115}$
„ 58. £1 4*s.* 6*d.*
„ 59. 19 yds.
„ 60. £812 10*s.* $7\frac{1}{2}$*d.*
„ 61. $\frac{115883}{124650}$
„ 62. £1 4*s.* $4\frac{1}{2}$*d.*
„ 63. 1000 men
„ 64. 164 yds. $2\frac{14}{15}$ ft.
„ 65. 924 yds.
„ 66. £220 13*s.* 4*d.*
„ 67. $76\frac{1}{2}$ boards
„ 68. $\frac{11}{19}$
„ 69. 12 minutes past 5
„ 70. 6 days
„ 71. £1500
„ 72. $16\frac{3}{8}$ miles from A., $23\frac{1}{3}$ from B, 40 min. $49\frac{1}{3}$ days.
„ 73. $49\frac{3}{5}$ days
„ 74. $9\frac{3}{8}$ min. past 12
„ 75. 46 min. $27\frac{1}{3}$ sec. past 9
„ 76. $49\frac{1}{11}$ min. past 10
„ 77 30 min. $10\frac{10}{11}$ sec. past 4

CHAPTER VIII.

DECIMAL FRACTIONS.

SECTION I.—NOTATION. Page 103.

x. 1. ·8; ·07; ·004; ·0006
, 2. $\frac{3}{10}$; $\frac{2}{100}$; $\frac{6}{1000}$; $\frac{4}{10000}$
„ 3. $\frac{7}{10}$; $\frac{7}{100}$; $\frac{7}{1000}$; $\frac{7}{100}$; $\frac{7}{10000}$; $\frac{7}{1000}$; $\frac{7}{10000000}$
„ 4. ·05; ·005; ·3; ·0009
„ 5. $\frac{6}{10}+\frac{8}{100}+\frac{1}{1000}=\frac{681}{1000}$; $\frac{37}{100}$; $2\frac{47}{100}$; $3\frac{1}{200}$
, 6. $37\frac{1}{2}$; $19\frac{13}{125}$; $\frac{19}{250000}$
, 7. ·03; ·0097; ·00257
, 8. ·3029; 41·09; 321·04; 29·1064; 37·39; 2·00309
, 9. $41\frac{21}{50}$; $3\frac{47}{1000}$; $42\frac{1}{10}$; $800\frac{1}{125}$; $111\frac{111}{1000}$

Ex. 10. 7·7
„ 11. 32·047; ·032047
„ 12. $\frac{7}{200}$
„ 13. 32764; 3276400
„ 14. ·03047200641; 3·047200641
„ 15. 320000; 6042; 90642·7; 410627·54
„ 16. ·00004; ·0000072; ·00000642694; 265·2741
„ 17. 20·4
„ 18. 1; ·0000000001

SECTION II.—ADDITION AND SUBTRACTION OF DECIMALS. Pages 105, 106.

x. 1. 8798·6388
, 2. 19·9998
, 3. 815·75774
, 4. 3·17574
, 5. ·111111
, 6. ·69259
, 7. 164·3239
, 8. ·934
, 9. 9895·2947
10. ·081
11. ·7

Ex. 12. One and nine million and nineteen ten-millionths
„ 13. 1*s.* 4*d.* and 1*s.* 2*d.*
„ 14. A, 11*s.* 6*d.*; B, 17*s.*; C, £1 2*s.* 6*d.*
„ 15. 2 cwts. 1 qr. 12·405 lbs.
„ 16. 2 ac. 2 ro. 28·86 sq. pls.
„ 17. 201·21737
„ 18. 3·683
„ 19. 33·99684
„ 20. ·091

SECTION III.—MULTIPLICATION AND DIVISION OF DECIMALS. Pages 108, 109, 110.

c. 1. (1) ·00370968
„ (2) 2·70459
„ (3) ·00252294
„ (4) ·000000998001
„ (5) 1883·927988
„ (6) 74073·925926
„ (7) 259·26020442
„ (8) 234792·89901
„ (9) 9828·734083345
„ (10) ·174096
„ (11) 335·54432
„ (12) 1·771561

Ex. 2. (1) 5·4
„ (2) ·55
„ (3) 6·01
„ (4) 19700
„ (5) ·000000000999
„ (6) ·000695
„ (7) 4·7
„ (8) 2290·0390625
„ (9) 7436·09963
„ (10) 11·54
„ (11) 237
„ (12) ·002054

Ex. 3. ·000000099̇
„ 4. 5·9048....
„ 5. 435·5
„ 6. ·001
„ 7. ·8
„ 8. 1368·8631
„ 9. ·66067....
„ 10. Sum, ·202404; difference, ·101404; product, ·007671152; quotient, 3·008
„ 11. ·001
„ 12. ·036742
„ 13. 4·9198....
„ 14. ·0016̇

Ex. 15. 8·91
„ 16. 203·178 and 101·589
„ 17. ·000099
„ 18. 1005·6979
„ 19. 3·17083̇
„ 20. ·223
„ 21. 98·99
„ 22. ·76886
„ 23. 21 times; remainder 5
„ 24. 36000
„ 25. 1000 sovereigns
„ 26. 12·25
„ 27. 1·225; 1·3; 1·725
„ 28. $2\frac{74}{303}$

Section IV.—To change Decimal Fractions to Vulgar Fractions, and *vice versâ*. Pages 112, 113.

Ex. 1. (1) $\frac{21}{25}$
„ (2) $5\frac{1}{32}$
„ (3) $\frac{1}{400}$
„ (4) $16\frac{2507}{5000}$
„ (5) $222\frac{2}{25}$
„ (6) $1\frac{3}{500}$
„ (7) $15\frac{37}{125}$
„ (8) $171\frac{129}{1000}$
„ (9) $\frac{1}{100000}$
„ (10) $\frac{1}{100000}$
„ (11) $\frac{803}{2000}$
„ (12) $20\frac{1}{5000}$

Ex. 2. (1) 11·1875
„ (2) 4·59375
„ (3) 8·24
„ (4) 1763·7
„ (5) ·1328125
„ (6) ·0389376
„ (7) 1·53125
„ (8) 9·171875
„ (9) 13·2

Ex. 2. (10) ·25
„ (11) 19·11875
„ (12) 17·6640625
„ 3. $\frac{1}{10000}$
„ 4. $\frac{1}{1000}$
„ 5. $\frac{4}{7}$
„ 6. 4; 6·6̇
„ 7. ·00546875; 11400
„ 8. ·433125
„ 9. 7·5125
„ 10. $4\frac{1863}{10000}$
„ 11. $\frac{25}{202}$
„ 12. ·425
„ 13. 1300
„ 14. $\frac{137}{440}$
„ 15. 10001; 100·01; 9·999
·1
„ 16. ·005859375
·00005859375
·000005859375

Section V.—Indeterminate Decimals. Pages 118, 119.

Ex. 1. ·3̇; ·428571̇; ·4̇5̇; ·6̇15384̇
„ 2. $3\frac{1}{3}$; $2\frac{17}{99}$; $51\frac{13}{90}$; $\frac{247}{660}$
„ 3. 1180·4596
„ 4. $\frac{1}{90}$ and ·01̇

Ex. 5. $1\frac{25}{144}$
„ 6. 563·18604
„ 7. $\frac{7}{9000}$
„ 8. $1\frac{124}{333}$

Ex. 9. $99\frac{7801}{10800}$
„ 10. $3\frac{2938}{3700}$
„ 11. $1\frac{42929}{874791}$
„ 12. Finite $\frac{6}{2^5}$; $\frac{19}{2^7}$; $\frac{675}{2^5}$; $\frac{512}{2}$
Indet. $\frac{5}{3\times 2^2}$; $\frac{3}{7\times 11}$; $\frac{17}{3\times 2^3}$
„ 13. $2\frac{871}{1250}$
„ 14. $\frac{104237}{199998}$
„ 15. $57\frac{1}{33}$
„ 16. $22\frac{11}{38}$
„ 17. $2\frac{7958}{11000}$ or 2·690$\dot{7}\dot{2}$
„ 18. 116·15139
„ 19. $\frac{1627}{8130}$
„ 20. ·25743
„ 21. 22·3$\dot{9}\dot{3}$
„ 22. ·30$\dot{0}\dot{2}$; ·0$\dot{0}\dot{2}\dot{4}$

Ex. 23. 65
„ 24. ·1876
„ 25. ·42$\dot{0}\dot{1}$
„ 26. ·9$\dot{1}\dot{8}$
„ 27. ·04329
„ 28. ·82$\dot{6}\dot{2}\dot{1}\dot{3}$; ·37$\dot{6}\dot{3}\dot{0}$
„ 29. 1·$\dot{0}\dot{0}\dot{1}$ or $1\frac{1}{999}$
„ 30. ·38 of the estate
„ 31. 2325
„ 32. $\frac{889}{900}$
„ 33. 1·07142857
„ 34. ·$\dot{6}$
„ 35. 600 men; 250 women; 1150 children
„ 36. ·78125
„ 37. 25
„ 38. ·34$\dot{5}$; ·3$\dot{4}\dot{5}$; ·$\dot{3}\dot{4}\dot{5}$; ·345
„ 39. ·015 or $\frac{3}{200}$

Section VI.—Reduction of Decimals. Pages 123, 124, 125.

Ex. 1. ·072916$\dot{6}$
„ 2. ·597$\dot{2}$
„ 3. 3·3*d.*
„ 4. 2·2197916$\dot{6}$
„ 5. 133·584 ft.
„ 6. 8*s.* 7.9*d.*
„ 7. ·$\dot{5}$
„ 8. £4 15*s.* $9\frac{3}{4}$*d.*
„ 9. 63 lbs.
„ 10. 3 lbs. 5 ozs. 10 dwt. 18·86 grs.
„ 11. 4919·86 sq. yds.
„ 12. £2 0*s.* 1*d.*
„ 13. ·0046875.
„ 14. 423·84*d.*
„ 15. £4 6*s.* 5·946*d.*
„ 16. 2.325
„ 17. ·1
„ 18. ·0864 seconds
„ 19. £30 18*s.* $8\frac{1}{4}$*d.*
„ 20. ·00975
„ 21. ·904576 nearly
„ 22. ·4375
„ 23. £36793 19*s.* 2*d.*
„ 24. *£6187 10s. 0d.*

Ex. 25. £1 15*s.* 0·03*d.*
„ 26. ·2685 . . . .
„ 27. ·0$\dot{5}$
„ 28. 7·8385583$\dot{3}$
„ 29. 3 cwts. 2 qrs.
„ 30. £170
„ 31. ·022418560 ml. ·17934$\dot{8}$ fur. 39·456$\dot{6}$ yds.
„ 32. ·720238
„ 33. ·00004236 . . .
„ 34. ·00091518 nearly
„ 35. 118·1538 . . .
„ 36. ·000016 . . .
„ 37. 3 cwts. 1 qr. 24 lbs.
„ 38. 33 seconds
„ 39. ·1492 ml.; 7 dwt. 9 grs.
„ 40. ·$\dot{3}$
„ 41. ·504 lb.
„ 42. ·0041875
„ 43. £5 12*s.* 4*d.*
„ 44. £2 2*s.* $9\frac{3}{5}$*d.*
„ 45. £1 0*s.* $10\frac{1}{4}$*d.*
„ 46. 4·2510416$\dot{6}$
„ 47. ·2865 nearly
„ 48. 11*s.* $4\frac{3}{4}$*d.*

Ex. 49. ·89671
„ 50. ·01775
„ 51. £24 19*s.* 5½*d.*
„ 52. 4*s.* 6·6*d.*
„ 53. ·1875
„ 54. ·003719 acres

Ex. 55. ·1979 nearly
„ 56. 1·3288 . . .
„ 57. £·17
„ 58. £4 9*s.* 7*d.*
„ 59. ·1 farthing
„ 60. ·19853

Miscellaneous Exercises following Chapter VIII.
Pages 127, 128, 129, 130.

Ex. 1. $3\frac{1}{2}$, $3\frac{49}{100}$, $3\frac{49}{99}$
„ 2. ·646875
„ 3. 179·34
„ 4. £4·35
„ 5. ·030476
„ 6. 2·89
„ 7. 2·21484375
„ 8. $1\frac{9283}{33300}$, $\frac{29983}{33300}$, $\frac{33083}{99900}$, $2\frac{9283}{11100}$
„ 9. £1 19*s.* 4½*d.*
„ 10. £97 19*s.* $5\frac{3}{5}$*d.*
„ 11. ·008728
„ 12. £3 19*s.* 10½*d.*
„ 13. ·0166375
„ 14. ·5̇
„ 15. $\frac{13}{320}$
„ 16. £39 6*s.* 8·1*d.*
„ 17. ·9734375
„ 18. £85
„ 19. £721 5*s.* $7\frac{1}{5}$*d.*
„ 20. 1411141·1955 nearly
„ 21. 37 yds. 2 ft. 1 inch
„ 22. $15\frac{2}{7}$, ·48
„ 23. 1·142857, ·125
„ 24. 3*s.* 5½*d.*, 4*s.* 5*d.*, 5*s.* $6\frac{3}{4}$*d.*
„ 25. 3 mls. 1 fur. 29 pls. 3 yds. 1 ft. 2·76 inches
„ 26. ·989
„ 27. $\frac{37}{240}$, $\frac{37}{45}$
„ 28. 7 cwt. 3 qrs.
„ 29. £935 1*s.* $3\frac{4}{7}$*d.*
„ 30. 5*s.* 3½*d.*; ·001024
„ 31. 4*s.* 4*d.* nearly (4*s.* $3\frac{23}{100}$*d.*)
„ 32. 80·12 acres
„ 33. £4 1*s.* 8*d.*
„ 34. 6 inches
„ *35.* *8·788*
„ *36.* $109\frac{1}{11}$

Ex. 37. 1 quart
„ 38. ·00648 . . .
„ 39. $5\frac{3}{4}$*d.*
„ 40. £2625
„ 41. £1 5*s.* 8*d.* nearly (7·9776*d.*)
„ 42. ·845
„ 43. $\frac{20947}{230505}$
„ 44. £27
„ 45. 8 yds. $2\frac{3}{7}$ ft.
„ 46. 218·1$\dot{8}$
„ 47. A £1, B £3, C £9, D £13
„ 48. 5·5 days
„ 49. $28\frac{3}{7}$ days
„ 50. 5·1139726
„ 51. 37 yds. $1\frac{16}{27}$ ft.
„ 52. ·82104
„ 53. 3 hrs. 28 min. 30·72 sec.
„ 54. 81 galls.
„ 55. £3 14*s.* $7\frac{35}{473}$*d.*
„ 56. (*a*) £1327 18*s.* 0*d.*, (*b*) 6*d.*
„ 57. 5*s.* 8*d.*
„ 58. $5\frac{5}{11}$ days
„ 59. 12·352083 acres
„ 60. 11·30 A.M.
„ 61. $23\frac{3}{8}$ days
„ 62. 19·36
„ 63. £225
„ 64. G.C.M. 1qr. 8 lbs., L.C.M. 6tons 9cwt. 2qr. 4lbs.
„ 65. $2\frac{16}{19}$ hrs.
„ 66. ·467 (nearly) lb. alcohol ·5323 lb. water
„ 67. 75 days
„ 68. 861 apples
„ 69. 120 crs. 600 sh. 840 sixd.
„ 70. 10*s.* $9\frac{8}{13}$*d.*

CHAPTER IX.

COMMERCIAL RULES.

SECTION I.—PERCENTAGE. Pages 133, 134.

Ex. 1. £20 16*s.* 8*d.* ; £13 15*s.*
„ 2. (*a*) 30 per cent.; (*b*) 30 per cent.
„ 3. £202 8*s.*
„ 4. £250 5*s.* 9$\frac{3}{4}$*d.*
„ 5. £79 18*s.* 7*d.*
„ 6. 20 per cent.
„ 7. 6272
„ 8. 21250
„ 9. £360
„ 10. 260 oats 91 barley 49 wheat
„ 11. 11$\frac{19}{173}$ per cent.
„ 12. 3$\frac{1}{7}$ per cent.
„ 13. 123
„ 14. £799 12*s.* 9·3*d.*
„ 15. 9·6 tons
Ex. 16. 50 per cent.
„ 17. 12 per cent. killed, 26 per cent. wounded, 38 per cent. hors de combat
„ 18. £17 3*s.* 4*d.*
„ 19. 30 per cent.
„ 20. 73·3 per cent. nitre, 15·5̇ per cent. charcoal, 11·1̇ per cent. sulphur
„ 21. 76$\frac{9}{16}$ per cent. coffee, 23$\frac{7}{16}$ per cent. chicory, 8 cwts. 3 qrs. 6 lbs. 2 ozs. coffee, 2 cwts. 2 qrs. 21 lbs. 14 ozs. chicory
„ 22. 80·000

SECTION II.—SIMPLE INTEREST, PRESENT VALUE, AND DISCOUNT. Pages 142, 143, 144.

Ex. 1. £29 12*s.*
„ 2. £60 15*s.*
„ 3. £3540 14*s.* 8*d.*
„ 4. £591 13*s.* 1*d.*
„ 5. £40 16*s.* 9*d.*
„ 6. £32 1*s.* 10*d.*
„ 7. £1792
„ 8. Int. £195 1*s.* 11*d.* ; Amt. £1435 13*s.* 11*d.*
„ 9. £33 9*s.* 7*d.*
„ 10. £616 14*s.* 0*d.*
„ 11. £109 15*s.* 7*d.*
„ 12. £10 18*s.* 2*d.*
„ 13. 5 per cent.
„ 14. 4 per cent.
„ 15. 5·249 years
„ 16. 8 per cent.
„ 17. £1604 5*s.* 0*d.*
„ 18. 6 years
„ 19. 8$\frac{1}{3}$ years
„ 20. 6$\frac{1}{4}$ per cent.
„ 21. £26 12*s.*
Ex. 22. 3$\frac{1}{2}$ per cent.
„ 23. £270
„ 24. £0 2*s.* 4*d.*
„ 25. £82 17*s.* 2*d.*
„ 26. 26·614 per cent.
„ 27. 289·682 per cent.
„ 28. 57$\frac{7}{9}$ per cent.
„ 29. £0 3*s.* 2*d.*
„ 30. 3$\frac{1}{3}$ per cent.
„ 31. 4$\frac{1}{4}$ years
„ 32. £143 15*s.* ; £5 5*s.* 5*d.*
„ 33. £5916 3*s.* 4*d.*
„ 34. £6 5*s.* 5*d.*
„ 35. £375
„ 36. 2$\frac{1}{2}$ per cent.
„ 37. £37 10*s.*
„ 38. £1820
„ 39. 2 years
„ 40. £935 1*s.* 4*d.*
„ 41. £2202 10*s.* 0*d.*
„ 42. 26$\frac{2}{3}$ years
„ 43. £1848

Ex. 44. 5¼ years
„ 45. $2\frac{8}{17}$ per cent.
„ 46. 5 per cent.
„ 47. £7877 10*s.*
„ 48. £171 8*s.* 7*d.*
„ 49. 1st better by £5 5*s.* 10½*d.*
Ex. 50. 19*s.* 6*d.*
„ 51. £1 1*s.* 8*d.* ; £1 1*s.* 3*d.*
„ 52. £49 7*s.* 10*d.* nearly
„ 53. £23645 16*s.* 8*d.*
„ 54. £96 8*s.* 0*d.* nearly

Section III.—Compound Interest. Pages 148, 149.

Ex. 1. £224 19*s.* 6*d.*
„ 2. £130 1*s.* 3*d.*
„ 3. £486 2*s.* 2*d.*
„ 4. £101 15*s.* 5*d.*
„ 5. £787 8*s.* 1*d.*
„ 6. £622 16*s.* 2*d.*
„ 7. £168 14*s.* 7*d.*
„ 8. £18 14*s.* 7*d.*
„ 9. £238 4*s.* 1*d.*
„ 10. £893 8*s.* 4*d.*
„ 11. 6*s.* 1½*d.*
„ 12. £520 5*s.*
„ 13. £217 4*s.* 1*d.*
„ 14. £246 0*s.* 9*d.*
„ 15. 10*s.* difference
„ 16. £7 12*s.* 6*d.*
Ex. 17. £61077 11*s.* 8*d.*
„ 18. £67 18*s.* 11*d.*
„ 19. £817 19*s.* 6*d.*
„ 20. 4*s.* 3*d.* difference
„ 21. £1389 3*s.*
„ 22. £1 7*s.* 8*d.*
„ 23. £1137
„ 24. £15 6*s.*
„ 25. £650
„ 26. £3023 8*s.* 8*d.*
„ 27. 3*d.*
„ 28. £366 3*s.* 3*d.*
„ 29. £1 5*s.*
„ 30. £139 5*s.* 10*d.*
„ 31. £83 6*s.* 8*d.*

Section IV.—Profit and Loss. Pages 152, 153, 154, 155.

Ex. 1. £20
„ 2. £82 10*s.*
„ 3. £12 10*s.*
„ 4. 3¾*d.*
„ 5. 20 per cent.
„ 6. 3*s.* 4*d.*
„ 7. 24⅘ per cent.
„ 8. 17*s.* 2½*d.*
„ 9. 30⅝ per cent.
„ 10. £40
„ 11. £7 8*s.* 9*d.*
„ 12. 33⅓ per cent.
„ 13. 3*s.* 4½*d.*
„ 14. £749 3*s.* 4*d.*
„ 15. 2*s.* 11*d.*
„ 16. £2160
„ 17. 33⅓ per cent.
„ 18. 40*s.* per doz.
„ 19. 5 per cent. gain
„ 20. 13*s.*
„ 21. £888 17*s.* 9*d.*
„ 22. 8*s.* 4*d.*
Ex. 23. 40 per cent.
„ 24. £24
„ 25. £3
„ 26. £120
„ 27. $10\frac{10}{11}$*d.*
„ 28. £993 4*s.* 6*d.*
„ 29. 5 for 3¾*d.*, or 100 for 6*s.*
„ 30. 25 per cent.
„ 31. 12½ per cent.
„ 32. Gained 6¼ per cent.
„ 33. 14⅜ per cent.
„ 34. Loss of £20
„ 35. £102 13*s.* 4*d.*
„ 36. £17 12*s.* 5*d.*, £546
„ 37. £1001 17*s.* 6*d.* absolute loss
„ 38. 1*s.* 3*d.*
„ 39. £2 2*s.* 6½*d.* nearly
„ 40. 3*s.* 3*d.*
„ 41. A £900, B £600
„ 42. 12½ per cent.

Section V.—Stocks. Pages 159, 160, 161, 162.

Ex. 1. £1041 13*s.* 4*d.*
„ 2. £5525, £4725
„ 3. £456
„ 4. £38 0*s.* 10*d.*
„ 5. $3\frac{11}{13}$ per cent.
„ 6. £720
„ 7. £800
„ 8. £94 12*s.* 3*d.*
„ 9. 4 per cents. at 102
„ 10. £100 17*s.* 8*d.* gain
„ 11. £29 7*s.* 2*d.*
„ 12. £3000. Income £97 10*s.*
„ 13. £1295
„ 14. £24 12*s.*
„ 15. £5003 5*s.* $1\frac{1}{4}$*d.*
„ 16. £17052$\frac{12}{19}$
„ 17. Bank Shares
„ 18. £192
„ 19. $4\frac{1}{8}$ per cent.
„ 20. $87\frac{1}{2}$
„ 21. £$1\frac{3}{7}$ loss
„ 22. £39
„ 23. £450450
„ 24. £985. Income £35 10*s.*
„ 25. $3\frac{1}{3}$
„ 26. £$33333\frac{1}{3}$, £31000
„ 27. £17135
Ex. 28. £18625
„ 29. No change
„ 30. £3599 15*s.*
„ 31. £5 loss
„ 32. £201 12*s.*
„ 33. £80
„ 34. Loses £77 10*s.* Gains £5 8*s.* income
„ 35. 11*s.* 8*d.* per cent. absolute loss
„ 36. $5\frac{1}{2}$ per cent.
„ 37. £7350, £251 17*s.* 10*d.* income
„ 38. £$62\frac{1}{2}$
„ 39. £97 5*s.* 11*d.* Gains £118 18*s.* 5*d.*
„ 40. £1660
„ 41. £6 5*s.* gain
„ 42. £31 5*s.* loss
„ 43. £247 9*s.* 5*d.*
„ 44. $83\frac{13}{14}$
„ 45. £$128\frac{19}{18}$
„ 46. £5 3*s.* $10\frac{2}{13}$
„ 47. £5527 10*s.*
„ 48. £5745 16*s.* 8*d.*
„ 49. £10 18*s.* 2*d.* nearly, 9·09

Section VI.—Equation of Payments, &c. Pages 166, 167.

Ex. 1. £98 0*s.* $8\frac{4}{7}$*d.*, £122, £145 19*s.* $3\frac{3}{7}$*d.*
„ 2. A £11 5*s.*, B £20, C £29 5*s.*
„ 3. £1560, £742 17*s.* $1\frac{5}{7}$, £297 2*s.* $10\frac{2}{7}$*d.*
„ 4. 95·25 % copper, 35.75 % tin, 5 % zinc
„ 5. 30 pigs
„ 6. 4 months
„ 7. 1*s.* $6\frac{3}{5}$*d.*
„ 8. 8 months
„ 9. 8838 francs 90 cents.
Ex. 10. By Holland. 14*s.*
„ 11. $5\frac{5}{8}$ months
„ 12. A £3175, B £5715
„ 13. 46 days
„ 14. £53 13*s.* nearly (12*s.* $11\frac{3}{14}\frac{1}{4}$*d.*)
„ 15. 60 per cent.
„ 16. £904 3*s.* 4*d.*
„ 17. Raisins 2*s.* 3*d.*, suet 4*s.*, flour 9*d.*
„ 18. A £1350, B 1200, C £1050

Miscellaneous Exercises on Commercial Rules. Pages 167, 168 169, 170, 171, 172.

Ex. 1. £4 15*s.* 8*d.* nearly
„ 2. £547 7*s.* 4*d.* nearly
„ 3. £350]
„ 4. 25 per cent.
„ 5. £3105 3*s.*
„ 6. $21\frac{83}{277}$ per cent.
„ 7. £192
„ 8. 6*d.*
„ 9. £719 7*s.* 6*d.*
„ 10. £75$\frac{1}{3}$ % spirit; 24$\frac{2}{3}$ % water
„ 11. £82 16*s.*
„ 12. £10180 3*d.*
„ 13. £216 4*s.*
„ 14. £14 12*s.* 3*d.* nearly
„ 15. 21$\frac{1}{3}$
„ 16. 6$\frac{1}{2}$ months nearly
„ 17. £2 10*s.*
„ 18. £851 1*s.* 3*d.*
„ 19. £4 11*s.* 3*d.* nearly
„ 20. 83$\frac{1}{3}$
„ 21. 20 years
„ 22. 12 per cent.
„ 23. £1.
„ 24. £120
„ 25. 28 lbs.
„ 26. 10 pence
„ 27. 2 pence
„ 28. 4$\frac{6}{8}$ years
„ 29. 9$\frac{1}{2}$ per cent.
„ 30. $22\frac{144}{263}$ per cent.
„ 31. £584 6*s.* 8*d.*
„ 32. 3 years
„ 33. £7 5*s.* 8*d.* nearly
„ 34. £1137 10*s.*
„ 35. £600
„ 36. 37$\frac{1}{2}$ per cent.
„ 37. 95
„ 38. 3*s.* 10*d.*
„ 39. 88$\frac{8}{9}$
„ 40. £4251
„ 41. £600
„ 42. £70 19*s.* 7*d.* nearly
„ 43. £5 5*s.* 7*d.* nearly
„ 44. £15000

Ex. 45. 6$\frac{13}{27}$ pence
„ 46. £1230 7*s.* 7*d.*
„ 47. 51$\frac{13}{37}$ per cent.
„ 48. 3$\frac{3}{20}$ per cent.
„ 49. £858
„ 50. A, £81 11*s.* 3*d.*; B £156 7*s.* 6*d.*; C, £174 7*s.* 6*d.*
„ 51. £31 10*s.* 6*d.*
„ 52. £3 1*s.* 8*d.* per cent.
„ 53. 28$\frac{4}{7}$ years
„ 54. £75
„ 55. £350
„ 56. 4*s.* 2*d.*
„ 57. 1*s.* 8*d.*
„ 58. £32 16*s.* 6*d.*
„ 59. £4$\frac{32}{127}$ per cent.
„ 60. £1$\frac{3}{7}$
„ 61. 4*d.* per lb.
„ 62. £150
„ 63. 1$\frac{1}{2}$ years
„ 64. 10 per cent.
„ 65. 26$\frac{2}{3}$ per cent.
„ 66. £17 3*s.* 9*d.* gain
„ 67. 80$\frac{1}{3}$
„ 68. £9750
„ 69. £126
„ 70. 185$\frac{5}{8}$ lbs.
„ 71. 50 shares
„ 72. A, £2550; B, £127 C, £850
„ 73. 7$\frac{59}{123}$ % gain
„ 74. 3$\frac{1}{3}$ per cent.
„ 75. 14*s.*
„ 76. 94$\frac{3}{8}$
„ 77. £3
„ 78. 18$\frac{3}{6}$ %
„ 79. 2 lbs. @ 2*s.* 3*d.*; 13 lb @ 3*s.* 6*d.*
„ 80. 1536$\frac{18}{65}$ yds.
„ 81. 12$\frac{13}{16}$ %
„ 82. $\frac{14}{25}$
„ 83. 1$\frac{24}{27}$
„ 84. £11252 2*s.* 4*d.* nearly
„ 85. £377\2 10*s.*

Ex. 86. £922
„ 87. $18\frac{2}{7}$ % gain
„ 88. 25 %
„ 89. £11 17*s.* 6*d.*; £15 15*s.*; £8 5*s.*
„ 90. £262 10*s.*
„ 91. $87\frac{1}{2}$
„ 92. £102; £104; £78

Ex. 93. £304
„ 94. $181\frac{1}{13}$ per cent.
„ 95. Railway Shares
„ 96. £41 16*s.* 9*d.* nearly
„ 97. £993 12*s.*
„ 98. $6\frac{4}{5}$*s.*, or 6*s.* 2*d.* nearly
„ 99. £833 6*s.* 8*d.*
„ 100. £980 2*s.*

CHAPTER X.

Miscellaneous Exercises.

Pages 172, 173, 174, 175, 176, 177, 178, 179.

Ex. 1. 60
„ 2. ·1212*d.*
„ 3. £39333 15*s.*
„ 4. $23\frac{1}{3}$ days
„ 5. $8\frac{7}{11}$ per cent.
„ 6. £1374 1*s.* 8*d.* nearly
„ 7. $\cdot\dot{1}\dot{3}$
„ 8. £1168 3*s.* 2*d.*
„ 9. £1105 3*s.*
„ 10. ·00001884; 3·67581
„ 11. 650
„ 12. £30 18*s.* $5\frac{1}{4}$*d.*
„ 13. £21 8*s.* $1\frac{1}{2}$*d.*
„ 14. 87
„ 15. 5 months
„ 16. 994·577 and 1000
„ 17. $3\frac{1}{2}$ per cent.
„ 18. 1 hr. $9\frac{2}{11}$ min.
„ 19. $\frac{1}{3}$ of a ton
„ 20. £3 4*s.*
„ 21. 57 : 20, or $\frac{57}{20}$
„ 22. 37·0017 and 1050·61
„ 23. $22\frac{1}{2}$ days
„ 24. Neither gained nor lost
„ 25. $2\frac{1}{2}$ hours
„ 26. $12\frac{1}{2}$ per cent.
„ 27. £200 15*s.* 10*d.*
„ 28. $\frac{14}{15}$
„ 29. £12800
„ 30. 801 acres
„ 31. A, £36; B, £12; C, £16
„ 32. £15

Ex. 33. $£7\frac{7}{9}$
„ 34. $\frac{65}{82}$
„ 35. $116\frac{2}{3}$ per cent.
„ 36. £80
„ 37. 50 days
„ 38. £141 7*s.* 10*d.* nearly
„ 39. 2
„ 40. $4\frac{1}{2}$ years
„ 41. 89 yds. 2 ft. $11\frac{1}{10}$ inches
„ 42. £308 6*s.* 8*d.*
„ 43. A, £64; B, £72
„ 44. 18 women; 22 men; 50 children
„ 45. A, £750; B, £400
„ 46. £50
„ 47. 60 ft.
„ 48. $4\frac{1}{2}$ per cent.
„ 49. $\frac{1}{4}$ day
„ 50. 92106 lbs.
„ 51. 26·6 per cent.
„ 52. $10\frac{1}{4}$*d.* nearly
„ 53. 612 bricks
„ 54. $\frac{97}{104}$
„ 55. $2\frac{1}{4}$ per cent. is gtr.
„ 56. $£^{2}_{45}3\frac{4}{0}1$ and £4·877083
„ 57. 4·684
„ 58. $7\frac{121}{180}$
„ 59. £124 10*s.*
„ 60. Estate £9000; 1st £3375; 2nd £2700
„ 61. £412
„ 62. £2 13*s.* $5\frac{1}{2}$*d.*

Ex. 63. 6*s.* 6*d.*
„ 64. $\frac{423}{2240}$
„ 65. 22
„ 66. 10
„ 67. £3 12*s.*, and 60 per cent.
„ 68. £236 5*s.*
„ 69. 272
„ 70. £5 6*s.* $10\frac{12}{23}$*d.*
„ 71. 3 days
„ 72. £1 19*s.* 6*d.* per doz.
„ 73. £592
„ 74. $6\frac{22}{88}$
„ 75. 12 hrs.
„ 76. $\frac{2}{7}$
„ 77. 25 days.
„ 78. £380 4*s.* 2*d.*
„ 79. 359·55 lbs.
„ 80. ·0099
„ 81. £6 14*s.* nearly
„ 82. 84 days
„ 83. £30 8*s.*
„ 84. £7510 12*s.* 7*d.* nearly; £258 12*s.* 4*d.* nearly
„ 85. A, 60 days; B, 120 days; C, 40 days
„ 86. 90 mins.
„ 87. C. Int. £700; 7 years
„ 88. 42 miles
„ 89. 3*s.* 6*d.*; 2*s.* 11*d.*; 2*s.* 6*d.*; 2*s.* $2\frac{1}{2}$*d.*
„ 90. $1\frac{1}{57}$
„ 91. 4*s.* $6\frac{18}{33}$*d.*
„ 92. £1114 18*s.* $0\frac{3}{4}$*d.*
„ 93. 8 hours
„ 94. $92\frac{4}{13}$
„ 95. 5*s.* per lb.

Ex. 96. £600; 25 %
„ 97. $4\frac{3}{7}$*s.*
„ 98. £3 11*s.* 8*d.* nearly
„ 99. $12\frac{13}{16}$ days
„ 100. 25
„ 101. A, $37\frac{1}{2}$ mls.; B, $26\frac{2}{3}$
„ 102. £1 9*s.* $6\frac{8}{7}$*d.* per gallon
„ 103. £828$\frac{4}{7}$; £787$\frac{1}{7}$; £662$\frac{2}{7}$; £621$\frac{3}{7}$
„ 104. 16*s.* $6\frac{3}{20}$*d.*
„ 105. 3·55 . . . years
„ 106. $179\frac{4}{5}$ yds.
„ 107. 5 per cent.
„ 108. £6 3*s.* $11\frac{1}{2}$*d.*
„ 109. 15552 bricks
„ 110. $\frac{15}{28}$; $1\frac{3}{14}$
„ 111. 8*s.* $11\frac{41}{167}$*d.*
„ 112. £24 12*s.* 11*d.* nearly
„ 113. 0
„ 114. £3
„ 115. $80\frac{25}{64}$ yds.; £17 2*s.* nearly
„ 116. $\frac{8}{57}$
„ 117. $3\frac{648}{925}$
„ 118. 1154
„ 119. 6*d.*
„ 120. 14 %
„ 121. £293 15*s.*
„ 122. ·43̇
„ 123. £40 14*s.* 10*d.*
„ 124. $1\frac{7}{30}$
„ 125. ·022916̇
„ 126. 8 %
„ 127. $115\frac{12}{24}$ acres
„ 128. $1\frac{1}{7}$ hours

CHAPTER XI.

Section I.—Square Root. Pages 182, 183, 184.

Ex. 1. (1) 31
„ (2) 40
„ (3) 65
„ (4) 307
„ (5) 401
„ (6) 527

Ex. 2. (1) 240
„ (2) 648
„ (3) 365
„ (4) 702
„ (5) 7209
„ (6) 7578

Ex. 3. (1) 459
„ (2) 307
„ (3) 6400
„ (4) 50701
„ (5) 62573
„ (6) 13509
Ex. 4. (1) 7·8895
„ (2) 9·134
„ (3) 5·9558 . . .
„ (4) 125·146
„ (5) ·04375
„ (6) 37·2
Ex. 5. ·23; ·0606; ·094
„ 6. 1·4142; 1·732 . .; 2·236. . .
„ 7. 32·25; 7·072
„ 8. 2·8284; 4·1683
„ 9. ·82314

Ex. 10. 1·3038
„ 11. 7; 12·288 . . .
„ 12. 28·3 feet
„ 13. 160 rods
„ 14. 3 yards
„ 15. (1) 77494
„ (2) ·239
„ 16. 437·962
„ 17. 8 miles
„ 18. 914 yds. 1 ft. 7 inches
„ 19. 28·284 feet
„ 20. 8 and 7
„ 21. 5030 and 8460
„ 22. 704
„ 23. 117·1111 feet
„ 24. 3840·67 rods

SECTION II.—CUBE ROOT. Pages 185, 186.

Ex. 1. (1) 24
„ (2) 35
„ (3) 65
„ (4) 57
„ (5) 74
„ (6) 97
Ex. 2. (1) 64
„ (2) 364
„ (3) 135
„ (4) 504
„ (5) 193
„ (6) 1061
Ex. 3. (1) 2·5
„ (2) ·4203
„ (3) 3·837
„ (4) 2·311
„ (5) 3·142

Ex. 3. (6) ·072
„ 4. (1) $2\frac{1}{4}$
„ (2) $\frac{11}{35}$
„ 5. 802
„ 6. 3002
„ 7. 27 cub. inches
„ 8. 34·5 cub. inches
„ 9. (1) 7
„ (2) 5
„ 10. (1) ·464
„ (2) ·585
„ (3) ·669
„ 11. (1) ·167
„ (2) ·046
„ (3) ·002
„ 12. ·667

CHAPTER XII.

MISCELLANEOUS EXAMPLES.

Pages 186, 187, 188, 189, 190, 191, 192, 193, 194.

Ex. 1. 12
„ 2. 7·177083
„ 3. 24, 48, 72, 96
„ 4. $2\frac{1}{2}$ per cent.

Ex. 5. $\frac{4}{13}$, $\frac{107}{260}$
„ 6. £1318 1*s.* 3*d.*
„ 7. $860\frac{20}{63}$ yds.
„ 8. 52

Ex. 9. 6 ozs.
„ 10. $15\frac{103}{170}$
„ 11. $5\frac{5}{11}$ min. to 11 or 10 hrs. $54\frac{6}{11}$ min.
„ 12. $3\frac{1}{2}$ per cent.
„ 13. ·000121
„ 14. A £62 10*s.*, B £104 3*s.* 4*d.*, C £166 13*s.* 4*d.*
„ 15. 120 miles
„ 16. $3\frac{989}{1322}$; $\cdot\dot{2}85714\dot{2}$
„ 17. $6\frac{1}{9}$ ft.
„ 18. $\frac{21}{25}$, $\frac{12}{24}$, $\frac{20}{27}$, $\frac{11}{16}$
„ 19. 729 yds.
„ 20. 4 per cent. gain
„ 21. $\frac{375}{506}$ or ·741
„ 22. $23271\frac{1}{3}$ cub. yds.
„ 23. £2 2*s.*
„ 24. £1281 13*s.* 6*d.* nearly
„ 25. 16,000,000 cub.ft. of water
„ 26. 36 hrs.
„ 27. $28\frac{1}{8}$ ft.
„ 28. £660
„ 29. 1
„ 30. $1\frac{1}{2}$ years hence
„ 31. 5 mls. down, 5 mls. back
„ 32. 5832 bricks
„ 33. 4 ft.
„ 34. A £75, B £40
„ 35. $£131\frac{9}{11}$
„ 36. 75 leaps
„ 37. 900 men
„ 38. £5 5*s.*
„ 39. £529 4*s.* nearly
„ 40. A $12\frac{4}{13}$ days, B $125\frac{5}{7}$ days, C $8\frac{56}{103}$ days
„ 41. Neither gained nor lost
„ 42. 1728 blocks
„ 43. 64
„ 44. 1320 ft. or $\frac{1}{4}$ mile
„ 45. Loss of $\frac{15}{17}$ per cent.
„ 46. A £90, B £30, C £50
„ 47. Sq. root = 10101 ; cube root 7·4
„ 48. $4\frac{4}{9}$*d.*
„ 49. £7462
„ 50. $\frac{232}{2202}$
„ *51.* 4 ft. $7\frac{10}{17}$ inches
„ *52.* B is paid £1 3*s.* 4*d.*

Ex. 53. $\frac{2}{3}$
„ 54. £1 1*s.* $1\frac{1}{4}$*d.*
„ 55. £11 17*s.* 5*d.*
„ 56. 234375 francs
„ 57. A £400, B £200, C £300
„ 58. $£545\frac{5}{11}$ or £545 9*s.* 1*d.* nearly
„ 59. 1
„ 60. £737 10*s.*
„ 61. G.C.M. £77 7*s.* 7*d.*
„ 62. 188 sq. yds. 6 sq. ft. 10 sq. in.
„ 63. $10\frac{10}{19}$ per cent. profit
„ 64. £21 0*s.* 3*d.*
„ 65. £1 3*s.* per yd.
„ 66. $8\frac{482}{995}\frac{17}{40}$
„ 67. 36 poles
„ 68. £31 19*s.* 8*d.* nearly
„ 69. $31\frac{21}{32}$ days
„ 70. 1*s.* $11\frac{11}{20}$*d.*, or 2*s.* nearly per yd.
„ 71. Sq. root 1·01; cube root 9·0101
„ 72. 4*d.*
„ 73. £25
„ 74. 25 florins 5 cents $45\frac{5}{8}$ milrees
„ 75. $90\frac{5}{13}$
„ 76. 4 : 3
„ 77. 10 of spirit to 89 of water
„ 78. 3·344
„ 79. £75
„ 80. $6\frac{6}{13}$ months
„ 81. Loss of £85 11*s.* 4*d.* nearly ; £5268 15*s.* nearly
„ 82. $11\frac{3}{7}$ hours
„ 83. 10*s.* 6*d.* per yd.
„ 84. 27·2 cub. inches
„ 85. $8\frac{3}{4}$ hrs. after B starts
„ 86. $5356\frac{13}{20}$ mls. ; 15·13 mls. per hr.
„ 87. A £1710, B £2280, C £8550
„ 88. £10 7*s.* 6*d.*
„ 89. 75 per cent. metal, 25 per cent. hemp

Ex. 90. £15 to each
„ 91. 320 cubes
„ 92. £9600
„ 93. 3 hrs.
„ 94. 101
„ 95. 50 tons 12 cwt. 3 qrs. 9 lbs. 8 ozs.
„ 96. 210 inches
„ 97. 7
„ 98. 767
„ 99. $£5\frac{2}{77}$
„ 100. £2400
„ 101. £5616
„ 102. 18·3942 sq. ft.
„ 103. 2256·96 dollars
„ 104. A £3750, B £3525, C £3280, D £2000, E £1385 14*s.* $4\frac{1}{2}$*d.*, F £1279 13*s.* 9*d.*, G £1219 14*s.* 6*d.*, H £1276 0*s.* $7\frac{1}{2}$*d.*, K £1078 11*s.* $10\frac{1}{2}$*d.*, L £780 4*s.* $10\frac{1}{2}$*d.*
„ 105. 469·89 galls.
„ 106. B would win by $21\frac{1}{2}$ yds.

Ex. 107. 2 : 15
„ 108. 50 ft.
„ 109. A £770, B £8470, D £2500
„ 110. 1
„ 111. 11 ft. 8 in., or 140 inches (no lid)
„ 112. 77°
„ 113. £$3\frac{1}{5}$, £3 4*s.* 5*d.*, £4 8*s.* 4*d.* (nearly)
„ 114 £18 8*s.* $3\frac{1}{2}$*d.*, £27 13*s.* $9\frac{1}{4}$*d.*
„ 115. 46 inches
„ 116. $\frac{3}{10}$ of a foot
„ 117. £440 ; income = £960
„ 118. £1 1*s.* $1\frac{1}{2}$*d.*
„ 119. 48 ft.
„ 120. £1 13*s.* 4*d.*
„ 121. 11*s.* $4\frac{1}{2}$*d.*, ·11375 of £5
„ 122. 3 ft.
„ 123. 14 January; 35 min. $43\frac{3}{5}$ sec. past 11 P.M.
„ 124. A £162, B £324, C £405, D £486, E £648
„ 125. $\frac{6}{11}$
„ 126. £24 4*s.* 2*d.*

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